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ACTION - AT - A - DISTANCE*

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Abstract: In a talk presented at the symposium "The Past Decade in Particle Theory" Prof. Sudarshan reviews the historical development of "action - at - a - distance" theories. He reviews the formulation of classical action - at - a distance theories and identifies the relevant conserved quantities. This theory is compared with a corresponding classical field theory. The lessons of this classical theory are applied to the formulation of a quantum theory of action - at - a - distance. The problem of unitarity and physical interpretation are described. The relationship with indefinite metric theories of elementary particles is clarified and finally a theory of interactions which includes self interaction but contains only finite computations is formulated.

INTRODUCTION

Physical theory deals with the nature of the elementary entities and their mutual interactions: all fundamental questions pertain to existence

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entities and their mutual interactions: all fundamental questions pertain to existence and change. In physics they correspond to the existence of "particles" and their "interactions." It has been the practice to assume existence of elementary entities (elements, molecules, atoms, nuclei, etc.) at a certain level and study their mutual interaction (chemical activity, collision characteristics, molecular formation, energy levels of atoms, etc.) The question of the nature and constitution of entities considered elementary at one level is to be tackled at the next level, where new interactions are postulated between the constituents of that object which was previously considered to be elementary. New levels of structure automatically signify new kinds of interactions, but this process of analysis must stop at the point where constituents lose their meaning. In the description of phenomena in particle physics one may well have reached this stage. The interactions between the particles must explain their structure as well, Existence and change became two facts of the same physical system. Not only are forces between particles obtained through exchange of particles, but the very existence of the particles is due to their interactions.

Such a state of affairs leads to very stringent con-

straints on allowable dynamical laws. If the electron is an elementary classical point charge and if it is to interact with the electromagnetic field in the usual manner deduced from the classical Maxwell equations, then there is a serious inconsistency: the electron should explode. There is no point in using some other force to hold the electron together if it is a point charge unless this force is also infinite. Perhaps the electron is not a point charge even though it is elementary; perhaps it has a finite extension, perhaps the point nature of the electron is only apparent; perhaps the electron does not interact with the electromagnetic field at the same point but interacts nonlocally. In any case the law of electromagnetic interaction when applied to the stability of the elementary effect itself leads inevitably to a contradiciton unless the local law of interaction is generalized to a non-local one.

One may ask whether the introduction of quantum mechanics elps to eliminate the difficulty. It ameliorates it in the ase of the Dirac electron, but does not resolve the problem. nd in the equally important case of weak interactions it eems to lead to worse troubles as long as one insists on the se of a local relativistic quantum field theory.

ACTION - AT - A - DISTANCE

From the earliest days of dynamics the question of action - at - a - distance versus action - by - contact has been discussed. One gets the impression that the question has been decided in favor of action - by - contact. For example the electrostatic force which is action - at - a - distance is replaced by a local interaction with the electromagnetic field. As we have already noted, Maxwell's theory is not really satisfactory since it contradicts the existence of point particles. Some form of effective non-local interaction is indicated. We can also raise the question: Do the observed phenomena of electromagnetism rule out an action-at - a - distance theory? Could we not reconcile the phenomena by using a suitable physical interpretation of a direct action - at - a - distance? This question was raised and answered in the affirmative by Wheeler and Feynman[1] in two papers (which have been uncritically cited by almost every writer since then). They show that, provided certain entities (collectively acting as the "absorber") are introduced, all the radiative processes can be equally well derived from a theory involving direct action - at - a - distance between charged particles. They confine their attention to electro-

magnetic interactions only, and their "absorber" is rather poorly defined. They do not solve the riddle of the existence of the electron; they do avoid the infinity due to the self energy of the electron, but at the price of complicating the effective interaction between the charged particles with the field. In other words their theory, even with the introduction of the absorber, is not equivalent to Maxwell's theory. In spite of all this, their work does show that the direct observation of a "propagating influence" such as the electromagnetic field in no way rules out the possibility of action - at - a - distance.

Let us recall that even earlier Whitehead[2] had attempted to construct a relativistic theory of gravitation as action - at - a - distance. He succeeded in showing that the Schwarzchild line element of general relativity describing the exterior solution for a massive gravitating body could be derived from this. This work has been given a satisfactory formulation in terms of an action principle by Schild[3] who derived the conservation laws for such a theory.

The logical questions involved in formulating a direct two-particle action - at - a - distance theory have been analyzed by Van Dam and Wigner.[4] They have shown that by suit-

ably restricting the interaction the ten fundamental conservation laws are satisfied. The individual particle contributions to the four components of the momentum and the six components of the angular momentum are nonvanishing asymptotically. Hence, the total momentum and angular momentum of the collection of particles are conserved in collisions. Van Dam and Wigner conclude that no physical field is involved since energy is neither absorbed nor radiated. As we shall see, this conclusion is not warranted.

Hence we shall be interested in theories which are in accord with the special theory of relativity. If the theory is manifestly covariant the nonlocality in space, implicit in the notion of a potential, automatically implies nonlocality in time. This comes in from the geometry of Lorentz transformations which make simultaneity frame-dependent, in contrast to a nonrelativistic potential. There is, however, a distinct source of nonlocality which came in from the constraint that the theory should explain the "propagation" of the interaction. Such a nonlocality (the "retardation" effect) should be present equally well in a nonrelativistic situation. Conversely, the retardation effect implies a finite speed of propagation. If this is generally true then

a (manifestly) relativistic theory must always involve propagated action.

The problem of the divergence of the self-interaction effects implies a need for modification of the local interaction in action - by - contact theories and to a restriction on the type of interaction in action - at - a - distance theroies. We shall see later on in this report that such a theory requires a cancellation of the high wave number (short distance) components of the propagated interaction; this implies in turn that there should be more than one simple coupled "field," and that some of these fields would have the wrong sign of the interaction energy ("negative probability") or be appreciable outside the light cone ("faster than light") or both. An acceptable relativistic action - at - a - distance theory appears to be equivalent to a suitable local interaction theory formulated in terms of an indefinite metric.

The problem of physical interpretation of indefinite metric theories has been studied over the past decade. We presented the status of the subject at that time at the XIV Solvay Congress.[5] Since that time the persuasive exposition (and diligent model construction) of Lee and Wick[6] have drawn

despite the failure of their conjecture, much attention to the uses of indefinite metric in field theory. The status of the problem has not changed in any essential aspect. We had stated that "...the proper identification of physical amplitudes is part of the dynamical problem in an indefinite metric quantum field theory." In an action - at - a - distance theory this problem is even more acute. We shall see that the methods of indefinite metric quantum field theory can be borrowed for developing the physical interpretation of relativistic action - at - a - distance theories.

In this report let us first deal with a classical theory of particles interacting through direct action - at - a - distance and show in what sense it is equivalent to a locally coupled field theory. We note that in classical theory the mediation is by a field rather than by exchange of particles! In a quantum theory, the quantized field is also a quantized particle assembly. Many of the essential questions can be raised and answered within the classical framework, particularly the question of the degree to which the mediating field is to be taken to be physical and the important role played by the choice of the boundary conditions. Attention is then turned to the quantum theory of the system and the structure

of the corresponding quantum field theory and the choice of physical amplitudes. In addition to the important question of the conservation of probability ("unitarity") we discuss also the questions of analyticity, the connection between spin and statistics and the TCP theorem.

CLASSICAL THEORY OF INTERACTING PARTICLES

Let M_a denote the proper masses of a collection of interacting particles and x_a^u, their coordinates. If

$$ds_a^2 = (dx_a^0)^2 - (dx_a^1)^2 - (dx_a^2)^2 - (dx_a^3)^2$$

are the proper times of the various particles, an action function for the particles with two-body interactions is:

$$A = \frac{1}{2}\int m_a \dot{x}_a^2 \, ds_a + \frac{1}{2}\iint K_{ab} \, ds_a \, ds_b,$$

where K_{ab} are invariant functions depending on the coordinates of the particles and their relative four-velocities. For a purely scalar interaction we may put

$$K_{ab} = K([x_a - x_b]^2),$$

while for a vector interaction we may put

$$K_{ab} = \dot{x}_a^\mu \, K([x_a - x_b]^2) \, \dot{x}_{b\mu}$$

and so on. For the electromagnetic direct interaction

$$K_{ab} = e^2 \dot{x}_a^\mu \, \dot{x}_{b\mu} \, \delta([x_a - x_b]^2),$$

while in the Whitehead theory of gravitation

$$K_{ab} = -G^2 \dot{x}_a^{\mu} \dot{x}_a^{\nu} \, \delta([x_a - x_b]^2) \dot{x}_{b\mu} \dot{x}_{b\nu} ,$$

the dots denoting differentiation with respect to the appropriate proper time labels. Without loss of generality we can assume that K_{ab} is symmetric in a and b. Manifest covariance of the theory demands that it be a Lorentz invariant quantity.

The variation of the action is

$$\delta A = ([\dot{x}_a \delta x_a] - \int \ddot{x}_a \delta x_a ds_a) + [\int \frac{\partial K_{ab}}{\partial \dot{x}} \delta x_a ds_b]$$

$$+ \iint \left\{ \frac{\partial K_{ab}}{\partial x_a} - \frac{\partial}{\partial s_a} \left(\frac{\partial K_{ab}}{\partial \dot{x}_a} \right) \right\} \delta x_a ds_a ds_b .$$

The equations of motion are obtained by considering variations which vanish at the end points. We obtain

$$m_a \ddot{x}_a = \int \left\{ \frac{\partial K_{ab}}{\partial x_a} - \frac{\partial}{\partial s_a} \left(\frac{\partial K_{ab}}{\partial \dot{x}_a} \right) \right\} ds_b .$$

The coefficients of the variation of the coordinates at the end points yield the expression for the conjugate physical quantities. By writing

$$\delta x_a = \varepsilon \quad \text{(independent of a)},$$

we obtain the expression for the total (independent of a) momentum of the system:

$$P_\mu = m_a \dot{x}_{a\mu} + \frac{1}{2}\int\left\{\frac{\partial K_{ab}}{\partial \dot{x}_a^\mu}\right\} ds_b + \frac{1}{4}\left(\int_{s_a}^{\infty}\int_{-\infty}^{s_b}-\int_{-\infty}^{s_a}\int_{s_b}^{\infty}\right)\left\{\frac{\partial K_{ab}}{\partial x_a^\mu}\right\} ds_a\, ds_b,$$

which is conserved by virtue of the equations of motion, and could have been obtained by direct integration of them. By choosing

$$\delta x_a^\mu = \varepsilon^\mu_\nu\, x_a^\nu \qquad (\varepsilon^\mu_\nu \text{ independent of a}),$$

we get the expression for the angular momentum

$$M_{\mu\nu} = m_a(\dot{x}_{a\mu}\, x_{a\nu} - x_{a\mu}\dot{x}_{a\nu}) - \frac{1}{2}\int_{-\infty}^{\infty}\left(x_{a\mu}\frac{\partial K_{ab}}{\partial \dot{x}_{a\nu}} - x_{a\nu}\frac{\partial K_{ab}}{\partial \dot{x}_a^\mu}\right) ds_b$$

$$- \frac{1}{4}\left(\int_{s_a}^{\infty}\int_{-\infty}^{s_b}-\int_{-\infty}^{s_a}\int_{s_b}^{\infty}\right)\left(x_{a\mu}\frac{\partial K_{ab}}{\partial x_a^\nu} - x_{a\nu}\frac{\partial K_{ab}}{\partial x_a^\mu} + \dot{x}_{a\mu}\frac{\partial K_{ab}}{\partial \dot{x}_a^\nu}\right.$$

$$\left. - \dot{x}_{a\nu}\frac{\partial K_{ab}}{\partial \dot{x}_a^\mu}\right) ds_a\, ds_b,$$

which is again conserved.

These equations simplify for the special interactions we have considered. For the scalar interaction

$$\frac{\partial K_{ab}}{\partial \dot{x}_a} = 0$$

$$P = m_a \dot{x}_{a\mu} + \frac{1}{4}\left(\int_{s_a}^{\infty}\int_{-\infty}^{s_b} - \int_{-\infty}^{s_a}\int_{s_b}^{\infty}\right)\frac{\partial K_{ab}}{\partial x_a^\mu}\, ds_a\, ds_b$$

$$M_{\mu\nu} = m_a (\dot{x}_{a\mu} x_{a\nu} - x_{a\mu} \dot{x}_{a\nu})$$
$$+ \frac{1}{4} \left(\int_{s_a}^{\infty} \int_{-\infty}^{s_b} - \int_{-\infty}^{s_a} \int_{s_b}^{\infty} \right) \left(\frac{\partial K_{ab}}{\partial x_a^{\mu}} x_{a\nu} - \frac{\partial K_{ab}}{\partial x_a^{\nu}} x_{a\mu} \right) ds_a \, ds_b$$

These expressions have a very simple interpretation. The first terms in P_μ and $M_{\mu\nu}$ are the free particle contributions. Remembering that K_{ab} is a function of $(x_a - x_b)^2$, it follows that the two double integrals really are the same terms except for the interchange of the particle labels. The double integral

$$\int_{s_a}^{\infty} \int_{-\infty}^{s_b} \left(\frac{\partial K_{ab}}{\partial x_a^{\mu}} \right) ds_a \, ds_b$$

refers to the total impulse that has "left" b by proper time s_b, but not yet "reached" a by proper time s_a (but will reach later!). It is therefore the impulse in transit. (Counting indiscriminately over all pairs accounts for the factor 2.) Similarly, in the expression for the angular momentum it is the moment of this "impulse in transit" which accounts for the second term. We shall assume that these mutual contributions to momenta and angular momenta vanish when the particles are spatially separated far enough so that we may neglect them asymptotically. We have already noted that

by virtue of the summation over all the particles, the kernel K_{ab} is symmetric. Because the various expressions have to be real, K should be real and symmetric. There is a "retardation," but it contains both retardation and advancement. Since action and reaction are equal, if we have retarded action, we must have (as viewed from the second particle) an advanced reaction. The only natural choice is to have both retarded and advanced impulse transmission in a time-symmetric fashion.

We note that the only dynamical entities are the particles themselves and the momentum and angular momentum reside either in the motion of the particles or in their mutual ("potential") contributions. There is no other vehicle for the dynamical quantities. In particular, there is no possibility of "radiation" leaving the system. If we want at all to consider energy leaving a collection of particles, there should be some other particles (to which it has been lost!) which act as absorbers or suppliers of energy. Isolated particle or particles do not, in the very nature of things, lose energy to empty space!!

Having paid attention to the essential question of the "absorber" to which the particles transmit energy and momentum

etc., we can now raise the question of "radiation," i.e. the energy momentum and angular momentum transmitted by the particle or particles of interest to the "absorber." We have assumed that the law of mutual interaction is such that asymptotically these mutual contributions may be neglected between any pair of particles. To be able to talk about radiation we must therefore make the following provisions: (1) There should be an arbitrarily large number of absorber particles which are at all times sufficiently far away from the first set of particles. (2) We should compute only the energy, momentum and angular momentum that are emitted from the particles of interest to the absorber particles, and the calculation of the radiated quantities of interest takes into account only these. (3) The reaction of the absorber particles on the particles of interest is to be totally ignored. The absorber is therefore not just any collection of particles, but of those that are in such a relation to each other as to produce no advanced field at any of the particles. Under these circumstances the reduction in the energy of the particles is, in fact, being transmitted to the absorber and the loss in energy is independent of any first hand specification of the composition of the "absorber."

INTRODUCTION OF THE FIELD

For many purposes it is advantageous to introduce a field as a suitable functional of the particle trajectories. We choose a specially simple case (to illustrate the principles) where the interaction is purely scalar and given by the real symmetric invariant function

$$K_{ab} = g_a g_b \, \bar{\Delta}(x_a - x_b)$$

where $\bar{\Delta}$ satisfies the equation

$$\left[\frac{\partial^2}{\partial t^2} - \underline{\nabla}^2 + m^2\right] \bar{\Delta}(x) = -\delta(x).$$

The action function is now

$$A = \int m \; ds_a + \tfrac{1}{2} g_a g_b \iint \bar{\Delta}(x_a - x_b) \; ds_a \; ds_b.$$

Let us <u>define</u> the function

$$\phi(y) = g_b \int \bar{\Delta}(Y - x_b) \; ds_b.$$

Then we have the two equations

$$m\ddot{x}_a = g_a \frac{\partial \phi(x_a)}{\partial x_a}$$

$$(\Box^2 + m^2)\phi(y) = -g_b \int \delta(y - x_b) \; ds_b,$$

provided the original interaction involved summation over <u>all pairs including self-interactions</u>. (We already see an inconsistency since $\bar{\Delta}(0)$ is undefined!) This is exactly the same form as the equations of motion of the particles coupled to a scalar field.

The Action for the system involving the coupled system of particles and fields which lead to the above equations is the following:

$$A = \frac{1}{2}\int m_a \dot{x}_a^2 ds_a + \frac{1}{2}\int [\partial_\mu \phi(y)\partial^\mu \phi(y) - m^2\phi^2(y)]d^4y + g_a\int :(x_a)^n:$$

In one sense there is a great difference between this system and the system of particles under direct interaction. In the present case the field carries its own energy and can act as the repository of energy, momentum and angular momentum. Despite this difference, which we will discuss presently, we note that the similarity between the two systems is more than appears in the equations of motion. The expression for the conserved quantities can be transformed from the action - at - a - distance form to the action - by - contact form. We shall do this for the conservation of momentum. In the field theory form, the total momenta are:

$$P_\mu = m_a \dot{x}_{a\mu} + \int \left\{ \partial_\nu \phi \cdot \partial_\mu \phi - g_{\mu\nu}(\partial_\sigma \phi \cdot \partial^\sigma \phi - m^2\phi^2) \right\} d\sigma^\nu$$

where $d\sigma^\nu$ is the 3-volume element in the y variable orthogonal to the ν direction. But

$$\int \left\{ \partial_\nu \phi \partial_\mu \phi - \frac{1}{2} g_{\mu\nu}(\partial_\sigma \phi \cdot \partial^\sigma \phi - m^2\phi^2) \right\} d\sigma^\nu$$

$$= \int \left\{ \partial^\nu \partial_\nu \phi \cdot \partial_\mu \phi - \frac{1}{2}\partial_\mu (\partial^\sigma \phi \cdot \partial_\sigma \phi - m^2 \phi^2) + \frac{1}{2}\partial_\mu (\partial^\nu \phi \cdot \partial_\nu \phi) \right\} d^4y =$$

$$\int (\Box + m^2)\phi \partial_\mu \phi d^4y = -g_b \int \partial_\mu \phi(x_b)\, ds_b .$$

Hence

$$P_\mu = m_a \dot{x}_{a\mu} - g_b \int_{s_b}^{\infty} \partial_\mu \phi(x_b)\, ds_b =$$

$$= m_a \dot{x}_a - \frac{1}{2} g_a g_b \int_{-\infty}^{s_a} \int_{s_b}^{\infty} \frac{\partial \bar{\Delta}(x_a - x_b)}{\partial x_a}\, ds_a\, ds_b ,$$

where the factor $\frac{1}{2}$ is inserted to avoid double counting. If we so choose we can further explicitly symmetrize the expression in a and b and obtain the form we had written before. Thus the two systems give the same result. But how are we to reconcile this with the fact that the field seems to contain its own autonomous vibrations independent of other particles?

The resolution of this dilemma lies in the observation that as we <u>defined</u> the field, the field had no autonomy. It was completely defined in terms of the particle variables. Hence the question of whether $\phi(y)$ vanishes at infinity cannot be set by us independently of the configuration of particles. If we have an "absorber," the contribution to ϕ from them must vanish at the position of the particles of interest. In other words, the apparent degrees of freedom of the field

are really the degrees of freedom of the myriads of particles constituting the absorber, and the boundary conditions imposed on the field are really disguised assumptions about the configurations of particles.

Let us then ask: Why bother with the fields? First, the introduction of the field enables us to restore, albeit formally, a local interaction structure. Second, in radiative processes we need not explicitly talk about the "absorber" but instead replace it by a boundary condition on the field. The conserved quantities can be seen simply as the sum of the free particle energies of the particles and the "free" field energy. Despite these advantages one must be cautious in the use of the field since the number of degrees of freedom in a <u>finite</u> universe for such a theory is <u>not arbitrarily large</u>; it is probably no more than the number of degrees of freedom of the collection of particles.

We may make this last remark more explicit by recalling the definition

$$\phi(y) = g_b \int \bar{I}(y - x_b)\, ds_b.$$

Suppose we separated the particles into two groups α which are in the absorber and β in the set of particles under study. We can now rewrite $\phi(y)$ in the form

$$\phi(y) = \phi_\alpha(y) + g_\beta \int \bar{I}(y - x_\beta)\, ds_\beta.$$

Remembering that

$$\bar{\Delta} = \frac{1}{2}(\Delta_R + \Delta_A) \; ; \; \Delta = \frac{1}{2}(\Delta_A - \Delta_R) \; ,$$

we could write

$$\phi(y) = -\frac{1}{2}g_\alpha \int \Delta(y - x_\alpha)ds_\alpha + g_\alpha \int \Delta_A(y - x_\alpha)ds_\alpha$$

$$+ \frac{1}{2}\, g_\beta \int \Delta(y - x_\beta)ds_\beta + g_\beta \int \Delta_R(y - x_\beta)ds_\beta ,$$

with the first and second terms satisfying the force field equation although they are completely determined by the particles. By definition, the second term does not contribute since an absorber does not produce an advanced field. We have therefore

$$\phi(y) = \phi_{in}(y) + g_\beta \int \Delta_R(y - x_\beta)\; ds_\beta .$$

which is the standard form for a general solution to the equation of motion satisfied by ϕ. But we find that the "in" field ϕ is not entirely arbitrary since it does depend upon the configuration and disposition of the particles, both in the absorber as well as elsewhere. If we genuinely had freedom to choose the "in" field, we would have had new degrees of freedom.

However, such a field manifests degrees of freedom and participates in statistical mechanical phenomena as if it had its own degrees of freedom. Of course it "borrows" these degrees of freedom from the "absorber" and the other particles; if the total number of particles involved is finite, the number of degrees of freedom of this field will also be finite,

reminiscent of the phonon modes of a Debye solid.

Since the Green's function for the differential equation is not uniquely determined, it follows that the decomposition of a given solution into a particular integral (proportional to a specific Green's function) and a complementary function (which is interpreted as the incoming free field) is not unique. In other words the correct boundary conditions to choose depend upon what is chosen to be the "in" field and the Green's function. In an action - at - a - distance theory the natural Green's function is the time-symmetric half-advanced, half-retarded kernel. For classical theory one likes to choose the retarded function as a rule. And finally quantum theory works with the causal Stückelberg-Feynman propogators which are retarded for positive energy particles but advanced for those of negative energy.

GENERAL INTERACTIONS IN CLASSICAL THEORY

The considerations given above used a very simple interaction structure. Most of these considerations can be effected if m^2 or g^2 were replaced by negative quantities. These cases correspond respectively to waves which travel faster - than - light and waves which carry negative physical variables. But despite these apparently strange properties

the calculations of the previous sections can be carried out.

The restriction to purely scalar interactions was made for the simplification of the algebraic steps. In fact the case $m^2 = 0$ has been treated by several authors both for electromagnetism and gravitation. Apart from a difference in appearance there is no significant difference. It is reassuring that the Maxwell-Lorentz equations of motion are recovered from the action - at - a - distance theory with the kernel

$$K_{ab} = e^2 \dot{x}_{a\mu} \dot{x}_b \delta([x_a - x_b]^2).$$

Not only are the successes reproduced but also the failures. Using an "absorber" we can get "radiation" which can equally well be obtained with the retarded boundary conditions and vanishing "in" fields.

The more significant generalization, one which is dictated by the requirements of consistency, is to consider a general two-body kernel. We shall again take up a scalar interaction model and simplify the calculation. Accordingly we start from a kernel K_{ab} which is symmetric, real and Lorentz invariant. If it is not factorizable in the form

$$K_{ab} = g_a g_b K([x - y]^2),$$

then there exists a collection of factorizable kernels of which K_{ab} is the resultant:

$$K_{ab} = g_a(\lambda) g_b(\lambda) K_\lambda([x - y]^2).$$

For every invariant function K_λ we introduce the spectral decom-

position (valid under relatively mild constraints);

$$K_\lambda\ ([x - y]^2) = \int_{-\infty}^{\infty} \rho_\lambda\ (k)\bar{\Delta}(x - y; \sqrt{k})\ dk.$$

where the weight function ρ_λ is real but not necessarily positive and the (K) integration goes from $-\infty$ to ∞. We have thus finally

$$K_{ab} = g_a(\lambda)g_b(\lambda)\int_{-\infty}^{\infty} \rho_\lambda(k)\bar{\Delta}(x - y; \sqrt{k})\ dk.$$

We must now introduce a family of fields $\phi(x,\lambda, k)$ which are coupled to the particles by

$$g_a = g_a(\lambda)\sqrt{\rho_\lambda(k)}.$$

This general form means that the direct action - at - a - distance can be replaced by a local coupling of the particles and the fields. Both faster - than - light and negative weight fields are in general included.

Not only may negative weight fields (i.e. those for which the spectral weight is negative) appear, but if the self-interactions are to be finite and thus are not to lead to an inconsistency, then they <u>must</u> be included. If we consider the static limit this is satisfied if the self-field at the origin vanishes; this is simply the integral over the total spectral weight. But if one allows for the propagation of the field before it is absorbed, it is clear that the detailed manner in which cancellations occur do depend on the velocity dependence (i.e. tensorial character) of the interaction.

We finally wish to note that an arbitrary velocity dependence could, at least formally, be expanded as a series of tensors in the velocities and would therefore correspond to a mixture of interactions. No essentially new point of principle seems to emerge from a consideration of these mixed forms

The problem of coupled systems can now be solved at least in a perturbative scheme. The complete trajectory can be obtained in terms of a power series expansion in the coupling constants. With suitable admixture of negative fields, the various orders of perturbations make sense. These are finite self-interactions effects; and in suitable cases such as the Whitehead-Schild model for gravitation, the inertia of the moving body may be changed ("curved space") by the interaction.

The conclusions we draw from the classical theory are the following:

(1) We can transform an action - at - a - distance theory of particles into an action - by - contact theory of particles coupled to fields.

(2) Self-interaction is a necessary adjunct to this demonstration of equivalence.

(3) There is no possibility of radiation emission from, or absorption by, a system of particles unless there are other particles present in the theory and they are arranged to have

no advanced fields at the location of the original set of particles.

(4) The degrees of freedom of these classical fields are really the degrees of freedom of the "absorber" particles which are of no interest to us otherwise.

(5) To make the general theory we may have to consider the coupling to a multitude of fields with all possible real values for the square of the mass form $-\infty$ to $+\infty$.

(6) Not only may some of the fields so introduced have negative weights, but the requirement of a meaningful self-interaction demands that there are many of them.

(7) Self-interaction effects are expected even within simple models of classical action - at - a - distance theories, but they are finite and calculable in perturbation theory.

QUANTUM MECHANICAL SYSTEMS WITH RELATIVISTIC ACTION - AT - A - DISTANCE

Our main interest is in relativistic quantum theory. In relativistic field theory local couplings have been used traditionally, but then we also have invariably the diseases of covariant local Lagrangian field theories. Consider the simple case of a spin $\frac{1}{2}$ (fermion) field ψ of (bare) mass M and a spin 0 (boson) field ϕ of (bare) mass m with local interaction; the action density is

$$\mathscr{L} = \bar{\psi}(i\not{\partial} - M)\psi + \frac{1}{2}(\partial^{\nu}\phi)(\partial_{\nu}\phi) - \frac{1}{2}m^2\phi^2 - \mathscr{H}, \quad \mathscr{H} = g\bar{\psi}\psi\phi.$$

In the interaction picture, the fields satisfy the free field equations and commutation relations:

$$(i\not{\partial} - M)\psi = 0, \quad \delta(x^0 - y^0)\,\{\psi_r^{+}(x), \psi_s(y)\} = \delta_{rs}\delta(\underset{\sim}{x} - \underset{\sim}{y})$$

$$(\partial^{\nu}\partial_{\nu} + m^2)\phi = 0, \quad \delta(x^0 - y^0)\,[\phi(x), \dot{\phi}(y)] = i\delta(\underset{\sim}{x} - \underset{\sim}{y}).$$

The transition amplitudes are defined in terms of the S-matrix

$$S = 1 + \sum_{n=0}^{\infty} \frac{i^n}{n!} \int d^4x_1 \ldots \int d^4x_n\, T(\mathscr{H}(x_1)\ldots\mathscr{H}(x_n)).$$

Using Wick's algebraic identities the time ordered products (denoted by T(....) above) are converted into a linear combination of normal products and these in turn lead to Feynman's graphical rules for computing scattering amplitudes. The amplitudes so calculated, at least in the lower orders of perturbation theory, can be obtained more directly by simply iterative substitution in the Heisenberg equations of motion obtained from the above action density.

The amplitudes so derived have two principal defects which must be resolved in some manner before physically meaningful conclusions can be drawn. The first concerns the fact that the interaction modifies the particle masses. The mass shifts themselves can, in principle, be computed as power series in the coupling constant. We must now view the action

density in the form

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{H}_0$$

$$\mathscr{L}_0 = \bar{\psi}(i\not{\partial} - M_0)\psi + \frac{1}{2}(\partial^\nu\phi)(\partial_\nu\phi) - \frac{1}{2}m_0\phi^2$$

$$\mathscr{H}_0 = g\bar{\psi}\psi\phi + (M_0 - M)\bar{\psi}\psi + \frac{1}{2}(m_0^2 - m^2)\phi^2,$$

and the S-matrix should be given with $\mathscr{H}_0$ considered as the interaction density. This mass renormalization is an <u>essential</u> step in the proper physical interpretation of the theory irrespective of the magnitude of the mass shifts.

The second and more serious difficulty is that except for some of the lowest order contributions, the contributions of the S-matrix are infinite in each order; and this comment also applies to the mass shifts. The source of this difficulty can be traced to the divergence of the integrals over the contraction function due to the confluence of the arguments, which in turn is directly to be traced to the local coupling postulated. And this appears in even more pronounced form for other interactions like the chiral four-fermion interaction in the theory of weak interactions.

There is, of course, a sense in which some physical predictions can be made in the theory. One shows that in a class of theories we can absorb all the divergence into renormaliza-

tions (albeit by an infinite amount) of the masses and the coupling constants. Quantum electrodynamics of electrons has been witness to the success of this program. But the method leaves too many questions unanswered. The mass shifts and effective coupling strengths are themselves physical quantities and should not be infinite: For example, it would be tempting to identify the mass difference between the charged and neutral pion to be due to the electromagnetic mass shifts. We must, accordingly, arrange to have a theory in which the renormalizations of the masses and coupling constants are themselves finite. This would involve a modification of the theory so that the propogation functions for the fields fall off faster for larger values of the momenta.

Similar conclusions would be arrived at if we started out with an action - at - a - distance theory. In this version we start from an action:

$$= \int \bar{\psi}(x)(i\not{\partial} - M)\psi(x)d^4x + \iint \bar{\psi}(x)\psi(x)K(x-y)\bar{\psi}(y)\psi(y)d^4x\, d^4y,$$

involving the coupling between the fermion fields at the points and y through the real symmetric kernel $K(x - y)$. For simplicity let us first consider

$$K(x - y) = g^2\, \bar{\Delta}(x - y).$$

In this case we can define the (auxiliary) field

$$\phi(x) = g \int \bar{\Delta}(x - y)\bar{\psi}(y)\psi(y)\, d^4y.$$

Then the equations of motion can be rewritten

$$(i\not{\partial} - M)\psi(x) = g\phi(x)$$

$$(\partial^\nu \partial_\nu + m^2)\phi(x) = g\bar{\psi}(x)\psi(x)$$

In other words, the symmetric interaction of the scalar densities at the points x and y is equivalent to the local coupling to the meson field $\phi(x)$.

We find, as in the classical theory, a Green's function for the propogation of the ϕ field which is different from the standard contraction function. We have already remarked that the identification of the suitable Green's function is equivalent to the specification of the boundary conditions. The standard Green's function for propogating meson fields is the causal Stückelberg-Feynman contraction function

$$\langle T\{\phi(x)\phi(y)\}\rangle_0 = \Delta_F(x - y) = \bar{\Delta}(x - y) + i\pi\Delta^{(1)}(x - y).$$

This Green's function can be obtained by starting with the $\bar{\Delta}$-function and choosing the boundary condition such that the "in" field is the free field plus a term

$$\phi_2(x) = ig\pi \int \Delta^{(1)}(x - y)\bar{\psi}(y)\psi(y)d^4y.$$

In other words, the S-operator

$$S = 1 + \Sigma_{n=0}^{\infty} \frac{i^n}{n!} \quad d^4x_1 \ldots \quad d^4x_n T\mathcal{H}(x_1)\ldots\mathcal{H}(x_n)$$

ields the scattering amplitudes provided the matrix elements re taken not for the field $\phi(x)$ but

$$\phi_{in}(x) = \phi(x) - i\pi g \int \Delta^{(1)}(x - y)\bar{\psi}(y)\psi(y)d^4y,$$

nd the quantization is taken with respect to $\phi_{in}(x)$ rather (x).

We are now in a position to discuss the general scalar ker-el.

$$K(x - y) = \int_{-\infty}^{\infty} \rho(k)\bar{\Delta}(x - y;\ k)dk.$$

long as $\rho(k)$ is positive, a field of mass $\sqrt{k}$ may be intro-ced along with it with the coupling constant

$$g(k) = \sqrt{\rho(k)}$$

$\rho(K)$ is negative we have to associate it with a field antized according to the indefinite metric:

$$\delta(x^0 - y^0)[\phi(x),\ \partial^0\phi(y)] = -i\delta(\vec{x} - \vec{y})$$

r such a quantization scheme, the states with an even number quanta of the field will have positive norm and the states th an odd number of quanta will have negative norm. The eory of the quantization according to the indefinite metric now too well known to be reviewed here. It suffices to say at the corresponding contraction functions will have oppo-te signs. An immediate consequence of this is that if we

have a linear superposition of fields of various masses making up a linear field

$$\phi(x) = \Sigma C_r \, \phi_r(x)$$

the effective contraction function is

$$\langle T(\phi(x)\phi(y))\rangle_0 = \Sigma_r |C_r|^2 \eta_r \Delta_F(x - y;\, m_r),$$

where η_r is the "metric" of the field ϕ_r which has the value +1 for ordinary fields and -1 for fields with indefinite metric. By choosing

$$C_r = |\rho(k)|^{1/2}$$

$$\eta_r = \rho(k)/|\rho(k)|,$$

we can therefore, reproduce the scalar kernel $K(x - y)$ that we started out with, at least as far as the real part is concerned. (We have already remarked that the imaginary part of Δ_F can be obtained by a suitable choice of boundary conditions).

For a fermion field of spin 1/2, apart from inessential numerical factors, the propogator is

$$S(p,m) = (p,m) = (\not{p} - m + i\varepsilon)^{-1}.$$

For a linear combination

$$\psi(x) = C_r \, \psi_r(x)$$

the effective propogator is

$$S(p) = |C_r|^2 \eta_r (\not{p} - m + i\varepsilon)^{-1}.$$

If we take

$$\eta_2 = \eta_3 = -1, \qquad \eta_1 = +1; \qquad (C_2/C_1)^2 = \frac{m_1^2 - m_3^2}{m_2^2 - m_3^2};$$

$$(C_3/C_1)^2 = \frac{m_1^2 - m_2^2}{m_3^2 - m_2^2},$$

$$S(p) = |C_1|^2 (m_1^2 - m_2^2)(m_1^2 - m_3^2)(\not{p} - m_1 + i\varepsilon)^{-1}(\not{p} - m_2 + i\varepsilon)^{-1} \times$$
$$\times (\not{p} - m_3 + i\varepsilon)^{-1}$$

so that $S(p) \sim p^{-3}$ for large values of p. This softening of the high momentum components leads to a softening of the small distance behavior of the kernel $K(x - y)$ and to convergence in perturbation theory of processes involving the fermion fields.

For boson fields (of spin 0) the situation is even simpler. Apart from inessential factors

$$\Delta(p,m) = (p^2 - m^2 + i\varepsilon)^{-1}.$$

Hence, if we take

$$\phi(x) = \phi_1(x) + \phi_2(x); \; \eta_1 = \eta_2 = +1,$$

we get the effective propogator

$$S(p) = (m_1^2 - m_2^2)(p^2 - m_1^2 + i\varepsilon)^{-1}(p^2 - m_2^2 + i\varepsilon)^{-1}.$$

Since this propogator also goes as p^{-4} asymptotically, convergence in each order for any interaction is guaranteed.

At this point, we may profitably compare our findings with the classical theory and verify that the situation is quite similar. In quantum theory we must pay particular attention to the question of the conservation of probability in all physical processes.

UNITARITY QUESTIONS AND THE THEORY OF SHADOW STATES

If the quanta of the abnormal fields were not present either in the initial or final state of any physical process there should be no difficulties with the probability interpretation. Even if they are neither present initially nor finally in a particular process the unitarity condition would demand in general that any intermediate state in which there are abnormal quanta should contribute its share to the imaginary part of the amplitude. We should therefore require that the imaginary part of any physical amplitude gets contributions only from states containing no abnormal quanta. Once this is done all questions of probability conservation ("unitarity") would be automatically taken care of, since the states containing abnormal quanta would now no longer be counted amongst physical states.

To illustrate the notion of shadow states and the role that they play in a quantum theory, let us take a simple quantum mechanical system with the Hamiltonian $\mathcal{H}=\mathcal{H}_0 + V$ where V is a suitable interaction. If we were to study the interaction representation, the state vector $|\phi(t)\rangle$ in the interaction representation has the time dependence

$$i \frac{\partial}{\partial t} \phi(t)\rangle = V_I(t)|\phi(t)\rangle$$

where

$$V_I(t) = \exp(i\mathcal{H}_0 t) V \exp(-i\mathcal{H}_0 t)$$

(We shall assume that the division between $\mathcal{H}_0$ and V is so arranged that $\mathcal{H}$ and $\mathcal{H}_0$ have the same spectra.) The formal solution to this equation is

$$|\phi(t)\rangle = T(\exp[i \int_{-\infty}^{t} V_I(t^1)dt^1])|\phi(-\infty)\rangle$$

$$= \{1 + \sum_{n=1}^{\infty} \lambda^n \frac{i^n}{n!} \int_{-\infty}^{t} dt_1 \int_{-\infty}^{t1} dt_2 \ldots \int_{-\infty}^{t_{n-1}} dt_n V_I(t_1)\ldots V_I(t_n)\}|\phi(-\infty)\rangle$$

$$\equiv \{1 + \sum_{n=1}^{\infty} \lambda^n \frac{i^n}{n!} \int_0^t dt_1 \ldots \int_0^t dt_n T(V_I(t_1)\ldots V_I(t_n)\}|\phi(-\infty)\rangle$$

If $|\phi(t)\rangle$ is the corresponding Schrödinger state vector then

$$|\phi(t)\rangle = e^{-i\mathcal{H}_o t}|\phi(t)\rangle,$$

then its solution is

$$|\phi(t)\rangle = \{e^{-i\mathcal{H}_o t}\sum_{n=0}^{\infty} i^n \int_{-\infty}^{t} dt_1\, e^{i\mathcal{H}_o t_1} V e^{-i\mathcal{H}_o t_1} dt_2 e^{i\mathcal{H}_o t_2} V e^{-i\mathcal{H}_o t_2} \times$$

$$\ldots \int_{-\infty}^{t_{n-1}} dt_n e^{i\mathcal{H}_o t_n} V e^{-i\mathcal{H}_o t_n}\} \, |\phi(-\infty)\rangle.$$

Let us now take the exact eigenstates of the total Hamiltonian

$$|E\rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{iEt}|\Psi(t)\rangle\, dt$$

$$= \sum_n i^n \int_{-\infty}^{0} e^{i(E-\mathcal{H}_o)\tau_1} V d\tau_1 \int_{-\infty}^{0} e^{i(E-\mathcal{H}_o)\tau_2} V d\tau_2$$

$$\ldots \int_{-\infty}^{0} e^{i(E-\mathcal{H}_o)\tau_n} V d\tau_n \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i(E-\mathcal{H}_o)\tau} V d\tau |\phi(-\infty)\rangle.$$

This could be rewritten in the form

$$|E\rangle = \sum_n \frac{1}{E-\mathcal{H}_o+i\varepsilon} V \frac{1}{E-\mathcal{H}_o+i\varepsilon} V \ldots V \frac{1}{E-\mathcal{H}_o+i\varepsilon} V \Pi_E \times |\phi(-\infty)\rangle$$

where

$$\Pi_E = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i(E-\mathcal{H}_o)\tau} d\tau,$$

is the projection operator to energy E of the unperturbed Hamiltonian. If we note

$$\Pi_E \, |\Phi(-\infty)\rangle = |\Phi_E\rangle,$$

we have the final expression

$$|E\rangle = \{1 + \sum_{n=1}^{\infty} \frac{1}{E-\mathcal{H}_o+i\varepsilon} V \quad {}^n\}|\Phi_E\rangle.$$

This expression, which looks typically "nonrelativistic" will be completely equivalent to the covariant Feynman rules if we choose V and $_o$ to be appropriate integrals of the action densities of a local field theory. The scattering amplitude is obtained by taking the energy conserving matrix elements of the summand in the expression for $|E\rangle$, so that we get

$$F_{\beta\alpha}(E) = \langle\beta|VGV|\alpha\rangle = \langle\beta|V \frac{1}{E-\mathcal{H}_o+i\varepsilon} V \frac{1}{E-\mathcal{H}_o+i\varepsilon} V..V|\alpha\rangle$$

We note that the scattering amplitude so obtained satisfies the unitarity relation which may be written in the symbolic form

$$F_{\beta\alpha} - F^*_{\beta\alpha} = \langle\beta|V \frac{1}{E-\mathcal{H}_o+i\varepsilon} V...V|\alpha\rangle - \langle\beta| V \frac{1}{E-\mathcal{H}_o-i\varepsilon} V..V|\alpha\rangle$$

$$= \langle\beta|V \frac{1}{E-\mathcal{H}_o+i\varepsilon} V...V|\nu\rangle\delta(E-E_\nu)\langle\nu|V \frac{1}{E-\mathcal{H}_o-i\varepsilon} V..V|\alpha\rangle$$

$$= \sum_\nu F_{\beta\nu} F^*_{\nu\alpha} \, 2\pi i\delta(E-E_\nu),$$

where, to avoid inessential elaboration, we have considered time-reversal invariance to be valid so that V is real; and

anomalous thresholds have been ignored. The imaginary part of the amplitude corresponds to physical intermediate states.

We observe that if we were interested only in constructing a solution to the fully interacting problem we could write

$$|E\rangle = \{1 + \sum_{n=1}^{\infty} (G_o(E)V)^n\}|\Phi_E\rangle,$$

where $G_o(E)$ is <u>any</u> Green's function for the Hamiltonian $\mathcal{H}_o$:

$$(E - \mathcal{H}_o)G_o(E) = 1.$$

The solution we deduced above corresponds to the retarded Green's function

$$G_o(E) = (E - \mathcal{H}_o + i\varepsilon)^{-1}$$

If we chose instead the time-symmetric Green's function

$$G_o(E) = \frac{1}{2}(E-\mathcal{H}_o+i\varepsilon)^{-1} + \frac{1}{2}(E-\mathcal{H}_o-i\varepsilon)^{-1} = P\,\frac{1}{E-\mathcal{H}_o},$$

the resultant "scattering amplitude" would be entirely real:

$$F_{\beta\alpha}(E) - F^*_{\beta\alpha}(E) = 0.$$

This would not satisfy unitarity if any physical scattering were to take place.

Suppose we partition the set of all states of the quantum mechanical system into two classes: those of the first class would be called the "physical states" and those of the

second the "shadow states;" we must make sure that this partitioning is compatible with the Hamiltonian $\mathcal{H}_o$. Hence the Green's function $G_o(E)$ is diagonal with respect to this partition:

$$G_o(E) = \left(\begin{array}{c|c} G_{op}(E) & 0 \\ \hline 0 & G_{os}(E) \end{array}\right)$$

The interaction V would in general have both diagonal and non-diagonal elements:

$$V = \left(\begin{array}{c|c} V_{pp} & V_{ps} \\ \hline V_{sp} & V_{ss} \end{array}\right)$$

Let us now choose the Green's function which is retarded with respect to the "physical states" and time-symmetric with respect to the "shadow states." We write:

$$G_o(E) = \begin{pmatrix} \left(\dfrac{1}{E - \mathcal{H}_o + i\varepsilon}\right)_{pp} & 0 \\ 0 & \left(\dfrac{P}{E - \mathcal{H}_o}\right) \end{pmatrix}$$

With this choice of Green's function we get an expression for the scattering amplitude which obtains contributions both from the "physical states" and the "shadow states." However, if we

compute the imaginary part of the amplitude in the physical domain we get the result

$$F_{\alpha\beta} - F^*_{\alpha\beta} = 2\pi i \sum_{\nu}{}' F_{\alpha\nu} F^*_{\alpha\nu}$$

with the primed summation going only over the physical states. In other words, the scattering involves only the physical states, even though the dynamics involve both the physical states and the shadow states.

Having arrived at this juncture we can now deal with the physical interpretation of indefinite metric theories. We use the usual diagrammatic approach to perturbation theory but any intermediate state which contains one or more abnormal particles should be treated as a shadow state. This implies that we subtract out the imaginary part coming in from any such intermediate state. This is a consistent procedure and may be carried out in any order of perturbation theory. The shadow states would automatically exclude themselves from the unitarity sum.

We may now remove the restriction that anomalous thresholds do not appear. Even in such a case, we must recognize that the scattering amplitude defined by the perturbation series is defined only for physical energies. In this domain the imaginary parts are given by the unitarity relation. The

analytic continuation of the scattering amplitude may contain imaginary parts elsewhere but they will not be in the physical domain. It follows that anomalous thresholds may exist even after the shadow states are eliminated.

It is remarkable to note that despite the radically new approach to the theory of the scattering amplitude in the low energy domain (before the opening of the thresholds for the shadow states) the local analytic properties of the scattering amplitude in this theory are exactly the same as in the usual (inconsistent!) local field theory. It follows that the usual results of quantum field theory like the connection between spin and statistics, the relative parity of particles and antiparticles, the TCP theorem as applied to masses and lifetime of particles, crossing symmetry in the small etc. are all equally true in the present theory. The essential difference lies in the fact that in the present theory the scattering amplitude is not a single analytic function, but is only piecewise <u>analytic</u>, it is continuous for physical values (since the shadow state constirubitons have the usual threshold behavior.)

Since the scattering amplitudes are only piecewise analytic (even though we have the usual unitarity relationships including the one between total cross section and the imaginary

part of the forward scattering amplitude), it is not easy to see to what extent dispersion relations should hold exactly. To the extent that the forward dispersion relations do hold, we have to assume that the onset of the shadow states is only at sufficiently high energies.

We are now in a position to deal with the quantum theory of relativistic action - at - a - distance! The auxiliary fields that we have introduced (whether they be of positive weight or of negative weight) are all associated with the time-symmetric (one-particle) propagators. They are therefore not candidates for being considered as physical quanta; we might consider any state in which at least one of these quanta is included as a shadow state. This is as it should be since we are interested in considering the theory of action - at - a - distance! We proceed as before and exclude the imaginary part coming from any intermediate state involving any of these auxiliary quanta. Since the contribution (in the physical domain) came only from states which can be realized, without any violation of the action - at - a - distance framework we can use the usual rules of Feynman diagrams, provided it be understood that the imaginary parts coming from shadow intermediate states is to be subtracted out.

We have now come to a new conclusion! We can construct finite relativistic quantum theories, the effects of interaction including self-interaction and obtain meaningful finite answers. Action - at - a - distance and indefinite metric theories turn out to have basically the same theoretical framework so long as it is realized that the identification of the physical amplitudes is part of the dynamical problem. We now have the solution to the basic problem posed centuries ago: Is it action - by - contact or action - at - a - distance? The answer is: they are only two ways of looking at the same theory. We have now moreover the complete framework for the theory. While the earlier work on finite quantum electrodynamics is a contribution in this general context much more detailed work is necessary to implement the programme in particle theory. Perhaps this will be accomplished during the next decade.

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