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Causality and Space-like Signals

ALTHOUGH we pointed out[1] in 1962 that space-like particles (signals) travelling "backward in time" carry negative energy—or must be reinterpreted—recent textbooks on the special theory of relativity (for example, refs. 2–7) still purport to show that causality arguments forbid the existence of faster-than-light particles. One book which is at least partially correct on this score is Ya. P. Terletskii's *Paradoxes in the Theory of Relativity*[8], but even this ignores the possibility of reinterpretation.

Here we present explicitly a proof that the sign reversal of the time interval between two events (for example, emission and absorption of a particle, or signal) is necessarily accompanied by sign reversal of the corresponding energy.

Consider two reference frames S and S′, with S′ moving at a velocity w in the x direction relative to S. Assume that at a time $t_0 = 0$, the reference axes of the two systems coincide and that the clocks are so synchronized that at $t_0 = 0$ the clocks in S′ indicate $t'_0 = 0$. Now consider the following hypothetical events. At $t_0 = t'_0 = 0$, a faster-than-light meta-particle (or tachyon, as it is now known[9]) is emitted in the x direction from the common origin of S and S′. Let the superluminal velocity of this meta-particle (or meta-signal) be $u > c$ relative to S, hence $v = (u - w)/[1 - uw/c^2] > c$ relative to S′. Assume now that at a time t_1, when it is at x_1 in the S frame, the meta-particle is absorbed. The corresponding time of absorption, as measured in S′, is given by

$$t'_1 = \gamma_w(t_1 - wx_1/c^2) \qquad (1)$$

where $\gamma_w = [1 - (w/c^2)]^{-1/2}$.

Because $x_1/t_1 = u$, the Lorentz transformation given here appears to be no longer orthochronous (preserving time direction) for a relative velocity $w > c^2/u$ (or for a tachyon velocity of $u > c^2/w$—we call such tachyon velocities ultraluminal). In these circumstances, the time interval $\Delta t = t'_1 - t'_0$ becomes negative. This is generally taken to imply that ultraluminal particles (or signals) must needs violate the causality principle. This argument was first advanced by Tolman[10] in 1917 and reproduced in various forms ever since.

To produce a measurable effect, any particle (or signal) must carry real positive energy. We assume that the tachyon in our example given here carries energy E as measured in frame S. Relative to S′, this energy is given by

$$E' = \gamma_w (E - wp_x) \qquad (2)$$

which may also be written as

$$E' = \gamma_w m_0 \gamma_u (c^2 - uw) \tag{3}$$

because $E = m_0\gamma_u c^2$ and $p_x = m_0\gamma_u u$, where $\gamma_u = [1 - u/c^2]^{-1/2}$. (For superluminal and ultraluminal particles γ^{-1} becomes imaginary, hence their proper mass m_0 must also be imaginary in order that E and p_x may remain real[1].) Equation (3) clearly shows that E' turns negative when the relative velocity w becomes greater than c^2/u (or when u becomes greater than c^2/w). The switching of signs occurs at $w = c^2/u$, which is exactly the value for which $\Delta t'$ changes sign. Hence any particle (or signal) propagating "backwards in time" in S′ carries negative energy in S′, which means that, for the observer in S′, it is a positive energy particle (or signal) propagating forward in time. We refer to this reasoning as "the reinterpretation principle". The observers in S and S′ disagree as to the direction of propagation of the meta-particle (meta-signal), but such disagreements are nothing new in relativity. Only lengths of the four vectors (x, y, z, ict) and $(p_x, p_y, p_z, iE/c)$ must remain invariant under Lorentz transformations, not their individual components.

As has been pointed out by Feinberg (ref. 9, see also ref. 11), our reinterpretation also provides the key to the understanding of negative energies of space-like particles (or signals), so that it is the crux of any consistent theory of faster-than-light particles[12]. In a sense our reinterpretation is "antiparallel" to the Stückelberg–Feynman view of positrons as negative-energy electrons travelling backwards in time[13].

All the presentations of causality paradoxes (including Terletskii's[8]) which we have so far come across fail to take this reinterpretation into account. As Hurst[14] pointed out, our reinterpretation invalidates this type of objection to the possibility of existence of faster-than-light particles.

Note added in proof. Since writing this article, several reports have appeared in which causality paradoxes involving faster-than-light particles are examined taking our reinterpretation principle into account. Some of the authors, for example W. B. Rolnick, seem to believe that causal loops would still remain and hence superluminal particles could not interact with ordinary matter. Others, for example B. S. DeWitt, seem to maintain that tachyons could not carry messages. Some other authors, however, for example R. G. Root and J. S. Trefil, point out that causal loops vanish if emission of tachyons from distant matter is considered. Furthermore, P. L. Csonka has discussed the necessity of carefully spelling out the appropriate boundary conditions. Inasmuch as the work of these authors shows that our reinterpretation (or switching) principle must be used in every careful examination of space-like signals, it corroborates the key contention of our communication.

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