ARKIV FÖR FYSIK Band 39 nr 40

3.68 14 3 Communicated 13 November 1968 by L. HULTHÉN and N. SVARTHOLM

The nature of faster-than-light particles and their interactions

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ABSTRACT

Faster-than-light particles, tachyons, proposed previously by the author, are described in a classical as well as a quantum theoretical picture using a Klein–Gordon equation with imaginary mass. For an invariant quantization of the associated field only annihilation operators are used. A particular case of tachyon-nucleon coupling with Yukawa interaction is considered. Detection experiments based on strong interaction of tachyons are suggested.

1. Introduction

It has generally been believed and explicitly stated by A. Einstein, that the Theory of Special Relativity puts an upper limit to the velocity of all particles and that velocities exceeding that of light are forbidden [1]. Almost a decade ago I re-examined this question and found that the usual arguments against the possible existence of faster-than-light particles (tachyons) are invalid. Crucial to their understanding is a reinterpretation of "negative energy particles travelling backward in time" to be positive energy particles travelling in the opposite direction. All the paradoxes and puzzles put up by consideration of faster-than-light particles can be resolved on the basis of this reinterpretation [2]. Our earlier work has stimulated serious experimental work by Alväger and collaborators and it seems appropriate to outline the theory of faster-than-light particles at the present time.

Classical theory of tachyons [3]

The invariant energy-momentum relation

$$E^2 - p^2 = m^2 \tag{1}$$

enables us to classify particles into three classes according to whether m^2 is positive (Class I: tardyons), zero (Class II: luxons) or negative (Class III: tardyons). Tachyons have imaginary rest mass and would speed up as they lose energy and attain infinite speed at zero energy. Given any Lorentz frame in which they move with a finite speed $v = p/E > c$ there exists another frame moving with a relative speed

$$u = c^2/v < c \tag{2}$$

in which the tachyon will appear to be moving with finite momentum, zero energy

Table 1. Spinning tachyons.

Type	Square of spin	Range of helicities	Statistics
I_0	0	0	Bose
$D_j^{\pm}$	$j(j+1)$	$m_j \geqslant j+1$ for D_j^+	Bose for $2j$ even
		$m_j \leqslant j-1$ for D_j^-	Fermi for $2j$ odd
C_σ^0	$-(\sigma^2+\frac{1}{4})$	m_j integral	Bose
$C_\sigma^{\frac{1}{2}}$	$-(\sigma^2+\frac{1}{4})$	m_j half integral	Fermi
$E_s(\frac{1}{2}>s>0)$	$S^2-\frac{1}{4}$	m_j integral	Bose

and infinite speed. For relative velocities greater than c^2/v the tachyon will appear to move backwards in time and arrive at its destination before it started. This sounds absurd, but this apparent paradox has a simple resolution: The world-line of a free relativistic particle is parallel to its energy-momentum vector and whenever the time-sense of propagation has reversed the "particle" has negative energy. The observer in such a frame would interpret negative energy propagation backwards in time as positive energy propagation forwards in time. There are no anomalies with particles arriving before they start [2].

The limiting case of infinite speed tachyon is interesting in that in this case a finite momentum but no energy is being transferred; and the question of emission and absorption of an infinite speed tachyon is indeterminate. This is entirely analogous to impulses transmitted by a rigid body, in which momentum is transmitted instantaneously without transfer of energy. In a sense, then, tachyons reintroduce instantaneous action-at-a-distance characteristic of a relativistic theory; an event interpreted as instantaneous action-at-a-distance in one Lorentz frame will appear to be a propagated action in another frame; classical rigid bodies do not make their appearance.

The possibility that emission and absorption can be interchanged gives rise to no anomalies with regards to closed cycles of causation.

A charged tachyon must emit Cerenkov radiation [2] in empty space since it is moving faster than light in empty space. The angle of emission is a simple function of the velocity; beginning with zero opening angle for a very energetic particle (with a velocity close to that of light) the Cerenkov angle increases monotonically as the energy decreases (and the velocity increases). The theory of Cerenkov emission by charged tachyons has some divergence difficulties: the rate of emission is infinite according to the usual theory. It therefore suggests that tachyons may be neutral. This is all the more natural when we consider that the luxons (neutrinos, photons) are all electrically neutral.

Some implications of the existence of tachyons, whether they are charged or neutral, and independent of quantum theory modification, is discussed towards the end of this note.

Quantum theory [4]

Spinless tachyons may be described by the Klein–Grodon equation

$$\Box^2+m^2)\,\varphi=0;\ m^2\ <0$$

Spinning tachyons cannot be represented by finite-component fields since all of

Table 2. Particle species and their interaction.

Particle	Spin	Primary interactions	Electric charge	Mass
Charged leptons (e, μ)	$\frac{1}{2}$	Electromagnetic, weak	± 1	
Nucleons (n, p)	$\frac{1}{2}$	Strong	$\pm 1, 0$	Class I
Mesons ($\pi, \varrho, \ldots$)	0.1	Strong, electromagnetic, weak	$\pm 1, 0$	$m^2 > 0$
Photon	1	Electromagnetic	0	Class II
Neutral leptons	$\frac{1}{2}$	Weak	0	$m^2 = 0$
Tachyons	0 ?	?	0 ?	Class III $m^2 < 0$

them must have infinite number of polarization states for each momentum; they are classified in Table 1.

Quantization uses the physical reinterpretation postulate: *In any transition amplitude viewed from any frame, a negative energy tachyon in the initial (final) state is to be replaced by an antitachyon in the final (initial) state with the opposite values of all additive charges.* This principle, which is a strong version of crossing symmetry, applies to the transition amplitudes and *not* to the states.

We quantize the spinless tachyon field by associating the entire field with annihilation operators:

$$\phi(x) = (2\pi)^{-\frac{3}{2}} \int a(k)\, \delta(k^2 - m^2)\, e^{-ikx}\, d^4k$$

since there is no invariant distinction between positive and negative frequencies. The annihilation operators and their adjoints, the creation operators satisfy the *commutation* relation

$$[a(k)\,\delta(k^2 - m^2), a^+(k)\,\delta(l^2 - m^2)] = \delta(k-l)\,\delta(k^2 - m^2) \in (k) \tag{4}$$

We shall also introduce the vaccuum state defined by

$$a(k)\ |0\rangle = 0 \tag{5}$$

This quantization is relativistically invariant though the contraction function

$$D(x, y) = \langle 0|\tau(\phi^+(x)\,\phi(y))|0\rangle \tag{6}$$

is not Lorentz invariant and is *not* the analytic continuation of the time-ordered Green's function for a scalar field with a real rest mass. This is reminiscent of the treatment of the electromagnetic field in the Coulomb gauge.

The usual method of associating annihilation and creation operators respectively with the positive and negative frequency parts of a field cannot be used in a relativistically invariant manner. Attempts to construct such a theory of tachyon fields lead to violations of Lorentz invariance as may be verified by a critical study. The situation is not remedied by attempting to employ anticommutation relations [5].

Theory of tachyon interactions

We have at present no reliable guide to the nature of tachyon interactions. Since each of the luxons have their own characteristic interactions, it is very plausible that tachyons would also have characteristic interactions.

A simple choice of interaction would be to have a (pseudoscalar) tachyon coupled to spin $\frac{1}{2}$ nucleons by a Yukawa interaction with the interaction density chosen to preserve the symmetry between emission and absorption:

$$V(x) = g\bar{N}\gamma_5\varkappa(\phi + \phi^+) = j\cdot\chi, \tag{7}$$

where N is the nuclear field and g is a (strong?) coupling constant. The equation of motion for the tachyon field

$$(\Box^2 + m^2)\,\phi(x) = j(x) \tag{8}$$

has the solution

$$\phi(x) = \phi_{in}(x) + \int G(x-y)\,j(y)\,d^4y \tag{9}$$

where $G(x-y)$ is a Green's function, which may be chosen to be manifestly covariant. The analytic continuation from the case of the usual Klein–Gordon equation to negative squared mass suggests the choice

$$G(x-y) = (2\pi)^{-4}\int \frac{e^{ik(x-y)}}{k^2 - m^2 + i\epsilon}\,d^4k \tag{10}$$

which is covariant and contains all momenta. This is to be contrasted with the structure of the contraction function $D(x-y)$.

A covariant perturbation theory for the Yukawa interaction (7) of tachyons can be developed by working in the interaction representation and rewriting the quantity

$$S = T\left\{\exp i\int W(x)\,d^4x\right\} \tag{11}$$

(where $W(x)$ is the interaction $V(x)$ re-expressed in terms of interaction representation fields) in normal-ordered form to obtain the S-matrix. For the tachyon field in interaction representation we choose

$$\varphi(x) = \phi(x) - \frac{1}{4}\int \Delta^{(1)}(x-y)\,j(y)\,d^4y \tag{12}$$

where $\phi(x)$ is quantized according to (3)–(5), and $\Delta^{(1)}\,(x-y)$ is the symmetric homogeneous function.

$$\Delta^{(1)}(x-y) = (2\pi)^{-3}\int \delta(k^2 - m^2)\,e^{ik(x-y)}d^4k \tag{13}$$

The interaction in the interaction representation now consists of the following contributions [6]. (*a*) The Yukawa coupling through $\varphi(x)$ which leads to an effective nucleon–nucleon interaction in the second order. (*b*) The second term of (13) leads to another such term to lowest order. (*c*) There is a direct action at a distance which corresponds to spatial momenta forbidden for free particles. The net result of all these effects is to produce an effective nucleon-nucleon interaction of the form

$$\tfrac{1}{2} g^2 \bar{N}(x)\gamma_5 N(x)\Delta_F(x-y)\bar{N}(y)\gamma_5 N(y) \tag{14}$$

This is the *same result* as we would have obtained if we wrote down the result for $m^2 > 0$ and considered its *analytic continuation* for negative squared masses.

Both in the free field theory and in the case of interacting fields we employ Bose statistics for the spinless case.

The transition amplitudes obtained by the procedure have to be renormalized according to the familiar renormalization scheme; and the resulting amplitude subjected to the quantum physical reinterpretation postulate.

Methods of experimental detection

The simplest method of identification of a tachyon is to measure its energy and momentum and verify that the momentum is larger than the energy; equivalently one may measure the velocity directly by a time of flight method. The first method has already been employed by T. Alväger and P. Erman [7] who used a magnetic deflection in a double focussing beta spectrometer to select the momentum of the particles and a semiconductor counter measured their energy. They concluded that in Thulium 170 there were less than 10^{-4} tachyon per electron if at all. Another experiment initiated by Alväger looks for a direct detection of faster-than-light particles by searching for their Cerenkov emission [2], this experiment is in progress.

Both these experiments presume that the tachyons are electrically charged; if the tachyons are neutral, both the experiments must give negative answers. In view of the fact that luxons are not electrically charged we should seriously entertain this possibility. Four experimental methods of searching for tachyons which do not require them to be charged particles are the following:

(*a*) Search for "decays in flight" of a stable particle: If we find that a particle which is stable in its own rest frame (like the proton) appears to decay in flight we can be sure that at least one of the "decays" products is a tachyon.

(*b*) Large angle scattering: If fast particles scatter through large angles with a pronounced resonance in the invariant momentum transfer, a tachyon is being emitted (or absorbed!).

(*c*) Poles in the scattering amplitudes: If the scattering amplitude between two ordinary particles exhibits a pole in the invariant momentum transfer variable for negative (space-like) values [8] we can conclude that a tachyon is being exchanged.

(*d*) Effective mass plots: The original method [9] of identifying pion-pion resonances can be adapted to the present case by plotting the effective 4-momentum squared of a collection of pions with some of the pions in the initial state and some in the final state. A peak in such an effective squared mass plot at a negative value would be evidence for a tachyon. One has to eliminate, in such an analysis, and purely kinematic enhancements.

All these methods presume that the tachyon, whether it is charged or neutral participates in strong interactions. A fifth method which may be employed consists of a search for the missing mass squared in a suitably selected set of processes. In principle missing mass spectroscopy can be used independent of the strength of the interactions of tachyons.

Discussion

If faster-than-light particles exist, we are provided with an almost instantaneous communication channel. While distant observers can communicate by tachyons there is a physical limitation to the speeds that can be employed: two observers in relative motion with velocity $u<c$, can employ only tachyons with speed less than c^2/u; compare eqn. (2).

An efficient space drive should maximize the available thrust per unit of energy lost. This ratio is best for a fast tachyon beam. Present day technology is not yet able to use even photon drives.

Finally the possibility that light quanta can spontaneously emit tachyons leads to a kind of Cerenkov effect in which the roles of tachyon and photon are interchanged. Such a possible energy loss mechanism would have to be taken into account in cosmology and could lead to a redshift of starlight [10] but such a redshift seems to imply a line broadening also of the shifted spectral lines.

The most important implications of tachyons would be to the architecture of the subnuclear world. The considerations outlined in this paper make their existence very plausible. If they do not exist, their absence itself would be a mystery!

ACKNOWLEDGEMENTS

This work was completed while the author* was on research leave from Syracuse University. He is grateful to his friend J. Mehra for his constructive criticism.

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4. Following the mathematical study of the Green's function for negative squared mass by SCHMIDT, H., Z. Physik *151*, 365; 408 (1958), the quantum field theory of tachyon fields was studied by S. Tanaka who did not want the tachyons to be observed as real particles, but only contribute to virtual processes.
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6. A more complete account is contained in DHAR, J., and SUDARSHAN, E. C. G., Phys. Rev. *174*, 1808 (1968).
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Tryckt den 18 september 1969

Uppsala 1969. Almqvist & Wiksells Boktryckeri AB