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What are Elementary Particles Made of?

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For many years scientists have been searching for the ultimate constituent of matter. At the turn of the century, the atom gave up its claim as the ultimate building block when atomic structure was understood. While we lost the heir apparent to the "fundamental building block" we got consolation prizes in terms of an understanding of the periodic system of elements and their chemical properties as well as understanding of atomic and molecular spectra. It then seemed that the photon, the electron, the proton, and the neutron had pretensions to the "throne." However, there were vaguely disturbing features like the existence of the neutrino and postulation and the discovery of the pion and the unwelcome but persistent muon. One had not fully understood the role of these various particles as elementary constituent of matter. For many practical purposes there was a satisfactory phenomenological quantum theory of their production and interaction.

Long before this state of affairs came to pass, we had changed our picture of the kind of things of which matter could be made. We did not expect any longer to build matter out of elastic billiard balls, coiled springs, or pulsating jellies. Instead, matter particles were described in terms of quantum mechanics. They were described in terms of suitable sets of canonical operator variables which were to be thought of as appropriate linear operators in a suitable vector space.

Another step in this silent revolution in the minds of men was undertaken when one had to describe a dynamical system in which the number of particles was itself a variable, so that creation and destruction of particles could be described. Such was the quatum theory of fields in which the field operators became dynamical variables. That is, fields whose "values" were no longer numerical quantities but were themselves linear operators. There were an infinite number of such operator variables corresponding to the infinite number of possible states of particles. In terms of such a theory, earlier attempts at the quantum theory of emission and absorption of radiation by atoms as well as the phenomena of beta radiativity and the decay of unstable elementary particles were brought under a unified scheme of description. In its implementation of quantum electrodynamics, the relativistic theory of fields scored its greatest triumph, but at the same time revealed some of its fundamental inconsistencies. In less successful situations like its application to particle decays, the theory could at best be thought of as a mode of description.

In the meantime the experimenters were busy discovering a variety of strongly interacting particles; most of them were extremely short-lived. These particles were, in their interaction properties, much more similar to the neutron, the proton, and the pion in that they participated in strong nuclear interactions and are now collectively known as hadrons. One may distinguish two sets of particles among these, those with integral spins called mesons, and those with half-integral spin called baryons. Many of the hadrons are so short-lived that it is more appropriate to describe their lifetime in terms of the width of the strong interaction resonance. Conventional quantum field theory of strong interactions could not make any quantitative predictions about the existence, much less the detailed nature, of the hadrons. In some cases special theories had been worked out within the framework of operator fields which accounted for some of the hadrons quantitatively, but the approximately conserved quantum numbers like isospin and hypercharge found no natural place in the theory. Nor was there anything appropriate to signal the multiplet structure of the hadrons. In a sense, the quantum field theory appeared as a purely phenomenological description of the hadrons. We are therefore left to conclude that elementary particles are not "made of" conventional quantized fields.

Another and more fruitful scheme of description of the hadrons was in terms of group representations. The use of group representations in strong interaction physics is as old as the nucleus and the pion. With the discovery of the strange hadrons, it was natural to extend the notion of charge independence, which was so successful in its application to nucleons and pions. Recent experimental investigation so far has borne out this hypothesis. Any departure from charge independence could be attributed to electromagnetic perturbations. One of the most immediate predictions was that hadrons must occur in multiplets which could be identified with various irreducible unitary representations of the isospin group $SU(2)$.

One does not stop at this point, since to the discerning eye there are other multiplet structures to be observed amongst the hadrons. It is now more or less established that the grouping of particles into octets and decuplets can be established and that these could be thought of as the 8- and 10-dimensional representations of the unitary group $SU(3)$. One can go further, somewhat in the manner of ancient mariners and astronomers, discerning the Great Bear and the Little Bear and so on amongst the stars. The modern mariners have found other groupings of fundamental particles in which they can discern other multiplets; these are now to be associated with spin-dependent groups like $SU(6)$. In some of these cases, the multiplet structure is remarkable. For example, the better established baryons constitute a single 56-dimensional irreducible representation of $SU(6)$ and the better established mesons constitute a single irreducible 35-dimensional representation. This is a great improvement over the very large number of $SU(2)$ multiplets which were involved in the organization of the hadrons. While the multiplet structure is thus striking, there is a problem. If we thought of the group as an invariance group, we would have expected that the particles which constitute a single representation all have equal mass. On the other hand, if the group is not an invariance group, then it is not clear why the particles should constitute the representation of the group. There was a departure from the precise equality of masses for the $SU(2)$ multiplets, but these differences were always blamed on the electromagnetic interaction. We thus have a paradoxical situation: On the one hand, the group seems to be basic in predicting the particle multiplets; on the other hand, the group does not seem to be an invariance group for the system.

We are therefore led to consider groups which are useful in classifying particles, but which are not invariance groups. We may refer to such groups as characteristic noninvariance groups. As an aid to their study, one may consider the internal dynamics of simple quantum mechanical systems and ask for multiplet structure of the levels. In the case of two particles bound by a central potential, we have the familiar angular momentum multiplets. In some special cases like that of Newtonian or harmonic potentials, there are further degeneracies. In addition we observe other regularities. For example, in the system with harmonic potential, the energy levels occur with definite multiplicity which increases with the increase of the energy. They could be thought of as constituting their representation of a noninvariance group $SU(3, 1)$. Here there is no question of the different energy levels being degenerate. In fact, the complete set of energy levels is to constitute a single irreducible representation of the group. This feature is reminiscent of the observation that all the states of a particle constitute a single irreducible representation of the algebra of three pairs of canonical variables. There is no question about this algebra of canonical variables being an invariant algebra of the Hamiltonian of the system. Rather, the interactions are described in terms of the canonical variables. In short, the particles are "made of" the three pairs of canonical variables.

Let us pursue this further. If the noninvariance group has all states of the system constituting a single irreducible representation, then we can identify the system as being made of the enveloping algebra of the group. That means that all dynamical properties of the system are to be identified with entities belonging to the enveloping algebra of the group. In other words, the elementary entity in terms of which the system is made is the noninvariance group. We have then come to an unfamiliar situation; the particles do constitute a multiplet representation of the group but only because the particles are themselves made of the "stuff called (the enveloping algebra of) the group."

Can we pursue this idea further? Can we say that the elementary entities in terms of which elementary particle physics is described must be the enveloping algebra of a suitable noninvariance group? Clearly something more than conventional groups have to be considered here, because we must have as many dynamical variables as were employed in the quantum theory of fields; we must still describe an infinite possible number of states and we must still be able to describe creation and destruction of particles. Therefore we must use group-theoretic "fields." That is, fields whose "values" are to be associated with a suitable group. It is familiar from the analysis of the rotation groups in classical and quantum mechanics that it is preferable to consider the Lie algebra elements, rather than the group elements which are more appropriately identified as dynamical variables. Here, also, we follow the same suggestion and identify the dynamical variables to be Lie algebra elements associated with space–time points. There are, therefore, an infinity of linear operators in a suitable vector space and, in addition, they satisfy postulated Lie algebra relations amongst themselves. This is the general scheme of current algebras in particle physics.

What are elementary particles made of? They are made of fields of Lie algebra. The multiplet structure is a reflection of this Lie algebra structure. The interactions are to be described in terms of these differences. The particles do not constitute irreducible representations of this algebra because, in general, the elements of the algebra connect states with different numbers of particles. We therefore do not have the simpler version of an invariance group. If we postulate that the interactions are to be simply described in terms of the "Lie fields," then instead of deriving simple relations between a finite number of matrix elements, we have relations between infinite number of such matrix elements. That is, we have *sum rules*. Recently, a variety of such sum rules have been derived using the postulate of a simple structure for weak, electrodynamic, and strong interactions in terms of these Lie fields ("currents"). In is too early to say whether this picture of the constitution of elementary particles is here to stay, but it appears to be a very intriguing and promising picture. It has been several decades since we changed our picture of matter as consisting of miniscule billiard balls and adopting a picture of matter being made out of linear operators. It seems the next step to adopt the view that all matter is made out of Lie fields. If it is so, then we will know this in no uncertain terms by the success in piecing together many of the pieces and bits of information about elementary particles that we now possess. We do not expect that this will be an ultimate picture. Perhaps elementary particles would provide us with yet another level of structure, but at the present time we have no indications to the existence of another level. There are still vexing problems which seem to exhibit no connection with this picture of matter, as for example, the existence of the photon and the leptons. Nor is there any understanding of the hierarchy of strength of strong, electromagnetic, and weak interactions. So perhaps at a more appropriate time we could again ask the question, "What are elementary particles made of?"