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PHYSICS OF COMPLEX MASS PARTICLES*

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INTRODUCTION

The last few decades have seen a remarkable enrichment of the concept of an elementary particle; they may now be endowed with spin (even half-integral multiples of $\hbar$, charge, magnetic moment, baryon number, strangeness, and so on. But the mass of the particle has, through all these modifications, retained its classical aspect of being a positive real number. Complex proper masses for particles are still beyond the pale of normal discussions in particle physics. We can identify two reasons for this reticence: First, in a theory where the four momentum components are represented by hermitian dynamical variables it is not unreasonable to choose the mass operator to be hermitian and hence to require the mass eigenvalues to be real. Second, it appears that Lorentz invariance of a theory is at variance with the notion of a complex mass since for real space momenta it leads to complex energies, and this appears to conflict with Lorentz invariance.

Several physical problems of more than passing interest persuade us to consider the possibility of complex mass particles. The most familiar one of these is the general problem of unstable particles and, in particular, the shape of Breit-Wigner resonances. But there are other such problems as well. The attempts to construct a finite quantum field theory of interactions invariably leads to a quantum theory with an indefinite metric,[1] and in such a theory complex mass states are permitted, provided that such states are represented by null vectors. Simple-minded suggestions like ignoring complex mass particles in conventional scattering amplitudes are incompatible with the conservation of probability. A logical analysis of the framework of relativity and of relativistic interactions lead one to consider tachyons (particles traveling faster than light) and their exchange in strong interaction phenomena.[2] These particles have non-real masses.

PROBLEMS OF PHYSICAL INTERPRETATION OF COMPLEX MASS PARTICLES

Before accepting complex mass particles into the fold we have to deal with the problem of physical interpretation. The first one concerning Lorentz invariance has already been alluded to: if we start with a particle of real momentum and complex mass its energy is complex, and in a Lorentz frame in relative motion both the energy and momentum become complex. But how can we deal with a complex momentum state? Such a state is not acceptable as a quantum-mechanical state! We recall that a similar situation was encountered in the representation theory for noncompact groups: for example, if we take an "eigenstate" of a noncompact generator of $O(2,1)$ and try to "raise" the eigenvalue by acting on it with another generator we end up with a state with a complex eigenvalue. This apparent difficulty is overcome by recognizing the complex eigenvalue "state" as a singular linear combination of normal states; and working with direct integrals of states with weight functions which are analytic.[3] We may do the same here: the complex momentum state is a singular linear combination of states of real momenta; and we ought to consider only integrals of momentum eigenstates with analytic weight functions. In this sense a complex mass particle simulates a collection of particles with real masses.

There is a second problem: we have already mentioned that a complex mass state of a pseudohermitian mass operator must have a self orthogonal (null) vector as its representative. This is easily demonstrated since

$$\Psi^{\dagger}\Psi = (m - m^*)^{-1}\{(\Psi,M\Psi) - (M\Psi,\Psi) = 0,$$

which proves the above assertion. Does this mean to say that all states containing complex mass particles have zero norm? No, not necessarily, If, in particular, we take a state containing two particles of complex conjugate masses and equal three-momenta the $(\text{norm})^2$ of the state would be positive or negative according as the state is symmetric or antisymmetric in the three-momenta. It is therefore not possible to simply ignore the production of such particles in field theory where such particles are coupled to ordinary particles.

LAW OF "ASSOCIATED PRODUCTION"

However with the sole exception of zero-width tachyons, all other complex mass particles have complex energies. It therefore follows that energy conservation in scattering forbids the production of a single complex mass particle. Complex mass particles obey "associated production." Recently Lee and Wick[4] studied a class of explicitly soluble models in which they found that complex mass particles are coupled dynamically but are not produced; but all the models they have studied simply verify the conclusion deduced from energy conservation above! Gleeson and I have shown that the Lee-Wick program fails for models where complex conjugate pairs of particles can be produced precisely because the quarantine of complex mass states furnished by the energy conservation law is no longer effective.[5] [In a separate paper Lee[6] has given some new arguments which amount to using the same energy conservation law to prevent such pairs of particles from contributing; Cutkosky, Landshoff, Olive and Polkinghorne[7] have dealt with essentially the same calculation to show that an ad hoc prescription, distinct from the Feynman rules in quantum field theory, must be employed to obtain Lee's "result".]

VIRTUAL COMPLEX MASS QUANTA AND NONLOCAL INTERACTIONS

The use of complex mass particles as single virtual particles exchanged between two normal particles is beset with no conceptual difficulties and would simply simulate a non-local interaction with an interaction kernel which depends upon the complex mass involved. Hence, as long as we confine our attention to the virtual quanta of a complex mass field we are led to no difficulty. In fact if we avoid having any real quanta of the complex field, we would have no problem of physical interpretation!

Such a state of affairs would have two merits: (i) It has been known for sometime that a finite quantum electrodynamics can be constructed using an indefinite metric and that it gives results in excellent agreement with experiment.[8] No obstacle appears in the path of generalizing it to other field theories. (ii) In some models of action-at-a-distance it is known[9] that there are no real particles associated with the non-local interaction. The colliding particles carry away all the incident energy and momentum and there is no provision for any "radiation".

For several years I have been investigating the interrelationship of action-at-distance and action-by-contact in relativistic quantum theories. Contrary to the usual belief that all nonlocal interactions were obtained by exchanges of particles which could be physically produced, I had been studying the possibility of a mediating field with no real quanta. A couple of years ago I succeeded in constructing such a "shadow field".[10] Its virtual quanta behave like the virtual quanta of a conventional field, but we may ignore its quanta. [Hence the term: "shadow".]

The fundamental clue resides in the observation that the interaction of a $j(x)$ with itself must be described by a symmetric, real kernel $K(x - y)$ so that the corresponding contribution to the action is:

$$\iint d^4x\, d^4y\, j(x)\, K(x - y)\, j(y).$$

If we now expand the translationally invariant symmetric real kernel K in the form

$$K(x - y) = \int d\rho(m)\, G(x-y;m)$$

(where the real Stiltje's measure $d\rho(m)$ is defined for complex m,) then $G(x-y;m)$ is a real symmetric Green's function satisfying:

$$\left(\Box + m^2\right) G\left(x-y;m\right) = \delta^4\left(x-y\right).$$

It follows that G must be the half-advanced half-retarded real Green's function. Such time-symmetric kernels have been proposed in connection with action-at-a-distance already in the early nineteen hundreds by Schwarzschild and by Ritz and subsequently by a number of other authors including Tetrode, Fokker, Wheeler and Feynman, Chretian and Peierls, and Van Dam and Wigner.

The fundamental problem is to find a set of mediating fields each of which is locally coupled to the source; but so arranged that we do not have to deal with real quanta associated with these mediating fields through their production in physical processes. This is accomplished by the theory of shadow fields outlined in the next section.

QUANTUM THEORY OF SHADOW FIELDS: QUANTIZATION

Let $\phi(x)$ be a (scalar) field with mass m. We now define a set of annihilation operators $b(\underline{k},\pm)$ according to the definition

$$\phi(x) = (2\pi)^{-3/2}\int \{(b(\underline{k},+)e^{(-i\omega x_o + i\underline{k}\cdot\underline{x})} + b(-\underline{k},-)e^{(+i\omega x - i\underline{k}\underline{x})}\}\frac{d^3k}{2\omega}$$

$$= (2\pi)^{-3/2}\int b(k)\delta(k^2-m^2)e^{-ikx}d^4k.$$

where ω is the principal square root of $m^2 + \underline{k}^2$. This field is not hermitian; we define its hermitian adjoint which contains only creation operators:

$$\phi^\dagger(x) = (2\pi)^{-3/2}\int \{b^\dagger(\underline{k},+)e^{(i\overset{\omega}{\wedge}x_o - i\underline{k}\underline{x})} + b(-\underline{k},-)e^{(-i\overset{\omega}{\wedge}x_o + i\underline{k}\cdot\underline{x})}\frac{d^3k}{2\omega}$$

$$= (2\pi)^{-3/2}\int b^\dagger(k)\delta(k^2-m^2)e^{+ikx}d^4k$$

With these definitions we get

$$[\phi(x),\phi(y)] = 0$$

$$[\phi(x),\phi^\dagger(y)] = \xi i\Delta(x-y);\xi^2 = 1$$

$$\Delta(x-y,m) = -\frac{1}{(2\pi)^3}\int \frac{d^3k}{\omega} e^{i\underline{k}(\underline{x}-\underline{y})}\sin\omega(x^\circ-y^\circ).$$

And, in particular, the equal time commutation relations:

$$\delta(x^\circ - y^\circ)[\phi(x), \phi^\dagger(y)] = 0$$

$$\delta(x^\circ - y^\circ)[\phi(x), \phi^\dagger(y)] = i\xi\delta(x-y); \xi^2 = 1.$$

These commutation relations are equivalent to the creation and annihilation operator commutation relations:

$$[b(k,\pm), b^\dagger(k,\pm)] = \xi(\omega \pm \omega')\ \delta(\underline{k}-\underline{k}').$$

The quanta of the field can be constructed in a suitable Fock space. Here ξ is the metric for the positive frequency quanta and $-\xi$ that of the negative frequency quanta.

The simplest interaction that we can consider is of the form

$$g\int d^4x\ j(x)[\phi(x) + \phi^\dagger(x)]$$

where $j(x)$ is a suitable source (which may be taken to be bilinear in a normal field) and g is a (real) coupling constant. In the lowest approximation the single exchange contribution to the effective interaction is given by

$$-\frac{i}{2}\xi g^2 \iint d^4x\ d^4y\ j(x)\ j(x)\ W(x-y)$$

where W(x-y) is the Wick contraction functions

$$W(x-y) = 0|T([\phi(x) + \phi^{\dagger}(x)][\phi(y) + \phi^{\dagger}(y)])|0$$

$$= i\ (x^{\circ}-y^{\circ})\quad (x-y) = \frac{i}{(2\pi)^4}\ P\int d^4k\ \frac{e^{ik(x-y)}}{k^2-m^2}$$

We have thus an interaction of the standard nonlocal form with the kernel

$$K(x-y) = 1/2\ \xi g^2 \varepsilon(x^{\circ}-y^{\circ})\ \cdot\ \Delta(x-y;m)$$

which is real (provided m is real) and symmetric. When m is complex we must have a corresponding contribution from m* so that their sum provides a real kernel.

QUANTUM THEORY OF SHADOW FIELDS: SCATTERING MATRIX

We are now in a position to write down the general S-matrix element in perturbation theory. We may use all the usual perturbation theory rules except that we must use the time-symmetric Green's function rather than the usual causal Green's function for the states containing shadow quanta. Both of them are Lorentz invariant; for one-particle intermediate states these are simply:

$$\bar{\Delta}(p) = P\ \frac{1}{p^2-m^2}$$

$$\Delta^C(p) = \frac{1}{p^2-m^2+i\varepsilon}$$

For all processes like single quantum exchange where the propagators have no singularity both kinds give identical results.

Under the circumstance that the denominators can vanish the shadow propagator yields a different result. As a simple example we may consider the self energy of a hadron of mass M coupled to a shadow field (both taken to have spin 0 for simplicity). The corresponding diagram is given in Fig. 1; and the leading singularity of the dressed hadron is obtained by computing the singularity of the self-energy bubble. We find that except for the point $p^2 = (M + m)^2$, there is no singularity for finite values of p^2. But unlike the case of normal particles, even for real values of M and m, there is no imaginary part (no branch cut!) along the real axis $(m + M)^2 < p^2 < \infty$.

Another example is given by the antihadron-hadron scattering process. The lowest order diagram is trivial; the next order corresponds to the so-called box diagram, Fig. 2. We can again calculate the imaginary part of the amplitudes. The intermediate states corresponding to the hadron pair yields the expected elastic unitarity condition. But there is <u>no</u> imaginary part corresponding to the shadow intermediate states. In other words we may ignore the shadow states altogether as far as unitarity is concerned. Nevertheless their exchange contributes to the actual scattering amplitude even in the lowest approximation.

APPLICATION TO PARTICLE PHYSICS

Such a situation should obtain quite generally. Using Cutkosky's general rule[11] for calculating the discontinuities associated with Landau singularities and the observation that

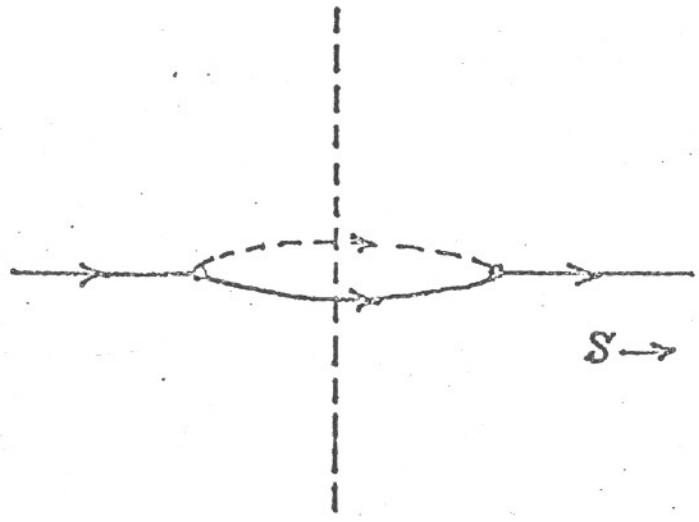

FIGURE 1 HADRON SELF ENERGY TO LOWEST ORDER

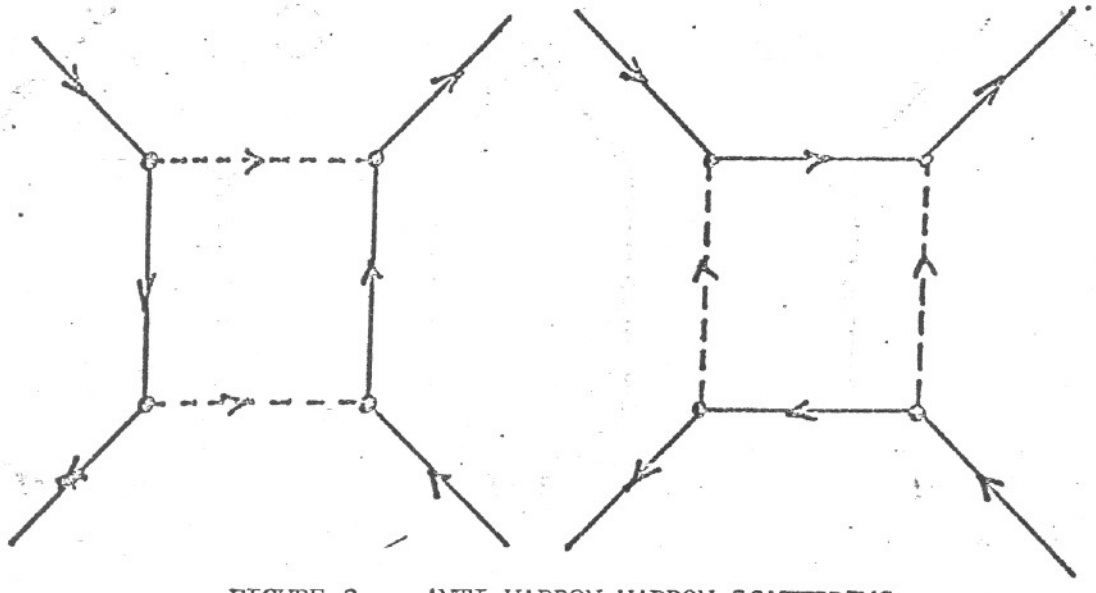

FIGURE 2 ANTI HADRON-HADRON SCATTERING

$$P\,\frac{1}{p^2-m^2} = \frac{1}{p^2-m^2+i\varepsilon} + i\pi\delta(p^2-m^2)$$

We can conclude that the discontinuity vanishes for a cut in which only a shadow line is involved. Only states containing physical particles exclusively contribute to such cuts. These and these alone can be considered as physical single particle states. The sole purpose of a shadow particle is to produce forces between ordinary particles.

For many-particle intermediate states we must make sure that those states which contain at least one shadow particle should not contribute to the unitarity sum. This, in turn, requires that the two-particle (and many-particle) Green's functions should have imaginary parts only for those channels in which no shadow particle is involved. For the channels which involve shadow particles there should be no imaginary part. A systematic construction of such Green's functions can be carried out, but here we shall be content with stating the general principles. Recently I have solved exactly several theoretical models (including the symmetric scalar and the symmetric pseudoscalar theories) in the one-meson approximation, and the results will be published elsewhere. In such exact solutions one proceeds from such a construction for the two-particle Green's function.

As an immediate application of shadow particles to particle physics we may consider resonances in hadron scattering. Most observed resonances in the direct channel are associated with poles on the second sheet and hence the resonance phase shift rotates in the counter-clockwise direction as the energy increases. They are not associated with complex mass shadow particles.

But if any resonances are observed in which the phase shift rotates in a clockwise direction as the energy increases, it is evidence for a complex mass shadow particle.

A related application is to resonances in momentum transfer. Tachyon exchanges would lead to such momentum transfer resonances in hadronic reactions, even if tachyons are not experimentally observed. If they continue to be undetected we must be aware of the possibility that they are shadow tachyons. Such a tachyon may have an arbitrary complex mass and spin. One of my associates is currently comparing such a theory with the available experimental data on pion-proton scattering and the preliminary finds are very encouraging.

At this point the question arises: Why insist on a complex mass, if it is to be a shadow? The complex mass arises naturally in an indefinite metric theory when a negative norm particle has a mass exceeding the threshold for dissociation into two or more positive norm particles. However, the basic need for convergence is already taken care of by the use of the indefinite metric. Could we not have real mass shadows? Indeed we could.

RELATIVISTIC FIELD THEORY OF ACTION-AT-A-DISTANCE

I assert that I have now solved the problem of constructing a relativistic quantum theory of action-at-a-distance. We have now all the ingredients for a finite quantum field theory of interactions. The quantization is accomplished by introducing auxiliary shadow fields which mediate the interaction. The perturbation theory rules can now be applied with the

proviso that we use the time-symmetric propagator rather than the causal Green's functions for those states with at least one shadow particle. Unitarity would be satisfied without including any of the states containing the shadow quanta. At all stages the theory is manifestly covariant.

Within the framework we have developed, action-at-a-distance and action-by-contact are two alternate ways of formulating the same system.[12] The fields satisfy local commutation relations and are coupled locally.

To achieve this complete equivalence we had to proceed beyond the conventional but arbitrary restriction to masses and positive-definite metric for the fields. We had to extend our study to include fields with indefinite metric and fields with complex mass, including tachyons. Nonlocal interactions and indefinite metric are closely related. In my Report to the Fourteenth Solvay Congress in 1967 on Indefinite Metric and Nonlocal Field Theories, I had expressed the hope that "the indefinite metric formulation of quantum field theory is the prelude to a new theory." This hope seems to be on the verge of fulfillment at the present time and the theory of shadow fields appears to be an essential step.

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