QUANTUM THEORY WITH SHADOW STATES: A SEPARATE REALITY

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ABSTRACT

Starting with an analysis of the natural (but not inevitable!) assumptions of conventional quantum theory the desirability of a generalization is highlighted. I give an exposition of a new scattering theory in which a new class of states enter; these states are relevant for the dynamical description but do not contribute to probability. The transition amplitude in this theory is calculated; it obeys the unitarity relation. Its relation to the standard scattering amplitude enables us to calculate it by simple methods and to study its piecewise analyticity. Certain causality questions are discussed and certain paradoxes resolved. A number of deserving candidates for the role of shadow particles are listed. It is suggested that perhaps "general interpretational principles of quantum theory" are not as general as they should be or could be.

INTRODUCTION: THE NATURE OF DYNAMICAL INTERACTIONS

It is a matter of everyday experience that bodies exchange energy and momentum on collisions. These impulsive interactions have led physicists to postulate action-by-contact as a fundamental law of interaction. The question naturally arises whether all interactions can be brought about in this fashion. Again, it is a matter of everyday experience that there are interactions between objects which do not collide: like the action of gravity or electricity. To bring these under the mantle of action-by-contact we introduce the gravitational or electric "field" which interacts with material bodies by contact. By thus enlarging the set of "physical objects" we can say all these interactions are only by virtue of action-by-contact.[1]

We observe that for these interactions apparently at-a-distance the mediation is by a field. And fields and particles are different kinds of objects in classical physics.

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In quantum theory fields and assemblies of identical particles describe the same physical situation; at this stage we may say that all interactions are "by contact" though now we include under the notion of contact creation and destruction of particles. Particle spectra and particle interactions are now no longer independent but are two aspects of the same set of phenomena. We may refer to the observation of particles both directly (by kinematic studies on suitable entities) or indirectly (by seeing their dynamical effects in interactions between other particles). It is then an article of faith, supported by some empirical evidence, that all particles seen indirectly must be seen directly. Attractive and economical is this item of faith: so much so that we usually raise it to the status of a guiding principle.

Despite this temptation there are several reasons to be cautious. Perhaps the most compelling one is that local relativistic field theory inevitably leads to infinities in any honest attempt to calculate the dynamical consequences of any postulated interaction including the quantum electrodynamic interaction. There are ways of extracting useful predictions regarding some physical quantities which are in excellent agreement with experimental results; but nevertheless there are quantities for which the predictions yield infinities. The root of these infinities can be traced to the infinite number of degrees of freedom[3] of fields and to the local coupling which involves all of them.

The kind of troubles one finds may be illustrated by a point particle in interaction with other fields, say the electron in interaction with the electromagnetic field. An honest assessment of the theory suggests that the electron should explode; in other cases instead of an explosion an implosion is predicted. Fortunately electrons and other particles neither explode nor implode! But we must search for a generalization of the local interaction model which incorporates this happy state of affairs. Possibly non-local field theories?

If the principle of deriving all interactions by exchange of particles embodied in local quantum field theory leads to difficulties we can ask: could we include additional forces not derivable from exchange of particles? Can we have genuine non-local interactions?

I have studied non-local theories within the framework of special relativity.[2] Since manifest relativistic invariance for a non-local theory implies both retarded and advanced interactions the conserved quantities of energy, momentum and angular momentum involve non-localized quantities. They can however be rendered local within an extended formalism in which new auxiliary fields are introduced which are coupled to the particles and fields of the theory. Thus, apart from certain additional boundary conditions pertaining to these auxiliary fields that are to be incorporated, the non-local theory becomes an extended local interaction theory.

But we might ask: what about the convergence that arises in theories with suitable non-locality? How does that come about in

the extended local field theory? The answer to this is that the extended field theory contains "negative weight" fields whose quantum theory is to be formulated in a linear vector space with an indefinite scalar product. Such "quantum theories with indefinite metric" have been studied for a long time,[3] but the resolution of the paradox of negative probabilities has come only recently.

Even more generally, we can ask the question: is the world of dynamics of interacting elementary entities composed of (observed) particles only? Are all shadows cast by entities with substance?

In the following talk Professor Stapp[4] refers to "...the primitive idea that the world consists only of physical objects, and that these objects act on each other only by direct contact." He then refers to the generalization in which we allow other interactions to be present provided they fall off with distance like a power law or an exponential law. He chooses the dynamical assumption: "It is assumed that all interactions not carried by physical objects fall off at least exponentially under space-time dilation."

But I like to consider the possibility that the world of elementary entities consists of both ordinary physical objects which are directly seen and non-ordinary objects which can be seen only indirectly. This "separate reality" simplifies the formulation of the dynamical picture of the world.

If we take such a world full of shadows with and without substance and insist on a pragmatic world-view with only the ordinary physical objects the price we pay for it is a non-local interaction which falls off only according to a power law. This leads to continuous but piece-wise analytic transition amplitudes in quantum field theory.[2,4]

I have a number of aesthetic reasons to prefer considering the world to include such nonordinary objects as shadows without substance, to consider the world to be full of mystery.[5,6] I cannot ask that you share this predilection with me. But I can, and I will, ask that you agree with me that this possibility is worth investigating if only for you to reassure yourself that it is not useful. But I hope that you find it otherwise.

In this paper I aim to show that not all particles "seen" need be seen directly. This departure from tradition is not to be undertaken lightly, and one should be critical of the prediction of only piece-wise analyticity. The departure consists in an altered perspective rather than in an unusual interaction structure. The possibility of such a nonordinary perspective relating to scattering and transition processes obtains since the determination of transition amplitudes involves not only the natural evolution (the "free Hamiltonian") but also the boundary conditions. It is in the recognition of this freedom that my theory of shadow states[2,7] has its foundation; and utilization of this freedom gives form to the theory. While local quantum field theory with indefinite metric[3] was the motivation I would like you to see the theory in its generality, to see the possibility that there could be particles which affect the dynamics without being seen directly irrespective of whether these particles have positive or negative norm states.

QUANTUM THEORY OF SCATTERING

Consider a quantum system described by a total Hamiltonian

$$H = H_o + gV.$$

For the time being we need not distinguish between quantum field theories and quantum mechanical systems with a finite number of degrees of freedom except to note that in the former case the various terms would be integrals over all space of corresponding densities. H_o is chosen so that it has the same spectrum as H and with the same multiplicity. [In the field-theoretic case this implies that all mass renormalization terms are included in V.] Let us now choose corresponding (improper) eigenvectors χ, ϕ of H and H_o:

$$H\chi = E\chi \ ,$$

$$H_o\phi = E\phi \ .$$

A formal choice of χ is as a solution of the equation:

$$\chi = \phi + gGV\chi$$

where G is Green's function obeying

$$(E - H_o)G = 1.$$

We verify that this is so by showing that

$$(H - E)\chi = (H - E - gV)\chi + g(E - H_o)\ GV\chi$$

$$= (H_o - E)(\chi - gGV\chi) = (H_o - E)\phi = 0 \ .$$

The "free state" ϕ and the "fully interacting state" χ differ and this difference $\chi - \phi$ may be thought of as a scattered wave:

$$\psi = \chi - \phi = gGV\chi \ .$$

It may be thought of as a free wave generated by a source:

$$\xi = (E - H_o)\psi = V\chi \ .$$

The "scattering" (transition) amplitude from an initial state i to a final state f is then given by

$$T_{fi} = \langle f|T|i\rangle = (\phi_f, gV\chi_i) \ .$$

This expression can be rewritten in a slightly different form by writing down the formal solution:

$$\chi = \phi + gGV\phi + g^2GVGV\phi + \ldots$$
$$= (1 - gGV)^{-1}\phi \ .$$

Then

$$\xi = V(1 - gGV)^{-1}\phi = (1 - gVG)^{-1}V\phi$$

and

$$T_{fi} = (\phi_f, V(1 - gGV)^{-1}\phi_i) = \langle f|V(1 - gGV)^{-1}|i\rangle$$

so that

$$T = V(1 - gGV)^{-1}$$

is the matrix of transition amplitudes. All these calculations are standard and they are the generalization of the usual method of taking the coefficients of the asymptotic diverging spherical waves in the usual elementary wave mechanical description of scattering.[8] The scattering is dependent on the interaction gV as well as the free Hamiltonian H_o.

What is important to recognize is that there is still some freedom in defining the correspondence $\chi \to \phi$ and hence of the "source of the scattered wavelets" ξ. This freedom stems from the freedom in the choice of the Green's function G. We may choose <u>any</u> Green's function; all the relationships would continue to hold as long as the defining relation

$$(E - H_o)G = 1$$

is maintained. As G changes so do ξ and T. <u>The scattering is defined not only by H_o and H but also the specific choice of G.</u> Since G may be identified with the propagation function, we can interpret this freedom by remarking that scattering depends not only on the interaction but also the manner in which the wavelets propagate[9]: and that is eminently reasonable!

It is natural to ask at this stage as to why this freedom does not seem to show up in the familiar elementary derivation of the scattering of a plane wave by a potential. The solution is unique since we take it for granted that the propagation of the scattered wavelets is forward-in-time and so it is mandatory to choose the so-called retarded Green's function. We choose as the solution χ of the exact Hamiltonian a state tending to a plane wave ϕ in the far past, and to the plane wave ϕ plus diverging spherical waves ψ in the far future. Here the choice is already made and no freedom is left.

The theory of shadow states[2,7] has its origin in the recognition of this freedom; and in the willingness to keep an open mind about whether all particles have to obey this boundary condition.

The transition amplitude matrix T satisfies certain non-linear conditions which incorporate the law of conservation of probability in the elementary theory of scattering. They are therefore called the generalized "unitarity" condition. These relations relate to the difference between T and $T^{\dagger}$. We have,

$$T^{\dagger} - T = gV\{(1 - gG^{\dagger}V)^{-1} - (1 - gGV)^{-1}\}$$

$$= gV(1 - gG^{\dagger}V)^{-1}(G^{\dagger} - G)gV(1 - gGV)^{-1} .$$

In writing down this relation we have made use of the hermicity of the interaction gV. The relation could be simplified to read:

$$T^{\dagger} - T = T^{\dagger}(G^{\dagger} - G)T .$$

This non-linear relation depends on the antihermitian part of the Green's function G. When we make a new choice for G we get a new relationship for T. This equation is the generalization of the "optical theorem" which relates the imaginary part of the elastic forward scattering amplitude to the cross section.[8] The optical theorem is directly related to probability conservation. Hence any freedom in the choice of the Green's function and the consequent change in the generalized unitarity relation entail a new probability interpretation. The possibility of discounting certain states from contributing to the probability arises out of this circumstance.

Let us, therefore, pay special attention to the choice of Green's function. The most familiar one is to choose the retarded Green's function:

$$G_R = (E - H_o + i\varepsilon)^{-1} .$$

For this choice

$$T_R^{\dagger} - T_R = 2\pi i T_R^{\dagger} T_R .$$

Together with time-reversal invariance this leads to the optical theorem. The probability interpretation is then the usual one with all states contributing to the physical probability. All states are physical states; and the physical probability that is summed over all these states is conserved. We may well say: all virtual states can become real. Or equally well: the world is made up of physical states only.

A NEW SCATTERING THEORY

Now let us be adventurous and explore other possibilities. Let us separate the vector space of states $\mathcal{W}$ for world into two subspaces $\mathcal{R}$ (for real) and $\mathcal{S}$ (for shadow) by means of a projection operator σ:

$$\mathcal{S} \quad \{\sigma\psi \mid \quad \psi\varepsilon\, \mathcal{W}\},$$

$$\mathcal{R} \quad \{(1-\sigma)\psi \mid \psi\varepsilon\, \mathcal{W}\}.$$

Since

$$\sigma^2 = \sigma = \sigma^\dagger ,$$

it follows that these two subspaces are orthogonal. We take care to have σ commute with the free Hamiltonian H_o so that $\mathcal{R}$ and $\mathcal{S}$ are invariant under the free Hamiltonian evolution. In a relativistic theory we may choose σ to commute with the generators of a larger invariance group including the "free" inhomogeneous Lorentz group. There is now the possibility of choosing the Green's function

$$\begin{aligned} G &= \sigma\bar{G} + (1-\sigma)G_R \\ &= 1/2\ \sigma(G_R + G_A) + (1-\sigma)G_R , \end{aligned}$$

where

$$G_A = (E - H_o - i\varepsilon)^{-1} .$$

Equally well we may write

$$G = (E - H_o + (1-\sigma)i\varepsilon)^{-1} .$$

The transition amplitude depends on σ in a non-linear manner and we must now seek out the proper probability interpretation associated with the framework.

Before doing this we note one point: the "scattered wave" ψ for an "incident wave" ϕ is now different from what it would have been with the standard (retarded) choice for G. This includes some advanced waves which are expected from the advanced components in the Green's function we have chosen. Hence the "free" state ϕ and the state χ in the far past no longer coincide; but the lack of this coincidence depends on the space $\mathcal{S}$ of shadow states. The correspondence between the "free states" ϕ and the "interacting states" χ is now more subtle: but it is of course well defined when we have settled on the projection σ.

We now complete the formulation of the new scattering theory by specifying the new probability interpretation. We compute the probability using $(1-\sigma)$ as the metric operator. The entire subspace of states $\mathcal{S}$ now correspond to zero contribution to the probability while the subspace of states $\mathcal{R}$ has the usual probability interpretation. There are no probability amplitudes connecting $\mathcal{R}$ and $\mathcal{S}$. The space $\mathcal{S}$ of states, devoid of probability interpretation, is a part of the world that is a mathematical auxiliary.

TRANSITION AMPLITUDES IN SHADOW STATE THEORY

One feature of the theory should be stressed though it should be clear: the Green's function G refers to the entire system and propagates the quantum state of the system. It is not the propagator of a particle, but the propagator corresponding to the system of particles. Misunderstanding on this point seems to prompt many authors including Professor Stapp[4] to criticize shadow state theory incorrectly. The probability interpretation is for the state not for a particle!

This completes the formulation of shadow state theory.

The discussion so far has been in terms of Hamiltonians and interactions and it looks non-covariant. Is the theory in fact, covariant? We may also wonder whether for the new theory we must develop an entirely new computational calculus similar to the one that we have developed, say in perturbation series for a relativistic quantum field theory. The theory is in fact relativistic. To show this and other features of the scattering amplitude in shadow state theory we study the scattering amplitude in perturbation theory carried out in the interaction picture.

Let $gV_I(t)$ be the interaction in the interaction picture. We rewrite the shadow theory Green's function in the form:

$$iG(E) = \int_{-\infty}^{\infty} \{\theta(t') - 1/2\ \sigma\} e^{-i(H_0-E)t'}\, dt'$$

Following the work of Richard[10] we then get the modified Dyson formula:

$$-iT = \sum_{n=1}^{\infty} \frac{(-ig)^n}{2\pi} \int_{-\infty}^{\infty} dt_1 \cdots \int_{-\infty}^{\infty} dt_n\ V_I(t_1)\ \{\theta(t_1-t_2)-1/2\ \sigma\}$$

$$V_I(t_2)\cdots\{\theta(t_{n-1} - t_n) - 1/2\sigma\}V_I(t_n)\ .$$

This can be simplified into the form

$$-iT = \sum_{\nu=1}^{\infty} (-\pi i)^{\nu}(-i\tau)(\sigma\tau)^{\nu}$$

where

$$-i\tau = \sum_{n=1}^{\infty} \frac{(-ig)^n}{2\pi} \cdot \frac{1}{n!} \int_{-\infty}^{\infty} dt_1 \cdots \int_{-\infty}^{\infty} dt_n\ T(V_I(t_1),\cdots, V_I(t_n))$$

is the Dyson expression for the standard transition amplitude computed using the fully retarded Green's function (for the system!). It is familiar to us in quantum field theory in terms of Feynman's diagrammatic computational calculus. (We stress that the interaction $gV_I(t)$ in the interaction picture does not depend on the choice of the Green's function.) We thus obtain the simple formula for the physical scattering amplitude:

$$\mathsf{T} = (1 - \sigma)T(1 - \sigma) = (1 - \sigma)\tau(1 - \pi i\sigma\tau)^{-1}(1 - \sigma)\ .$$

While we have used perturbation theory and interaction representation to calculate T in terms of τ we could derive this relationship in other ways not dependent on perturbation theory. We may take this to be an exact relation.

It is clear that T so defined is relativistically invariant and possesses invariance under internal symmetry operations that were common to σ and V. Though it is not explicit in the appearance of the projection operator, σ is a <u>piecewise constant</u> operator considered as a function of the four-momenta of the particles. Hence the transition amplitude is piecewise analytic since τ itself is an analytic function.

But let us show that T conserves probability. By direct calculation we obtain:

$$\begin{aligned}\mathsf{T}^{\dagger} - \mathsf{T} &= (1-\sigma)\{(1+\pi i\tau^{\dagger}\sigma)^{-1}\tau^{\dagger} - \tau(1-\pi i\sigma\tau)^{-1}\}(1-\sigma)\\ &= (1-\sigma)(1+\pi i\tau^{\dagger}\sigma)\{\tau^{\dagger}-\tau-2\pi i\tau^{\dagger}\sigma\tau\}(1-\pi i\sigma\tau)^{-1}(1-\sigma)\\ &= 2\pi i\,\mathsf{T}^{\dagger}\,\mathsf{T}\,.\end{aligned}$$

In other words <u>the probability</u> with the metric $(1 - \sigma)$ in the space W (or equally well with metric 1 in the space R and 0 in the space S) <u>is conserved</u> by the transition amplitude. And this is equally true whether we are talking about nonrelativistic multichannel systems, relativistic multichannel systems or relativistic field theory. The only word of caution is that in the last case we must ensure that we have a theory which yields a finite amplitude τ.

PIECEWISE ANALYTICITY

The scattering amplitudes in shadow state theories are only piecewise analytic. The conventional amplitude computed from a local relativistic field theory is, apart from phase space factors, an analytic function with singularities which are themselves dictated by physical considerations. In the present theory wherever the shadow states are energetically forbidden the shadow theory amplitude coincides with the conventional amplitude; but for domains where they are energetically allowed the two amplitudes differ thus exhibiting clearly the discontinuity in analyticity. In each piece we have a non-linear function of τ which is therefore analytic.

Since the discontinuity in analyticity enters only through phase space factors the analytic jump starts out with zero. Hence though we have only piecewise analyticity we do have continuity.

Such violations of global analyticity should be experimentally tested. M. G. Gundzik and I have shown that such discontinuities cannot be ruled out even in such well studied cases like pion-nuclear forward scattering.[11] More systematic study of this question would be desirable.

CAUSALITY

The problem of "causality" in such theories is of great interest. The philosophic principle of causes having well defined effects following them, itself, though taken for granted in physics, has been severely criticized by careful and wise thinkers.[12] A discussion of these questions is beyond our scope here: but it does pertain to incompatibility of the twin requirements of logical distinction between the cause and effect and of their invariable association. Seeking for causal connections involves identifying components of a total reality, as autonomous and modifiable, identifying them as causes, and looking for responses to such causes. We have remarked above that the Green's functions we choose are for the whole system (and not for individual particles). If we insisted on incorrectly identifying individual shadow particles as autonomously propagating then one is led to apparent paradoxes. The state has to be treated as a whole. (For states involving shadow particles we do not have a pole factorization theorem of the kind that Professor Stapp[4] presupposes.)

In a previous section we had seen that the piecewise analytic physical scattering amplitude T had only the discontinuities connected with the physical states. The Landau rules would surely indicate a singularity even for shadow intermediate states: but examination of the nonlinear relation between T and the amplitude τ shows that if τ had a shadow Landau singularity the corresponding discontinuity is eliminated from the physical amplitude τ.

The moral of the story is that we can use the geometric picture of colliding billiard balls for the globally analytic mathematical amplitude τ but one must not get misled into making incorrect physical deductions from the elegant geometrical picture. Shadow state theory does not admit the same geometrical pictures permitted by the old theory. There are other entities apart from physical particles that transmit interaction. Incidentally, by direct construction, the shadow theory amplitude is continuous. (It seems to be one of the possibilities considered but then abandoned by Iagolnitzer and Stapp.[4])

The basic reason for things to be this way is that the shadow states can be thought of as an energy-dependent "size" for particles and the corresponding non-locality falls off only by a power law. Such a geometric fall off is at variance with our notion of forces falling off exponentially. It also makes it necessary for us to take account of detectors and other apparati and their modification of the quantum mechanical process with such a long range interaction. The clear but nontrivial declaration of shadow state theories is that under conditions where the interactions are between well-defined particles the transition amplitude to physical states alone conserves probability.

There is one technical point which bears directly on the theme of this conference: the question of reversal of cause and effect. In an indefinite metric theory it is possible to have poles on the physical sheet at points $m_o + i\Gamma/2$ which may correspond to physical

channels. With poles on the unphysical sheet at $m_0 - i\Gamma/2$ we have a simple physical picture, namely that of an unstable particle. If we consider the collision of two particles head-on to produce the unstable particle which subsequently reemits these particles their space-time trajectory can be drawn as in Fig. 1.

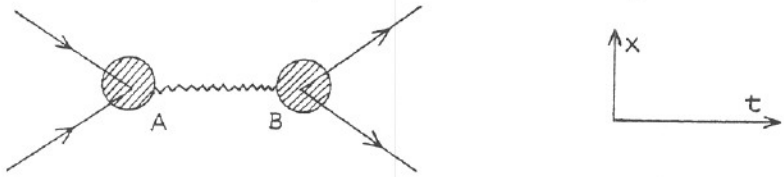


Fig. 1. Collision of two particles propagated by an unstable particle corresponding to a pole at $m - \frac{i}{2}\Gamma$. (After Stapp.[4])

In contrast for the collision in case the pole is on the physical sheet it appears that we have a causality violating sequence shown in Fig. 2.

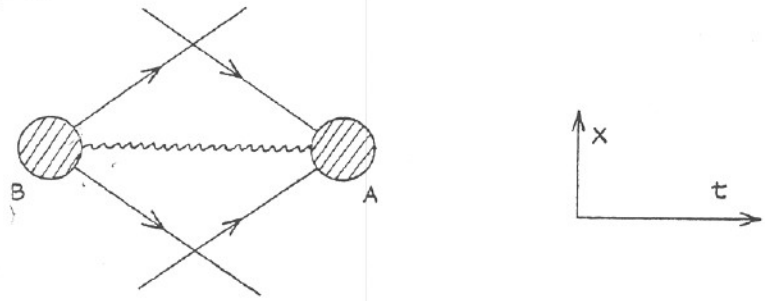


Fig. 2. Collision of two particles advanced by an unstable particle corresponding to a pole at $m + \frac{i}{2}\Gamma$. (After Stapp.[4])

Actually this picture is incorrect; instead what we obtain is the situation depicted in Fig. 3. The dotted portions do not take

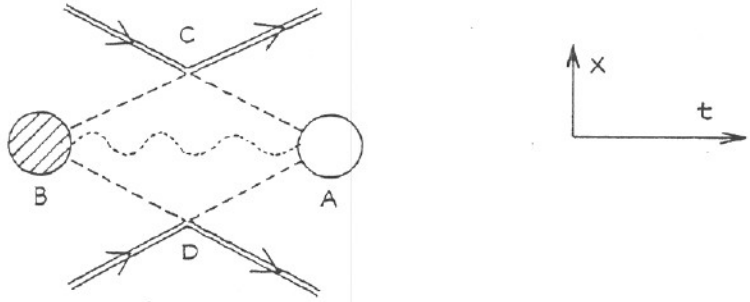


Fig. 3. Collision of two particles in a channel with a pole at $m + \frac{i}{2}\Gamma$.

place at all and the causal anomaly vanishes. We may, of course, think of this as due to a size for the particles. (A size for either one of them will do.) The situation may be illustrated by the "orbit diagrams" Fig. 4a,b. In the second case the particles

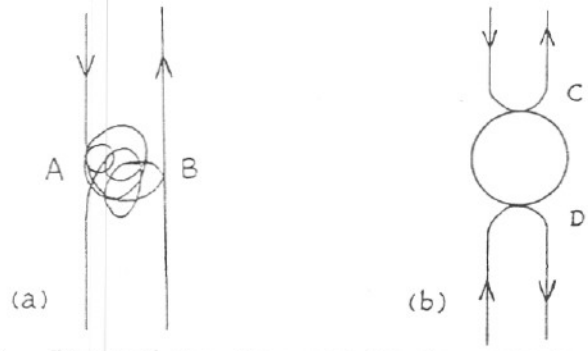


Fig. 4. "Orbits" for the particles in cases described in Figs. 1 and 3.

never really "meet" each other. The case therefore corresponds to a long range repulsive interaction. The behaviour of the phase shifts through the "resonances" in the two cases bear out this physical interpretation.

Even if there are no causal anomalies we may want to restrict the "size" of the interacting particles to microscopic dimensions. This would require that in Fig. 3 the dimension CD is "small". This entails that AB also is small; in other words the imaginary part $\Gamma/2$ of the "mass" of the unstable pole be "large", that the width be of the order of strong interaction widths. Such large complex masses do not arise from real masses in an indefinite metric theory of weakly coupled satellite quanta like for lepton satellites in quantum electrodynamics or in weak interactions. But in an indefinite metric theory we may, if we choose, start with pairs of complex conjugate masses. We must not lose sight of the possibility, however, that "large sized objects" are compatible with shadow state theory; and we may consider it worthwhile to be alert to the possibility of macroscopic impact parameter collisions. They would appear as disconnected scattering events in a bubble chamber.

WHO NEEDS SHADOW STATES?

Even if we are convinced about the consistency of the physics of shadow states one may raise the question: why bother? Why should we include such an additional complication into the theory? There are two enticing reasons that appeal to me: First, there are a number of assorted particles that have been postulated in particle physics from time to time which have not been found. Included in this "missing particles list" are magnetic monopoles,[13] intermediate vector mesons,[14] tachyons[15] and quarks.[16] Depending on one's

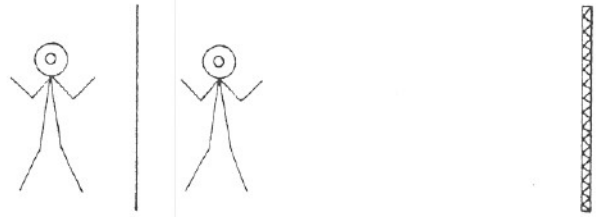
prejudices, background, philosophy and friends one or other of these species of particles appear more essential than the others. But they all share the distinction of not only being not discovered but it appears as if they are as plentiful as the unicorn. It is tempting to suggest that some or all these particles are shadow particles. They enter dynamics but are "not really there"! Since my aim has been to display the logical structure of shadow state theories I will not attempt discussing any models in this presentation.

Second, local relativistic quantum field theory inevitably leads to infinities in any direct physical computation from a Lagrangian. As long as one persists in local interaction structures the infinities persist. The only way out seems to be to introduce fields whose fundamental equal-time commutator (or anticommutator) has the opposite sign to the usual theory so that the corresponding propagators differ by a sign.[3] Such "negative weight" fields correspond to fields defined over an indefinite metric space. By suitably blending positive and negative metric fields we can obtain finite relativistic local field theories. But there was always a difficulty with probability interpretation of the indefinite metric state space. The restriction of the real state space $\mathcal{R}$ to states containing only positive metric quanta resolves this problem once and for all. I have discussed these questions in detail elsewhere[2,5] and I shall have to be content with that.

RECAPITULATION

In conclusion allow me to state the following: (i) Quantum theory allows a generalization to shadow state theories with a consistent probability interpretation. (ii) Shadow states enter dynamics but do not enter unitarity. (iii) Modified Green's functions of shadow state theory do not allow consideration of autonomous particles in the shadow subspace. (iv) Considerations of shadow states as if the individual particles are propagating autonomously are incorrect; and they lead to incorrect conclusions about the consistency of shadow state theories. (v) Some interactions not carried by physical objects fall off no faster than a power of the distance. (vi) Shadow states are useful to give a consistent interpretation of indefinite metric theories. (vii) If we have the conventional scattering amplitude computed using the retarded Green's function, we can simply obtain the physical scattering amplitude. (viii) Magnetic monopoles, tachyons, or quarks could all be accommodated as shadow particles and this may make them totally unobservable. (ix) Piecewise analyticity is inevitable in shadow state theories.

The question of piecewise analyticity which upsets many is not so far removed from our experience after all. Whenever we look into a mirror we see a causal world, obeying a number of simple laws. For every object placed in front of the mirror we see another one, laterally inverted, in the mirror. If we have a field of illumination the illumination is a smooth function of position. But if we actually proceed to look behind the mirror we see a very different



Before the looking glass (a) Behind the looking glass (b)

Fig. 5. A Piecewise "analytic" world view.

physical situation. Until we get to know about mirrors we will have an unusual world view. What I have presented here is just another case of a possible piecewise-analytic world view.

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DISCUSSION

Editor's Note: The following was edited by both the editor and the speaker from tapes of the question and answer period which followed the talk.

NEWTON: The scattering amplitude is defined physically either by means of wave packets, as a relation between a state at time $-\infty$ and a state at time $+\infty$, or else in a time independent way in terms of what you send in, the beam you sent in, and what comes out. You've defined in a formal sort of way something that has the virtue of being unitary but what has it got to do with physics? In other words, what has this amplitude got to do with any observation of that kind?

A. Let me restate the two ways of defining the scattering amplitude. In the first case you choose a state of the exact Hamiltonian which strongly converges to a preassigned solution of the free (comparison) Hamiltonian in the infinite past and call it the "in" state; the corresponding state which converges to the preassigned state in the infinite future and call it the "out" state. If we now consider the scalar product of an "in" and an "out" state that gives the scattering matrix from which the scattering amplitude can be obtained. Another definition of the scattering amplitude is as the matrix element of the interaction between the "in" state and the states of the free Hamiltonian. In my theory I do very similar things. It is easier to draw the correspondence with the first method: we simply make a different definition of the "in" and "out" states! This, of course, reflects the different boundary conditions employed. It must be stressed that the state χ is an (ideal) eigenstate of the exact Hamiltonian; and it would be the state obtained by summing the perturbation series for the eigenstates of the exact Hamiltonian. This is a physical choice (unless you forbid me to use the word physical in my theory). That means that when I calculate the scattering amplitude I choose to calculate it using this identification of the states. And if you do experiments I would urge you to compare your data with the predictions. They may or may not agree but, then, this is to be found out. The difference comes in, if you like to view it that way, from the contribution from an elementary scatterer being chosen differently by the "establishment" and in this theory. This freedom of choice is possible because there are solutions of a free wave equation which are proportional to a source.

NEWTON: But it's not just proportional to the source. It's also that a choice of sigma means that it is a very specific choice of phases at large distances. Because the choice of σ amounts to a principle value Green's function in that region, which means that for those shadow particles, the phases have to be just exactly like you chose them at infinity; which means that the particles that you don't want to acknowledge as particles nevertheless have to be controlled by infinity.

A. Exactly, exactly, but I would not control them but instead I identify the states in which they are controlled! In fact this will formally come up in Stapp's discussion: I don't want to anticipate too much of the discussion. If you consider a multiparticle scattering amplitude, when you "cut it" out you see that there is something which appears like a singularity, corresponding to free particles, but it contains incoming and outgoing waves and therefore a nonvanishing real factor but no imaginary part corresponding to the shadow intermediate states.

DRESDEN: Am I correct in believing that the fundamental formula you wrote down is essentially an expression for the T-matrix, which has the σ's in it?

A. Right.

DRESDEN: But I did not quite understand your claim. Because is the assertion that if I now start computing matrix elements of processes with that new T-matrix (for whatever reasons) no matter what choice I make of σ I will get finite results?

A. No, what I said was that no matter what σ you choose, you get a consistent answer in the sense that when restricted to the nonshadow subspace you get a unitary amplitude. Now, if you want a finite amplitude you start with a finite theory! If the original theory was not finite there is no way of getting one by this method. The only thing that the theory of shadow states enables you to do is to make a finite unitary theory in terms of interacting particles, by introducing indefinite metric, negative probabilities complex mass and other such "unphysical" features into your theory.

DRESDEN: Yeah, but I thought that effectively you had some kind of a compensation mechanism and σ allows you to make, construct....

A. In a sense the only thing that σ does is to allow you to have things which you don't then have to pay for later on. You can have negative probabilities in the virtual states, and it is always the virtual states which create trouble, not the real states.

DRESDEN: I see, so there are really two quite separate elements.

A. Right, in fact I wanted to distinguish between them, but a shadow state need not necessarily be a negative metric state. Even though that is the context in which I developed the formalism.

FADNER: I would like to suggest that you've got a very elegant and particular example of a more general principle. And that is, this of course was stated by you, that the probabilistic interpretation of Ψ is not necessary. In fact it is not necessary in a Heisenberg sense in that we never observe it. The only thing we ever do observe is an interaction probability, and so although it is handy and useful to normalize Ψ through letting the integral of $(\Psi^{\dagger}\Psi)$ over all space go to one, in many cases at least, this is not necessary. But it is sufficient to give us a good and reasonable theory for things we actually observe, which are things such as scattering. So I would like to suggest that this is a more general principle which is not really stated very often. The second thing I would like to suggest is it's possible that the theory you are presenting on the basis of shadow states, is also sufficient, but not a necessary theory in order to get the thing we are really after, which is a probabilistic interpretation of interactions.

COOKE: Could you compare what you have done with the Gupta-Bleuler ideas in electrodynamics?

A. The celebrated Gupta-Bleuler work, of course, is much more elegant in a certain sense, perhaps because it was inevitable. In electrodynamics, the good Lord said "Let there be light" and then there was light; and then there was Maxwell's equations and you have to make a quantum theory, and the simplest most elegant way of doing it is the formalism developed by Gupta. On the other hand, one also recognizes that the Gupta-Bleuler method does not lead to a finite theory. It's very elegant, but too conventional to be able to eliminate divergences. It is the most beautiful theory of that kind: but it does not do the job of making the theory finite.

COOKE: I know, but I was asking not so much about the questions of the theory being finite....

A. Perhaps you're asking about the negative metric aspect of it? But in some ways the negative metric aspect in the Gupta-Bleuler thing may be stated very simply. (I suppose that neither Gupta nor Bleuler would be very happy with the way I put it: but I hope that both of them are absent from the audience.) If you had Maxwell's equations, you may write down the equations in the form:

$$\partial_\nu F_{\mu\nu} = J_\mu$$

and

$$\partial_\mu A_\nu - \partial_\nu A_\mu = F_{\mu\nu}$$

From these two equations, it follows that:

$$\partial_\mu \partial_\nu A_\nu - \Box^2 A_\mu = J_\mu$$

In the Gupta-Bleuler form, that is with all the four components taken in this particular fashion, we have: $\Box^2 A_\mu = -J_\mu$ and therefore, we have the equation that

$$\Box^2 \partial_\mu A_\mu = -\partial_\mu J_\mu$$

And if you so choose the sources such that the right-hand side is equal to zero, it follows that $\partial_\mu A_\mu$ obeys a free field equation. Now it is up to you whether it should be zero, finite or any number that you want, and it doesn't affect the rest of the problem; it's an independent field so you can choose to have no quanta of this kind to start with. If you start with such a thing then automatically in that theory, all the states that you have, have either zero norm or positive norm. But this is possible only because of the fact that the source that you had here in this particular case was conserved, and therefore this particular combination obeys the free field equation. You have a supplementary condition and in a certain sense the supplementary condition that you have is not that there are no negative norm particles, but in fact, the negative norm particles and the positive norm particles have to come in a certain phase, if they come in at all. That condition

would be propagated for all times. The Hamiltonian consisted of something which would commute with this condition. This condition was something which is preserved. In our case the situation is not so. σ is not a constant of the total Hamiltonian. Consequently, any supplementary condition that was imposed on σ was going to affect the time development of the system. σ did not commute with V and therefore did not commute with the total Hamiltonian. In Gupta-Bleuler electrodynamics the supplementary condition is something which was consistent with the equations of motion, therefore, this choice did not affect the problem whatever.

MEYER: Talking about particles, you appear to use the term particle in some general sense and also in a more specific sense. Now, am I correct in supposing that when you are alluding to particles specifically, you mean such well established particles as photons, electrons, protons, neutrons, atoms, and so forth?

A. In this discussion, I specifically did not include atoms because I was speaking more in terms of a field theory of fundamental interaction. But on the other hand, if at the level on which I was thinking the atom is a single entity, I could include atoms also. But I also include such objects, if you like, (in solid state) called 'phonons' which are good, longitudinal photons which are nonexistent but very important, and a whole lot of other things like that--rho mesons which are very useful, but don't exist, and tachyons which don't exist and are probably not useful.

MEYER: When you use the phrase, visible particles, are all of these particles that we have just been mentioning, are they visible particles?

A. Let me be a little less rhetorical and a little more precise. When I said visible particles, I meant a particle which can be seen. 'Seen' in terms of neutrons being seen or protons being seen, in the sense that I can detect it. I can talk about a state (forget all about quantum mechanics) and finally when I am "observing" I observe the particles by direct or indirect means.

MEYER: And therefore everyone of these particles....

A. Everyone of these particles. But there are other particles like, for example the rho meson, which you do not detect.

MEYER: You mean photons are invisible particles?

A. No, no, rho mesons. Some photons are seen, some photons are not seen. The virtual photons are not seen. But they are seen indirectly because they are very important.

MEYER: OK. Now in the general sense that you are using the term particle, do you mean that all of these particles are particles of matter, some of them are particles of matter, or none of them are particles of matter?

A. The 'matter' is to be defined in terms of what you want it to be. Let me bring the discussion to a more physical level. Are phonons that are occurring in a Debye solid, particles of matter or are they fiction? For all particle purposes, they are particles of matter, because they do contribute to specific heat and entropy and all such things; they scatter against each other; and yet you know that if you take a bar of iron there are no phonons "inside". Iron is very solid. And yet it is very useful in some contexts to think of it as a phonon gas. I believe that all statements about particles are fiction, just like all statements about material objects. There are no tables! Not even people!!

ROLNICK: When you have these negative metric states taking part in the dynamics, can that lead to an acausal type of preacceleration or any kind of acausal effects.

STAPP: I will discuss that in my talk, so now that I have the floor: Can I just ask something different? You wanted to relate quarks to shadow states or you suggested that perhaps there might be a relationship between quarks and shadow states. You also wanted to make the relationship between shadow states or that the shadow states were not observable and you also made the point that any combination, or any state that had at least one shadow state in it would be a shadow state. That would seem to say that any combination of quark states would be a shadow state, and hence unobservable.

A. No, no. First of all I was not pushing quarks, I am neither for them nor against them. You can have it if you want. I'm suggesting that here are particles which seem to be very useful for certain dynamical calculations but it seems to be a very great embarrassment for certain other experiments because you don't seem to find free quarks. Yet it looks as if there are free quarks participating. I'm saying that one possibility is to consider the three quarks state, the bound state of quarks, to be physical and only these to be physical and the other things to be shadow states. These are energetically separated and therefore you can do this. Now it is not quite the same thing that I said with regard to other particles because I was not thinking of constructing any bound states. The simplest choice is to identify certain particles and call them the shadow particles and anything which the shadow particle touches, also becomes a shadow. But you are quite right. If I wanted to apply this for the three quark state I would certainly not be doing this, I would be making a little more devious choice. I should also mention, while I have the floor, that in principle, especially on the basis of what Professor Newton

and various other people said yesterday, if you want tachyons to exist, one way of controlling them--keeping them from becoming a pest--is to say that they are also shadow particles, that they are only responsible for forces and not seen.

SHPIZ: The expansion that was written down by Richard, usually comes from writing a Schroedinger picture, then passing to the interaction picture. If you do that in this theory then, and you start out with real states, won't you then from time development pass into shadow states?

A. Only in the intermediate states.

SHPIZ: Why not in the final states?

A. Because without them everything is alright; so you don't pass into them.

SHPIZ: You can actually eliminate the possibility--you project out when you take the matrix elements.

A. Yes indeed. Look at the expression for the transition amplitude. It satisfies all the requirements that you want to have, but at the intermediate stages of calculation, you have the shadow states also coming in. So you have a whole system which is developing in time; if you are not looking on the mass shell then you have a whole lot of other things. In the intermediate states, you would find a whole lot of these things. The most economical way of interpreting them, in terms of the only observed restricted set of particles is to say that there are energy dependent forces which look as if it is caused by exchange of particles. But there are no such particles.

BELINFANTE: Is that only so in the development from $-\infty$ time to $+\infty$ time or can you also do it between finite times?

A. It is only valid for the total development from $-\infty$ to $+\infty$ because at finite times you get things off the mass shell which are not necessarily energy conserving and then there are effects coming in from shadow state admixtures.

BELINFANTE: Didn't Haller and Landovitz show that, by working rather differently, mathematically, you could (that is in the photon case) work between finite times. I never read that paper in detail, though. They dealt with the same problem in connection with quantum electrodynamics with the longitudinal photons and the usual method, when applied between finite times, leads to unphysical states. But Haller and Landovitz stated in their paper that one could avoid these unphysical states.

A. I suppose that you would avoid the problem if you include non-local interactions; but I am not familiar with the paper to make any useful comments on it.

HAVAS: It seems to me that your work is exactly the quantum analogue of what I discussed yesterday, that what you are calling shadow states, is talking somehow in the language of field theory of action-at-a-distance interactions. I think that you stated that much at the beginning of your talk. What was said just at the end was precisely the analogue, say, of talking of a closed system. You have to go from $t = -\infty$ to $t = +\infty$, to come up with a consistent result.

A. That is gratifying.

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