A Renormalizable Theory of Leptons with Intrinsic Symmetry Break-Down (*).

J. P. HSU and E. C. G. SUDARSHAN
Center for Particle Theory, University of Texas at Austin - Austin, Tex.

(ricevuto il 9 Giugno 1975)

We wish to point out that one can construct a renormalizable unified theory of weak and electromagnetic interactions for leptons with intrinsic symmetry break-down. The theory constructed in this way differs from the usual gauge theory (1) with spontaneous symmetry break-down in basic concept and in the physical implications concerning the scalar field involved. For example, the masses of vector fields have nothing directly to do with spontaneously broken gauge symmetry (2). The theory involves a massless scalar boson with a very weak coupling which is consistent with experiments. It is manifestly renormalizable (in the same sense as the Weinberg theory (3)) by standard power counting.

In the unified theory the Lagrangian for the leptons (say, electron and electron-neutrino) is

$$\mathscr{L} = \mathscr{L}_b + \mathscr{L}_e \,, \tag{1}$$

$$\mathscr{L}_b = -\tfrac{1}{4}\boldsymbol{W}_{\mu\nu}^2 - \tfrac{1}{4}B_{\mu\nu}^2 + |(\partial_\mu + ig\boldsymbol{\tau}\cdot\boldsymbol{W}_\mu/2 - ig'B_\mu/2)S|^2 \,, \tag{2}$$

$$\mathscr{L}_e = \bar{L}i\gamma^\mu(\partial_\mu + ig\boldsymbol{\tau}\cdot\boldsymbol{W}_\mu/2 + ig'B_\mu/2)L + \bar{R}i\gamma^\mu(\partial_\mu + ig'B_\mu)R - (gm_e/\sqrt{2}\,M)(\bar{L}SR + \bar{R}S^\dagger L) \,, \tag{3}$$

$$S = \begin{pmatrix} S^+ \\ S_0 + \sqrt{2}\,M/g \end{pmatrix}, \qquad L = \frac{1}{2}(1+\gamma_5)\begin{pmatrix}\nu_e \\ e\end{pmatrix}, \tag{4}$$

where $\boldsymbol{W}_{\mu\nu} = \partial_\mu \boldsymbol{W}_\nu - \partial_\nu \boldsymbol{W}_\mu + g\boldsymbol{W}_\mu \times \boldsymbol{W}_\nu$, $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and $R = (1-\gamma_5)e/2$. The Lagrangian (1) is not invariant under usual gauge transformation because M in S given

(*) Work supported in part by the U.S. Atomic Energy Commission.

(1) S. WEINBERG: *Phys. Rev. Lett.*, **19**, 1264 (1967); A. SALAM: *Proceedings of the Eighth Nobel Symposium* (New York, N. Y., 1968).

(2) See, for example, J. P. HSU and E. C. G. SUDARSHAN: *Phys. Rev. D*, **9**, 1678 (1974); J. P. HSU: preprint CPT 194 (1974).

(3) For the problem of anomalies and renormalizability, see B. W. LEE: *Perspectives on theory of weak interactions*, in *XVI International Conference on High-Energy Physics* (1972).

by (4) is not zero. From the expression (4) for S and (2) we see that the fields $W^{\pm}_{\mu}=(W_{1\mu}\mp iW_{2\mu})/\sqrt{2}$ and $Z_{\mu}=(gW_{3\mu}+g'B_{\mu})/(g^2+g'^2)^{\frac{1}{2}}$ have masses M and $M_Z=Mg^{-1}(g^2+g'^2)^{\frac{1}{2}}$ respectively; while $A_{\mu}=(gB_{\mu}-g'W_{3\mu})/(g^2+g'^2)^{\frac{1}{2}}$ is massless and hence identifies with the photon field. We shall not consider the leptonic Lagrangian for the muon and the muon-neutrino for it has precisely the same form as that of (3), except that m_e is replaced by m_{μ}.

The Lagrangian $\mathscr{L}_{M=0}$, that is (1) with $M=0$ in (4), is invariant under the $SU_2\times U_1$ gauge transformation. Its symmetry structure is exactly the same as that in the Weinberg Lagrangian [1]. We note, however, that the Lagrangian (1) does *not* contain the quartic potential $V(S)=-M_1^2S^{\dagger}S+h(S^{\dagger}S)^2$, which is necessary for spontaneously broken symmetry in the Weinberg theory. Thus, the present theory has little to do with the concept of spontaneously broken gauge symmetry. Furthermore, the masses for the vector fields are built in the theory by the substitution $S^0\to S^0+\sqrt{2}M/g$ in the gauge-invariant Lagrangian $\mathscr{L}_{M=0}$. This is just the same as adding a mass term in the Lagrangian without other deeper reason. In this sense the existence of the masses for the vector fields in (1) may be viewed as due to an ***intrinsic*** symmetry breaking.

For manifestly covariant formulation we may introduce the Lagrange multiplier field χ's in the theory and quantize the Lagrangian

$$\mathscr{L}_{\text{tot}}=\mathscr{L}+\mathscr{L}_L, \tag{5}$$

where

$$\mathscr{L}_L=M\chi_A\partial_{\mu}A^{\mu}+\frac{1}{2}\alpha M^2\chi_A^2+M_Z\chi_Z[\partial_{\mu}Z^{\mu}-iM_Z(S_0-\overline{S}_0)/\sqrt{2}\eta]+\frac{1}{2\eta}M_Z^2\chi_Z^2+$$
$$+\xi^{-1}M^2\chi^+\chi^-+M\chi^-(\partial^{\mu}W^+_{\mu}+iMS^+/\xi)+M\chi^+(\partial^{\mu}W^-_{\mu}-iMS^-/\xi),\qquad \xi>0,\ \eta>0, \tag{6}$$

according to the canonical procedure within the framework of the indefinite-metric quantum field theories [2]. The quantities α, $\eta>0$, $\xi>0$ are real parameters. The spin-1 part of the 4-vector fields $W^{\pm}_{\mu}$, Z_{μ}, A_{μ} and the scalar field $\varphi\equiv(S+\overline{S}^0)/\sqrt{2}$ in (5) have definite masses (*i.e.* M, M_Z, 0, 0 respectively) and are physical by definition. All other fields, *i.e.* the longitudinal and the timelike photons and other scalar fields with masses depending on η or ξ, are unphysical; they are not allowed to appear in the external states of physical processes.

The Lagrangian (5) leads to field equations for $W^{\pm}_{\mu}$, Z_{μ}, A_{μ}, S and $S^{\dagger}$ and constraints. Using these field equations alone, we have

$$\Box\chi_A+t_A=0, \tag{7}$$
$$t_A\equiv ie(W^-_{\mu}\partial^{\mu}\chi^+-W^+_{\mu}\partial^{\mu}\chi^-)-eM(S^-\chi^++S^+\chi^-)/\xi,$$

$$(\Box+M_Z^2/\eta)\chi_Z+t_Z=0, \tag{8}$$
$$t_Z\equiv iG\cos^2\theta(W^+_{\mu}\partial^{\mu}\chi^--W^-_{\mu}\partial^{\mu}\chi^+)+GM_Z\varphi\chi_Z/2\eta+GM\cos2\theta(\chi^+S^-+\chi^-S^+)/2\xi,$$

$$(\Box+M^2/\xi)\chi^++t^+=0, \tag{9}$$
$$t^+\equiv GM\cos\theta(\varphi-i\varphi')\chi^+/2\xi-\partial_{\mu}\chi^+(ieA^{\mu}-iG\cos^2\theta Z^{\mu})+$$
$$+ieW^+_{\mu}\partial^{\mu}\chi_A-iG\cos^2\theta W^+_{\mu}\partial^{\mu}\chi_Z-GM_Z\cos\theta S^+\chi_Z/2\eta,$$
$$G=(g^2+g'^2)^{\frac{1}{2}},\quad g/G=\cos\theta,\quad e=-gg'/G,\quad G_F=g^2/8M^2,\quad \varphi'=(S^0-\overline{S}^0)/\sqrt{2}i,$$

and the adjoint of (9). These equations are obtained by tedious but straightforward calculations. Equation (7) is expected because the Lagrangian $\mathscr{L}_L$ in (5) is not Abelian gauge invariant. The presence of source terms in (7)-(9) indicates that the interactions of the unphysical particles will contribute an extra amplitude in the intermediate states of physical processes and upset unitarity. We should isolate and remove the extra amplitude from the theory, leaving the physical S-matrix unitary. After this, the resultant theory should be independent of the parameters α, η, ξ. This is accomplished by constructing, on the basis of the equations (7)-(9) and the adjoint of (9), a Lagrangian involving complex fictitious scalar fermions D_A, D_Z, D^+ and D^- ([4,5]):

$$(10)\qquad \mathscr{L}_{ff}(D) = \{-\bar{\chi}_A[\Box\chi_A + t_A] - \bar{\chi}_Z[(\Box + M_Z^2/\eta)\chi_Z + t_Z] - \\ - \bar{\chi}^+[(\Box + M^2/\xi)\chi^+ + t^+] - \bar{\chi}^-[(\Box + M^2/\xi)\chi^- + t^-]\}_{\chi'_a \to D'_a} .$$

The unitary amplitude A_u of the theory is then completely defined by

$$(11)\qquad A_u = \int \exp\left[i\int d^4x\,(\mathscr{L} + \mathscr{L}_L + \mathscr{L}_{ff} + \mathscr{L}_s)\right] d[\chi_A, \chi_Z, \chi^\pm, F] = \\ = \text{const}\int \exp\left[i\int d^4x\,(\mathscr{L}_{\text{eff}} + \mathscr{L}_s)\right] d[F] ,$$

where $\mathscr{L}_s$ is the external source terms for physical particles, F stands for a set of fields $\{W_\mu^\pm, Z_\mu, A_\mu, \varphi', \varphi, S^\pm, D, \bar{D}\}$ and the effective Lagrangian is given by

$$(12)\qquad \mathscr{L}_{\text{eff}} = \mathscr{L} + \mathscr{L}_{ff}(D) - \frac{1}{2\alpha}(\partial_\mu A_\mu)^2 - \frac{\eta}{2}(\partial_\mu Z^\mu + M_Z\varphi'/\eta)^2 - \xi|\partial^\mu W_\mu^+ + iMS^+/\xi|^2 .$$

We have integrated over $\chi^\pm$, χ_A, χ_Z in (11), and the constant factor is irrelevant to physics. The effective Lagrangian $\mathscr{L}_{\text{eff}}$ completely defines the theory for leptons. The propagators for the fields in (12) asymptotically fall off as the *second* power of the momentum and the theory based on (12) is manifestly renormalizable ([3]). (We may remark that the terms $i\partial^\mu W_\mu^+ MS^- - i\partial^\mu W_\mu^- MS^+$ coming from the last term in (12) are cancelled by those from the first term $\mathscr{L}$ in (12) and, therefore, there is no W-S transition propagator. Similarly, there is no Z-φ' transition propagator.) The Feynman rules of the theory can be derived from (12). In this theory the unitarity and the independence of the parameters α, ξ, η have been verified by explicit calculations at the one-loop and the two-loop levels ([6]).

We have also calculated (using n-dimensional regularization) some observable quantities to check the consistency of the theory and experiment. For example, the anomalous quadrupole moment ΔQ of the vector particle W at rest is

$$(13)\qquad \Delta Q = \frac{1}{9\pi}\frac{e^2}{4\pi} + \frac{G_F M^2}{\sqrt{2}\,4\pi^2}\left(1 + \frac{M^2}{6M_Z^2}\right) + \frac{G_F M^4}{\sqrt{2}\,6\pi^2 M_Z^2}\int_0^1 dx\,\frac{x^3(8 - 8x - x^2)}{x^2 + (1-x)M_Z^2/M^2} .$$

([4]) J. P. Hsu and E. C. G. Sudarshan: *Phys. Lett.*, **51** B, 349 (1974), and preprint CPT 175 (to be published in *Nucl. Phys.*).

([5]) For the Georgi-Glashow type Lagrangian, cf. J. P. Hsu: *Phys. Rev. D*, **9**, 1113 (1974), Appendix.

([6]) The details will be published elsewhere.

The anomalous magnetic moment Δa_e of the electron is, aside from $\alpha/2\pi$,

$$\Delta a_e = -\frac{1}{3\pi}\frac{e^2}{4\pi}\left(\frac{m_e}{M_Z}\right)^2 + \frac{G_F m_e^2}{\sqrt{2}\pi^2}\left(\frac{23}{24}-\frac{M^2}{4M_Z^2}\right). \tag{14}$$

These results are the same as those in the Weinberg theory of lepton with $m_\varphi = 0$ ([7]). From these calculations one sees that there is no genuine infra-red divergence like that in the massless Yang-Mills field although the φ mass is zero. The W mass is greater than 37 GeV. The e-φ coupling constant is

$$\frac{gm_e}{\sqrt{2}\,M} \sim 2\cdot 10^{-6}\,, \tag{15}$$

which is too small to be detected in the laboratory ([8]). However, since φ is massless, (15) may have important consequence in the evolution of stars, similar to the role played by the massless neutrino with a very weak coupling ([9]).

Finally, we may remark that the independence of ξ, η, α in this theory is due to the fact that the Lagrangian $\mathscr{L}$ in (1) is strictly invariant under a «distorted» gauge transformation ([10]) (although it is not invariant under the usual gauge transformation). The same situation occurs in gauge theories, namely, their Lagrangians (after spontaneous symmetry break-down) are invariant under «distorted» gauge transformations ([10]).

We note that, within the framework of *perturbation* theory, the Feynman rules derived from (11) are the same as those from the Weinberg theory ([1]) in the limit $M_1 \to 0$, $h \to 0$ but M_1^2/h finite. However, the present formalism shows that one can set $M_1 = h = 0$ in the Weinberg Lagrangian from the outset and generate vector-boson mass via an intrinsic symmetry breaking. Furthermore, we have the interesting possibility that such a theory of massive vector bosons could be asymptotically free. The reason being that, in a gauge theory of scalar and vector bosons, with no fermions and no $h(S^\dagger S)^2$ coupling, the S-S scattering obtained by *nonperturbative* computation is actually finite ([11]).

([7]) W. A. BARDEEN, R. GASTMANS and B. LANTRUP: CERN TH-1485 (1972).

([8]) Such a coupling of the electron to massless φ-quanta would imply a long-range attractive force between electrons and hence imply, in turn, substantial attractive forces between matter several orders of magnitude larger than the gravitational forces. To avoid such an absurd circumstance we must have compensating forces coming in from the coupling of φ-quanta with protons. A coupling of equal magnitude and opposite sign of the φ field with protons would accomplish this.

([9]) See, for example, H. Y. CHIU and P. MORRISON: *Phys. Rev. Lett.*, **5**, 573 (1960); D. A. DICUS: *Phys. Rev. D*, **6**, 941 (1972).

([10]) G. 'T HOOFT and M. VELTMAN: CERN preprint, TH-1571; J. P. HSU: *Lett. Nuovo Cimento*, **11**, 525 (1974). The latter contains general formal proof of unitarity and gauge independence of the renormalized S-matrix for the Lagrangian with distorted gauge symmetry.

([11]) A. SALAM and R. DELBOURGO: *Phys. Rev.*, **135**, B 1398 (1964); R. DELBOURGO, A. SALAM and J. STRATHDEE: *Scalar multiplets and asymptotic Freedom*, ICTP/73/29 (1974).