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An Associative Algebraic Model for Weak Meson Decays (*).

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Summary. — An algebraic model describing symmetry breaking is applied to the weak meson decay. The suppression of the strangeness-changing weak decay is obtained as a consequence of symmetry breaking.

In two previous papers [1,2] we have introduced a finitely generated associative algebra $\mathscr{B}$ as a model for hadrons and obtained a meson mass spectrum in good agreement with the observed masses. The characteristic property of the algebra is that the generators H_i, $E_{\pm\alpha}$, G_i, $F_{\pm\alpha}$ of the internal group $SL_{3,c}$ do not commute with the generators P_μ of the Poincaré group $\mathscr{P}$ but they do commute with the four-velocity operators $M^{-1}P_\mu$. We are interested in extending the model to describe weak meson decays.

For this purpose we consider the possibility of factorizing the interaction in terms of a four-vector operator H_λ (hadronic part), which causes transitions between different components of the $SL_{3,c}$ multiplet and another four-vector L^λ which describes the leptonic part of the interaction but acts as an invariant with respect to the hadronic $SL_{3,c}$. The natural candidate for the operator H_λ is the operator

$$H_\lambda = \sum_{\alpha=\pm1,\pm2,\pm3} \{P_\lambda, E_\alpha + F_\alpha\} . \tag{1}$$

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(1) A. BÖHM: *Phys. Rev.*, **158**, 1408 (1967).
(2) N. MUKUNDA, E. C. G. SUDARSHAN and A. BÖHM: *Phys. Lett.*, **24** B, 301 (1967).

This operator is bilinear in the generators and is an element of the associate algebra $\mathscr{B}$.

This ansatz is suggested by the fact that the generators of $SL_{3,c}$ or $SU_3 \otimes SU_3$, E_α and F_α, correspond to the integrals over the weak hadron currents. For the matrix element of L^λ we require that it describes the properties contained in conventional leptonic current. Then we consider writing an effective transition operator of the form

$$T = GL^\lambda H_\lambda \tag{2}$$

(where G is a constant characterizing the strength of the interaction) to describe the weak decay ([3]).

However the ansatz for the matrix element of L^λ has an unconventional form ([3]) due to the following. It is in the spirit of this approach that H_λ changes the energy-momentum of the states. Therefore—as the operator T is to conserve energy-momentum—this is to be compensated in the leptonic part such that it results in over-all energy-momentum conservation. As (2) is not in the usual form of a space-time integral of a suitable density this compensation requirement leads to the unconventional form for the matrix element of L^λ. To obtain our results in a more familiar way, we replace (2) by

$$T = G\int \mathrm{d}^4x L^\lambda(x) H_\lambda(x)\,, \tag{2'}$$

where $L^\lambda(x)$ and $H_\lambda(x)$ are suitable field operators corresponding to L^λ and H_λ respectively.

We want to consider the (semi)-leptonic decays

$$\beta \to \alpha_1 + \alpha_2 + \ldots + \alpha_N + l + \nu \tag{3}$$

where β and $\alpha_1, \ldots, \alpha_N$ $(N = 0, 1)$ are the various mesons and $l\nu$ is the lepton pair. Consequently we have to calculate

$$\begin{aligned} \mathscr{A} &= \int \mathrm{d}^4x \langle l\nu\alpha_1 \ldots \alpha_N p_l p_\nu p_{\alpha_1} \ldots p_{\alpha N} | L^\lambda(x) H_\lambda(x) | \beta p_\beta \rangle = \\ &= \sum_\gamma \int \frac{\mathrm{d}^3 q_\gamma}{2c_\alpha E_\alpha(q)} \int \mathrm{d}^4x \langle l\nu\alpha_1 \ldots \alpha_N | L^\lambda(x) | \gamma, q_\gamma \rangle \langle \gamma q_\gamma | H_\lambda(x) | \beta p_\beta \rangle\,. \end{aligned} \tag{4}$$

The hadronic matrix element $\langle \gamma | H_\lambda(x) | \beta \rangle$ can be calculated in the frame of

([3]) A. BÖHM and E. C. G. SUDARSHAN: *Phys. Rev.* (in press).

our algebraic model. If we define the local operator $H_\lambda(x)$ by

$$H_\lambda(x) = \exp[-iP_\mu x^\mu] H_\lambda \exp[iP_\mu x^\mu] \tag{5}$$

we obtain [3]

$$\left\{\begin{aligned} &\langle \gamma q_\gamma | H_\lambda(x) | \beta p_\beta \rangle = \exp[i(p-q)x](p_\lambda + q_\lambda) C_{\gamma\beta} \langle \gamma, q_\gamma | \frac{m_\gamma}{m_\beta} p\beta \rangle = \\ &\qquad = \exp[i(p-q)x](p_\lambda + q_\lambda) C_{\gamma\beta} 2c_\gamma E_\gamma(q) \delta^3\left(q - \frac{m_\gamma}{m_\beta} p\right); \\ &E_\alpha(q) = \sqrt{m_\alpha^2 + q^2}. \end{aligned}\right. \tag{6}$$

The quantities $C_{\gamma\beta}$ are $SL_{3,c}$ (or $SU_3 \times SU_3$) Clebsh-Gordan coefficients, depending upon the position of γ and β in the $SL_{3,c}$ (or $SU_3 \times SU_3$) multiplet (Appendix A of ref. [3]); the quantities C_γ are normalization constants due to our normalization of the hadron states with respect to the « invariant » measure of $\mathscr{B}$ (Appendix B of ref. [3]).

For the leptonic matrix element we make—in accordance with the usual assumptions about the leptonic current—the following ansatz

$$\begin{aligned} \langle \bar{\nu} l \alpha_1 \dots \alpha_N p_\nu p_\ell \dots p_{\alpha_N} | L^\lambda(x) | \gamma q_\gamma \rangle &= \frac{\sqrt{C_\gamma}}{2m_\gamma} \frac{\sqrt{C_{\alpha_1} \dots C_{\alpha_N}}}{2m_{\alpha_1} \dots 2m_{\alpha_N}} \cdot \\ &\cdot \frac{1}{(2\pi)^4} \exp[-i(p_\nu + p_\ell + p_{\alpha_1} + \dots + p_{\alpha_N} - q_\gamma)^\alpha] \bar{u}^\ell(p_\ell) \gamma^\lambda (1 + \gamma_5) u^\nu(-p_\nu). \end{aligned} \tag{7}$$

Inserting (6) and (7) into (4) we obtain

$$\begin{aligned} \mathscr{A} = \sum_\gamma G C_{\gamma\beta} p_\gamma^{(\beta)} \left(1 + \frac{m_\gamma}{m_\beta}\right) \frac{\sqrt{C_\beta}}{2m_\beta} \frac{\sqrt{C_{\alpha_1}}}{2m_{\alpha_1}} \dots \frac{\sqrt{C_{\alpha_N}}}{2m_{\alpha_N}} \cdot \\ \cdot \delta^4(p_\beta - p_\nu - p_\ell - p_{\alpha_1} - \dots - p_{\alpha_N}) \bar{u}^\ell(p_\ell) \gamma^\lambda (1 + \gamma_5) u^\nu(-p^\nu). \end{aligned} \tag{8}$$

Here $\sum_\gamma$ means the sum over all $SL_{3,c}$ states γ, which have the same internal $SL_{3,c}$ (or $SU_3 \times SU_3$) quantum numbers as $\alpha_1 \dots \alpha_N$. *E.g.* for the special case of (3): $K^+ \to \pi^0 + \ell + \bar{\nu}$, γ can be only π^0; for $K^+ \to \ell + \nu$, γ can be only σ, the state with the quantum numbers of the vacuum $I = 0, I_3 = 0, Y = 0$ $(y, = Y) = 0$ (for which we assume $m_\sigma \approx 0$); for $K^+ \to \pi^+ \pi^- e^+ \nu$, γ could be a resonance of the 27-plet with the quantum number $Y = 0$, $I_3 = 0$, $I = 0, 1, 2$.

Let us now calculate the transition rates for the leptonic decays of the kaon and the pion. Consider the two-body decay $K^- \to \mu^- \bar{\nu}$. Then, from (8), we get

$$\langle \bar{\nu} \mu p_\nu p_\mu | L^\lambda \left| \frac{m_\sigma}{m_K} p_K \sigma \right\rangle = \frac{\sqrt{C_\sigma}}{m_\sigma} \delta^3(p_\nu + p_\mu - p_K) \bar{u}^\mu(p_\mu) \gamma^\lambda (1 + \gamma_5) u^{(\nu)}(-p_\nu).$$

This together with (6) implies

$$\langle \bar{\nu}\mu p_\nu p_\mu | T | p_K K\rangle = G C_\sigma p_\lambda^{(K)} \frac{1}{2m_K}\left(1+\frac{m_\sigma}{m_K}\right)\sqrt{C_K}\cdot\delta(p_\nu+p_\mu-p_K)\cdot$$
$$\cdot\,\bar{u}^{(\mu)}(p_\mu)\gamma_\lambda(1+\gamma_5)u^{(\nu)}(-p_\nu)\,.$$

Making use of the relation

$$\sum_{\text{pol}} |\bar{u}^{(\mu)}(p_\mu)\gamma^\lambda(1+\gamma_5)(p_\lambda^{(\nu)}+p_\lambda^{(\mu)})\,u^{(\nu)}(-p_\nu)|^2 = 8m_\mu^2\, p_\lambda^{(\mu)}\cdot p^{(\nu)\lambda}$$

the time rate of change of the kaon wave function is

$$\left\{G C_{\sigma 2}\frac{1}{2m_K}\left(1+\frac{m_\sigma}{m_K}\right)\right\}^2 \frac{8m_\mu^2}{2E_K(q)}\iint \frac{d^3p_\mu}{2E_\mu}\frac{d^3p_\nu}{2E_\nu}\delta^4(q-p^{(\mu)}-p^{(\nu)})p^{(\mu)}\cdot p^{(\nu)}\,.$$

The partial-decay width is then deduced to be

$$\Gamma(K\to\mu\nu) = \frac{\pi}{4}\left(\frac{G}{m_K}\right)^2 C_{\sigma K}^2\left(1+\frac{m_\sigma}{m_K}\right)^2 m_K m_\mu^2\left(1-\frac{m_\mu^2}{m_K^2}\right)^2.$$

In a similar manner we can compute the width for the corresponding decay of the pion

$$\Gamma(\pi\to\mu\nu) = \frac{\pi}{4}\left(\frac{G}{m_\pi}\right)^2 C_{\sigma\pi}^2\left(1+\frac{m_\sigma}{m_\pi}\right)^2 m_\pi m_\mu^2\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2.$$

The C.G. coefficient $C_{\sigma K}$ replaces $C_{\sigma\pi}$ since the transition is caused by the strangeness conserving operator F_1 rather than by the strangeness changing operator F_2. The $SL_{3,c}$ C.G. coefficients $C_{\sigma K}$, $C_{\sigma\pi}$ are

$$C_{\sigma K} = C_{\sigma\pi} = \frac{1}{6\sqrt{2}}\sqrt{q-b^2}\,,$$

where b is a parameter characterizing the $SL_{3,c}$ representation chosen.

From (9) and (10) we obtain the suppression of the strangeness-changing decay

$$S_{\ell_2}^2 \equiv \frac{\Gamma(K\to\mu\nu)}{\Gamma(\pi\to\mu\nu)} = \frac{m_\pi}{m_K}\frac{(1-m_\mu^2/m_K^2)^2}{(1-m_\mu^2/m_\pi^2)^2}\,.$$

If we express this suppression by an (axial vector) Cabibbo factor [4,5] we get

$$\operatorname{tg}\theta_A = S_{\ell_3} = \frac{m_\pi}{m_K}\frac{(1+m_\sigma/m_K)}{(1+m_\sigma/m_\pi)} = 0.28\,, \tag{11}$$

which is to be compared with the value 0.27 deduced experimentally [5,6].

Let us now calculate the three-body leptonic decay rates. For the process $K^+ \to \pi^0 e^+ \nu$ we get

$$\begin{aligned}\langle e^+\nu\pi^0 p_e p_\nu p_\mu|T|p_K K\rangle &= \int \frac{d^3q}{2c_\pi E_\pi(q)}\langle e\nu\pi p_e p_\nu p_\pi|L^\lambda|q\pi\rangle\langle\pi q|H_\lambda|p_K K\rangle = \\ &= GCp_\lambda^{(K)}\left(1+\frac{m_\pi}{m_K}\right)\langle e\nu\pi p_e p_\nu p_\pi|L^\lambda\left|\frac{m_\pi}{m_K}p_K,\pi\right\rangle = \\ &= GCp_\lambda^{(K)}\left(1+\frac{m_\pi}{m_K}\right)\frac{\sqrt{C_\pi}}{2m_\pi}\frac{\sqrt{C_K}}{2m_K}\delta^3(p_\nu+p_\mu+p_\pi-p_K)\,\overline{u}^{(\nu)}(p_\nu)\gamma^\lambda(1+\gamma_5)u^{(\ell)}(-p_e)\,,\end{aligned} \tag{12}$$

where C is a suitable linear combination of C.G. coefficients of $SL_{3,c}$. It is conventional to write the invariant matrix element for this process in the form [5]

$$\frac{G_K}{\sqrt{2}}\{(f_+ + f_-)p_\lambda^{(K)} + (f_+ - f_-)p_\lambda^{(\pi)}\}\,\overline{u}\gamma^\lambda(1+\gamma_5)u^{(e)}\,,$$

where f_+ and f_- are suitable functions of the invariant momentum transfer to the leptons. Comparing with our result (12), we see that our model predicts a constant form factor with both f_+ and f_- equal to each other. Our constant form factor is to be compared with the experimental determination [7,8]

$$\left|\begin{array}{l} f_+(q^2) + f_+(0)\left(1+\lambda\dfrac{q^2}{m_\pi^2}\right), \\ \lambda = 0.02 \pm 0.01\,. \end{array}\right. \tag{13}$$

For $\xi = f_-/f_+$ we predict the value unity which is not inconsistent with the experimental determination. [7]

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(4) N. Cabibbo: *Phys. Rev. Lett.*, **10**, 531 (1963).

(5) R. H. Dalitz: *Varenna Lectures* (1964); N. Cabibbo: *Rapporteur's talk, Proceedings of the XIII International Conference on High Energy Physics* (Berkeley, 1966), p. 29.

(6) N. Brene, M. Roos and A. Sirlin: CERN preprint TH. 872 (1968).

(7) J. W. Cronin: *Proceedings of the International Conference on Particles and Fields, Rochester, 1967*; R. W. Garland, K. Tsipis, S. Devons J. Rosen, D. Ty: *Phys. Rev.*, **167**, 1225 (1968).

(8) W. J. Willis: *Proceedings of the Heidelberg International Conference of Elementary Particles* (1967), p. 271.

We can also calculate the matrix element for the decay $\pi^+ \to \pi^0 e\nu$ by the same method. The quantity of interest is the suppression factor

$$S_{\ell_3} = \frac{m_\pi}{m_K} \frac{(1 + m_\pi/m_K)}{(1 + m_{\pi^0}/m_\pi)} \simeq 0.18 \,, \tag{14}$$

which is to be compared with the experimental value [5,6] 0.21.

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RIASSUNTO (*)

Si applica al decadimento debole del mesone un modello algebrico che descrive la rottura della simmetria. La soppressione del decadimento debole che cambia la stranezza è conseguenza della rottura della simmetria.

(*) *Traduzione a cura della Redazione.*

Ассоциативная алгебраическая модель для распадов слабых мезонов.

Резюме (*). — Алгебраическая модель, описывающая нарушение симметрии, применяется к распаду слабых мезонов, как следствие получается нарушение симметрии.

(*) *Переведено редакцией.*