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A world of Bose particles

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Editor's Note: Satyendranath Bose, known primarily as one of the co-founders of quantum statistics, died on 4 February 1974, a few weeks after a symposium in honor of his 80th birthday was held at the Saha Institute for Nuclear Physics in Calcutta. The following paper, originally prepared in that connection, reviews some of the more important developments in particle physics which followed from the fundamental insight contained in a four-page paper published by Bose exactly fifty years ago. At the request of the editors of the AJP, *Professor Sudarshan has kindly consented to adapt his paper for reproduction here. We would like to thank William Blanpied for bringing this paper to our attention.*

The study of the nature of light has been essential to our understanding of the physical universe. Planck[1] heralded quantum theory at the turn of the century. Einstein[2] built on that foundation and advanced the notion that light propagated and interacted as quanta. Bohr[3] showed that the quantum hypothesis leads to an understanding of the characteristic line spectrum of hydrogen. Following the observation of Ehrenfest[4] that the energy of field excitations should be quantized, Debye[5] rederived Planck's radiation law. Einstein[6] gave yet another derivation of Planck's law based on radiative equilibrium resulting from the simultaneous consideration of induced and spontaneous emissions, a method which was an adaptation of the Boltzmann[7] derivation of the equilibrium distribution from the collision equation. Debye[8] used the notion of quantization of elastic vibrations to account for the specific heat of solids. Altogether it was increasingly suggestive that the radiation inside a cavity should be thought of as a photon gas.

In 1923 Compton[9] discovered the change in wavelength of x-rays scattered by free electrons; Compton and Debye[10] were both quick to identify the effect as the demonstration of the elastic collision between a photon and an electron.[11] Pauli[12] had even used the Compton effect in discussing the equilibrium of electrons in interaction with radiation. But none of them had a theory of the photon gas.

The crucial step in the identification of black body radiation as a photon gas was taken by thirty-year-old Satyendranath Bose.[13] In a short (four-page!) paper[14] published in 1924 he showed that photons had a startlingly "new" property of being strictly identical! This led to a new expression for the thermodynamic properties of an assembly of photons in contrast to what one would have calculated on the then accepted basis, the so-called Maxwell–Boltzmann statistics. With the modification that Bose introduced into the calculation we obtain the correct thermodynamics of the photon gas. That one step was the basis of the new synthesis between the wave and particle properties of photons, and with it, the foundations of quantum field theory. After Bose's paper came an avalanche of developments: the extension of Bose's theory to particles of arbitrary mass and non-zero chemical potential by Einstein,[15] the Fermi[16]–Dirac[17] statistics for electrons, the quantization of the electromagnetic field by Heisenberg and Pauli,[18] and quantum electrodynamics by Dirac.[19]

To put Bose's synthesis in its proper setting it is good to recall that there were two items of unfinished business, one regarding photons as particles and the other concerning statistical mechanics of identical particles. In both cases uneasy make-shift solutions were generally accepted instead of definitive solutions. In the old dichotomy between particles and waves there were highly persuasive arguments on both sides, but it was thought that a crucial experiment was the determination of the relative speed of light in two media with different refractive indices. It appeared that if light consisted of particles, the speed in the optically denser medium should be greater; if it consisted of waves, this speed should be lesser. Fizeau's[20] experiment on the speed of light in water thus seemed to find definitively for the wave theory! Yet, what of the postulated photons? How do we reconcile the notion of a photon with its discrete momentum and energy to its lesser speed in water than in air? We must conclude that photons may be particles, but they do not behave as particles are naively expected to behave. A revision of the concept of a particle ought to be made.

The other item of unfinished business is even older. It concerns itself with the statistical mechanics of identical particles. In calculating the partition function and the entropy, one finds that the entropy is not a strictly additive quantity: when we "mix" two volumes of an ideal gas at the same temperature and pressure the resultant entropy is larger than the sum of the two entropies. This Gibbs paradox[21] shows that such a collection of identical particles is not a satisfactory model for an ideal gas. Instead of heeding this warning signal[22] people "fixed" the trouble by an *ad hoc* procedure in dividing the partition function by the factorial of the number of particles, thus condoning the Maxwell–Boltzmann statistics. The genuine need for a reexamination of the implications of the strict identity of the particles was not appreciated until Bose, three decades yet to be born.

Elementary particles were originally introduced as the stuff from which the world was made. They were to be immutable entities. But the photon was clearly an entity which could be created or destroyed. Where does a photon come from and where does it go? And how can we really understand creation and destruction? What is the implication of strict identity of photons? In what sense and to what extent can we think of light as a collection of photons? All these questions were answered at one stroke by Bose, who asked us to consider the many-photon states to be counted as states with equal probability. Photons were thus particles all right, but particles for which strict identity was to be recognized by considering as distinct only those cases in which the distribution of photons over phase cells were distinct.

Photons thus became nothing but levels of our underlying field. Creation or destruction of photons is merely a "movement" of the field. Photons are then manifestations of the potentialities of the radiation field: the dichotomy between the field and the particle thus ceases. Two have become one. In the words of the *Bhagavad Gita,* for which Bose, along with so many other Indians of his generation, would have felt an almost intuitive affinity:

> At the close of many births the man full of wisdom cometh unto me; "The Lord Kṛṣṇa is all," saith he, "the supreme One who is supremely difficult to find."[23]

Automatically the embarrassment of the Gibbs paradox is resolved: the paradox was just telling us that the strict identity of particles must be taken into account. But if photons are but the difference between levels of the radiation field they are identical! And the processes of creation and destruction of photons is to be thought of as the change in the state of the field, the "motion" of the field. If we have equations of motion of the field, we have the means of describing the creation and destruction of photons. It took two more years for Heisenberg and Pauli[18] to write down the equations of motion for the radiation field and another year for Dirac[19] to construct a theory of the emission and absorption of photons. In Dirac's work the oscillators of Planck were at last identified. The formulation of the equations of motion of the electromagnetic field had still unsatisfactory features. Many others contributed to the resolution of this problem, among them Dirac, Heisenberg and Pauli, Fermi, and Gupta.[24] Bose must have been pleased that it was one of his countrymen, Gupta, who was able to work out a final form of the dynamics of the radiation field, eighteen years later.[25]

In the course of his work on the quantum theory of radiation Dirac introduced the now familiar notions of creation and destruction operators which increase or decrease the number of quanta in a state. These creation and destruction operators, introduced as the operator coefficients of the quantized field operator, do not commute with each other but instead satisfy a commutation relation which transcribes the commutation relations between field quantities as formulated by Heisenberg and Pauli.[18] Dirac[26] had already discovered that the commutator bracket in quantum mechanics was the natural analogue to the Poisson bracket in classical mechanics. It then reaffirms the Bose hypothesis that photons obey the Bose statistics. The quantized radiation field describes the same system as the totality of many-photon states provided photons obey Bose statistics.

A year after Bose discovered his statistics Pauli[27] enumerated the Exclusion Principle obeyed by electrons: strict identity did not automatically imply Bose statistics. In another year Jordan[28] laid the foundations of a quantum field theory for particles obeying the Exclusion Principle and in the succeeding year Jordan and Wigner[29] completed this work. The corresponding statistics had already been worked out by Fermi[16] in 1926. Einstein[15] amplified and extended Bose's work to make a statistics for ideal gases. He included the chemical potential appropriate for a gas in which the total number of particles are conserved; and allowed for a more general energy–momentum relation for the particles.

Both Bose's theory, including Einstein's extension, and Fermi's theory were essentially statistical theories and dealt with complexions of the field. The actual construction of the wave functions which are respectively symmetric and antisymmetric in the many-particle labels was done by Dirac.[19] To find the relation between particle type and the statistics which obeyed it was left to Pauli,[30] who showed that integral spin particles obeyed Bose statistics and half-integral spin particles obeyed Fermi statistics. In modern relativistic quantum field theory this spin-statistics theorem is one of the most fundamental results.

In 1935 Yukawa[31] advanced the meson theory of nuclear forces and the meson field. In the four decades since that time, mesons of a variety of kinds have been identified and seen to play essential roles in nuclear interactions. They are all Bose particles. Wherever they form multiplets constituting representations of internal symmetry and groups like isospin the Bose property holds for the multiplet provided we include the internal symmetry labels, also.

As far as modern particle physics is concerned, the language employed is unreservedly quantum-mechanical and the symmetry or the antisymmetry is built into the structure of the theory. This is automatically assured by employing the quantized field to formulate the kinematics of the theory. If the dynamics is also formulated in terms of quantized fields the required symmetry conditions on the many particle states automatically obtain. If some other formulation of the dynamics is employed we must make sure that the symmetry properties of the many particle states are guaranteed.

This symmetry has the immediate consequence that in any state containing two or more particles of the same field the state should be symmetric under the interchange of identical particles. Thus two spinless Bose particles, say two pions of the same charge, can be only in a state of relative even angular momentum. A spin one object cannot therefore decay into two spinless identical particles. Two photons of the same polarization cannot be in a state of orbital angular momentum one. In this manner a number of "selection rules" can be deduced for particle reactions and decays.

Already at the statistical level the identification of only complexions as states of equal probability implies that an ideal Bose gas would have a tendency to have two particles in the same phase cell more often than in the case of a classical gas in which the particles are distinguishable. This positive distance-correlation can be quantitatively computed. A similar situation obtains in particle physics. Other things being equal, like-Bose particles tend to be positively correlated; the angular correlation of pions of like charge in the multipion annihilation of nucleon–antinucleon systems exhibits such an effect.[32]

The manifestations of Bose statistics in particle physics goes beyond this. In relativistic quantum field theory the creation and destruction operators are the coefficients in the expansion of the field operator in terms of a complete set of negative and positive frequency wave functions. Therefore, we expect a symmetry under the interchange of the positive and negative frequency parts of a field operator. It turns out that indeed such a formal symmetry exists in the transition amplitudes in quantum field theory: it is called Crossing Symmetry.[33] If we make use of Crossing Symmetry we see an extension of Bose symmetry even in the modified interchange of particles in the initial and final states!

This extension of Bose symmetry is very closely related to the notion of causality in relativistic quantum field theory.

Earlier we referred to the work of Heisenberg and Pauli[18] and subsequent generalizations in which the field quantities themselves were treated as dynamical variables and commutation relations formulated for them. The implications of these commutation relations for measurability of field quantities was studied in detail by Bohr and Rosenfeld[34] in 1933. The fact that field quantities, separated by space-like distances, commute, was interpreted to mean that such measurements are compatible and do not disturb each other. This is at times interpreted also in terms of the unavailability of any particles in this theory which travel faster than light which could carry disturbances from one location to another. This circumstance is referred to, by abuse of language, as "causality." A close examination of the mathematical steps involved show that the extension of Bose statistics through crossing symmetry is necessary to maintain causality.

Bose statistics is not an impressed property of some species of particles but something which is inevitable if on the one hand the particles are truly identical and on the other the theory is to be approximated by a classical field theory in the limit of large number of quanta.[35] This second condition is a natural extension of Bohr's Correspondence Principle.[36] We expect therefore that intense light beams should behave more or less classically.

What is however remarkable is that apart from this "classical limit" there is a remarkable possibility of a classical description of optical phenomena even in the case of weak illumination. Thus the behaviour of a field of illumination, even if it is so weak as to contain no more than a single photon at any one time, can nevertheless be described in the language of classical coherence theory.[37] In particular the classical theorems of partial coherence and intensity interferometry are valid for quantum optics with only minor alterations. With the discovery of the Optical Equivalence Theorem[38] the synthesis of the field theory and the theory of photons obeying Bose statistics is completed.

When Bose advanced his hypothesis the only species of elementary particles that were identified were the electron, the proton, the neutron, and the photon. Of these only the photon obeys Bose statistics. The photon number is not conserved and it is a zero mass particle. Both of these impart special characteristics to the statistics of photons. Among the Bose systems available then was Helium. In this case the particle number is conserved; and the particles are non-relativistic. So we need to extend Bose's ansatz. To conserve the particle number we have to introduce a non-zero chemical potential. For photons the chemical potential is zero. In the case of an ideal Bose gas with non-zero chemical potential there is a critical temperature below which a finite fraction of the gas condenses into a single quantum state. This condensed phase, discovered by London,[39] should exhibit superfluid properties; and London suggested that superfluid helium should be related to this Bose–Einstein condensation phenomenon.

Einstein arranged for the translation and publication of Bose's paper[14] on the statistics of photons and added a remark endorsing it as "substantial progress." Both in his original letter to Einstein and in his subsequent correspondence Bose addressed the great man as "teacher" and accords him great respect; and that is as it ought to be. It is in the definition of the teacher, as understood in the classical Indian tradition, that he remove all the doubts of the student and weld his understanding into a harmonious unity: such a teacher is the one worthy of adoration.

> To that Teacher who removes all my doubts, welds my vision into a unity and thus enables me to gaze on secret knowledge; to that One my homage.[40]

Einstein does not seem to have told Bose how his theory could be extended to a theory of ideal Bose gases by introducing a chemical potential and making use of a general energy–momentum relation.[41] Instead, Einstein formulated this extension in one of his papers.[15] And when Bose sent him another paper[42] on the equilibrium of matter in interaction with radiation he got that, too, translated and published, but this time with a critical comment. This question concerns the equilibrium of radiation when it is in interaction with matter and being continually emitted and absorbed. In 1915 for the special model of a Bohr atom coupled to radiation, Einstein[43] had shown that we could get Planck's distribution provided the emission rate contained one term proportional to the number of photons in the phase cell times the spontaneous transition rate. This additional term is called stimulated or induced emission. This special model was extended to many-level systems by Einstein and Ehrenfest[44] in 1923. Pauli,[12] on the other hand, had studied the scattering of light by electrons and showed that if the Compton effect was taken into account, the Planck distribution was steady. Bose[42] proposed to take up the general case and showed that in his formulation both the Pauli processes and the Einstein–Ehrenfest processes were included as special cases. This by itself should have met with general acceptance and acclaim. But, Bose did point out that instead of Einstein's assumption of a stimulated and a spontaneous transition rate for absorption it is possible to consider only the spontaneous transition rate for emission *provided* the absorption rate is taken to be not proportional to the number of quanta per phase cell but this number divided by this number plus one. Now, as far as radiative equilibrium is concerned this ansatz is as good as the Einstein ansatz,[43] since only these ratios do come in! So it would have been quite possible for Einstein to add such a footnote to Bose's paper and call attention to the positive general features of Bose's formulation. But he chose otherwise. Three years later Dirac[19] constructed his theory of emission and absorption of radiation; and the Einstein ansatz of stimulated and spontaneous emissions was seen to be a natural consequence of the matrix elements of creation and destruction operators. But Bose did not write on radiation theory ever again.

Bose's work stands out as one of the central columns supporting the edifice of modern physics. His great achievement has inspired and fostered us all. And Bose, until his recent passing, continued to inspire and foster creativity and class amongst all of us who were his students and followers. But most of us, in remembering this giant amongst us, rarely ever think of the courage and dignity of one who must have felt such keen disappointment in the lack of generosity and appreciation from him whom he considered his master. Nor have other great men been too generous in their appreciation of the work of a man who did not complete even thirty years when his finest work was announced. In a nation where the intellectuals are not often eager to recognize and honour originality it requires a courageous man to be ahead of his peers. To such a person Bose is an inspiring example in dignity and courage:

> He whose mind is free from anxiety amid pains, indifferent amid pleasures, loosed from passion, fear, and anger is called a *muni,* a sage of stable mind.[45]

Bose laid the foundations for our understanding of strictly identical particles and for our recognition of the many boson states as simply different states of the field. Creation and destruction become then modifications of this field; and the field enters the center stage. The field-particle dichotomy is ended, the Gibbs paradox and the Boltzmann *ad hoc* assumptions are both resolved and a new comprehensive notion of indistinguishability emerges. In Dirac's work three years later we see the fulfillment of the vision engendered by Bose.[46]

> Understand Me as the Knower of the Field in all Fields, oh Bharata. Wisdom regarding both the Field and the Knower of the Field—that in my opinion is the greatest wisdom.[47]

This is the fiftieth anniversary of Bose's great contribution; and such is its greatness that it is difficult to think of a world without Bose quanta. Melpattur Nārāyana Bhattatiri, a medieval South-Indian poet-devotee of Kṛṣṇa, has aptly described this situation:

> The ecstacy of enlightenment is inherent in that eternally liberated, liberating, and all pervading principle we call the Brahman. That essential principle shines through the words of a hundred thousand of our sages; yet it cannot be bounded by time and space. If seen it would be unrecognized; and yet its comprehension remains the primary goal of humanity.
>
> Oh my countrymen! Great indeed is our good fortune! For that Brahman has appeared incarnate among us.[48]

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