Dipole Form Factors*

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It is shown that a mechanism recently proposed by Hammer and Weber to obtain dipole behavior for the nucleon electromagnetic form factors is incorrect.

The complex poles which describe the effects of resonances on propagators and scattering amplitudes have long been known to occur in complex-conjugate pairs. However, as long as one is interested in a domain above threshold on the physical sheet, it has been recognized that only one of these poles is relevant to an approximation by nearest singularities. The reason for this lies in the fact that while one of the two resonance poles lies close to the physical region, the route to the second is considerably longer, requiring that one pass through the unitarity cut, travel to the unitarity threshold, encircle it, and finally return along the opposite side of the cut to the second pole. In view of this considerable disparity in the lengths of these paths (assuming one is not too close to threshold) dominance by a single pole is an entirely reasonable phenomenon.

Recently, however, a somewhat different region of the physical sheet has been examined by Hammer and Weber.[1] They consider the small-momentum-transfer region of the nucleon electromagnetic form factors and study the effect of an unstable vector meson (e.g., the ρ) on the vector-meson propagator. Since the low-momentum-transfer region lies below the unitarity threshold, one can reach each of the two complex-conjugate poles by paths of equal length and the assertion of Ref. 1 that the two poles are of equal importance is therefore certainly correct. It is nontheless the aim of this note to point out that the suggestion of these authors that dipole form factors follow from this observation is not tenable.

In order to display this result we consider the propagator $G(p^2)$ of a scalar field[2] whose lowest threshold at $4M^2$ refers to a two-particle cut. It will be convenient to consider only this two-particle contribution to the unitarity cut and thus write for $-p^2 > 4M^2$

$$\frac{1}{2\pi i}[G^{-1}(-p^2=s-i\epsilon)-G^{-1}(-p^2=s+i\epsilon)]=Z_2{}^2\frac{g_0{}^2}{16\pi^2}\left(\frac{s-4M^2}{s}\right)^{1/2}|\Gamma(s)|^2, \tag{1}$$

where $\Gamma(s)$ is the unrenormalized vertex function. Equation (1) allows one to write for $G^{-1}(p^2)$ the representation

$$G^{-1}(p^2)=p^2+\mu_0{}^2-Z_2{}^2\frac{g_0{}^2}{16\pi^2}\int_{4M^2}^{\infty}\frac{ds}{p^2+s}|\Gamma(s)|^2\left(\frac{s-4M^2}{s}\right)^{1/2},$$

which upon performing a subtraction at $p^2=-4M^2$ becomes

$$G^{-1}(p^2)=p^2+\mu_0{}'^2+Z_2{}^2\frac{g_0{}^2}{16\pi^2}(p^2+4M^2)\int_{4M^2}^{\infty}\frac{ds}{p^2+s}\frac{|\Gamma(s)|^2}{[s(s-4M^2)]^{1/2}}.$$

For the low-energy phenomena of interest here it is sufficient to replace $\Gamma(s)$ by $Z_1{}^{-1}$ and $s^{1/2}$ by $2M$. One then obtains after straightforward integration

$$G^{-1}(p^2)=p^2+\mu_0{}'^2+g_0{}^2\frac{Z_2{}^2}{Z_1{}^2}\frac{(p^2+4M^2)^{1/2}}{32M\pi}, \tag{2}$$

which is seen to be an entire function in the complex $(p^2+4M^2)^{1/2}$ plane.[3] The quadratic expression (2) in this variable thus has two zeros which are either both real or a complex-conjugate pair. Following Ref. 1 we assume the parameters which appear in (2) to be such that one has complex-conjugate poles, and consequently that[4]

$$G^{-1}(p^2)=[(p^2+4M^2)^{1/2}-\alpha][(p^2+4M^2)^{1/2}-\alpha^*], \tag{3}$$

where the condition

$$\alpha + \alpha^* = -g_0^{\,2} \frac{Z_2^{\,2}}{Z_1^{\,2}} \frac{1}{32M\pi^2}$$

implies

$$\mathrm{Re}\alpha < 0,$$

thereby assuring that the poles of $G(p^2)$ lie on the unphysical sheet $\mathrm{Re}(p^2+4M^2)^{1/2}<0$.

For p^2 small and real as considered in Ref. 1, one readily sees from (3) that the two poles in the variable $(p^2+4M^2)^{1/2}$ contribute equally. On the other hand, $G^{-1}(p^2)$ can also be written

$$G^{-1}(p^2) = [(p^2+4M^2)^{1/2} - \mathrm{Re}\alpha]^2 + [\mathrm{Im}\alpha]^2. \tag{4}$$

Since the poles of $G(p^2)$ occur for

$$-p^2 = 4M^2 + (\mathrm{Im}\alpha)^2 - (\mathrm{Re}\alpha)^2 \pm 2i\,\mathrm{Re}\alpha\,\mathrm{Im}\alpha,$$

it follows that for a pole above the two-particle threshold

$$|\mathrm{Im}\alpha| > |\mathrm{Re}\alpha|,$$

and that a narrow resonance corresponds to small $\mathrm{Re}\alpha$. Although $G(p^2)$ cannot now have a finite-order pole structure in the variable p^2 (as there is no stable single-particle state), it is clear that for small (or spacelike) p^2 and a relatively narrow resonance the form (4) is approximated by

$$G^{-1}(p^2) = p^2 + 4M^2 + (\mathrm{Im}\alpha)^2,$$

i.e., the usual single pole approximation. This result thus contradicts the conclusions of Hammer and Weber and shows that the mechanism they propose cannot yield the desired dipole form factor.[5]

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[1]C. L. Hammer and T. A. Weber, Phys. Rev. Lett. 28, 1675 (1972).

[2]The fact that the authors of Ref. 1 consider the vector-meson case has no bearing on the general conclusions obtained here.

[3]It may be helpful to note here that $(p^2+4M^2)^{1/2}$ plays the role of $-ik$ in the nonrelativistic case.

[4]It should be emphasized that although the reduction of the propagator to the form (3) has been carried out by a series of approximations in order to proceed from first principles, these approximations are well known to be exact in the case of certain soluble theories such as the Lee model. On the other hand, for purposes of discussing Ref. 1, one could just as well have started by assuming (3) provided that due regard is given to the fact that the poles are in the variable $(p^2+4M^2)^{1/2}$ rather than its square. Viewed in this way one sees that the derivation of (3) was primarily intended to emphasize the important role of that variable.

[5]We have not considered here the possibility that a second pole could arise from the vertex function in such a way that it, together with the propagator pole, could lead to an effective dipole form. This, however, is in keeping with the procedure in Ref. 1 in which only propagator singularities are considered.