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Unified Theory of Weak and Electromagnetic Interactions Without Spontaneously Broken Gauge Symmetry I*

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SUMMARY

We construct a class of renormalizable and "unified" theories of electromagnetic and weak interactions without Higgs mesons and without spontaneously broken gauge symmetry. We employ Lagrange multiplier fields in the construction of these theories, starting from a Yang-Mills type Lagrangian whose isospin gauge symmetry is broken by mass terms. These mass terms may be chosen to violate even the global isospin transformations; we need this generality to deal with massless photons and massive intermediate vector bosons within the same "unified" theory. The construction relies on a highly symmetric <u>interaction</u> structure (which is not shared by the total Lagrangian) to bring about renormalizability. The theory so constructed is <u>renormalizable</u>, <u>covariant</u> and <u>asymptotically free</u>. Once a family of fictitious particles with well-specified interactions are introduced into the theory the S-matrix may be calculated using a set of simple Feynman rules and is seen to be explicitly unitary. In the definition of the Lagrangian of the interacting fields we have two free parameters α and ξ; the propagators and the vertices do depend on these parameters. The parameter α corresponds to a gauge change in the photon propagator; and ξ is the ratio of the square of the mass of the charged intermediate boson to that of its scalar counterpart so that the limit $\xi \to 0$ corresponds to infinitely heavy scalar quanta and a manifestly unitary S-matrix.

1. Introduction

It is an attractive proposal that electromagnetism and weak interactions be unified so that there is essentially only one coupling constant, the weakness of weak interactions being related to the large mass of the charged intermediate vector mesons. With such theories there is also the possibility that the unified theory is renormalizable. The present paper is the first of a series dealing with this problem and the general theory of particle interactions.

It is usually believed that the Weinberg-Salam type models of weak and electromagnetic interactions are renormalizable because the vector mesons get their masses from spontaneously broken gauge symmetry, not from mass terms put in at the beginning.[1] We shall show that, by introducing a Lagrange multiplier field,[2] we can construct unified theories of electromagnetic and weak interactions which are renormalizable and in which the vector mesons get their masses from mass terms put in at the beginning. The unified theories constructed in this way are much simpler because there is no scalar Higgs field with complicated interaction terms. This simplicity drastically changes the nature of theory. In fact, the unified theory without spontaneously broken gauge symmetry is, in contrast to the unified gauge theories, asymptotically free.[4]

Asymptotically free theories are of great interest physically because they exhibit Bjorken scaling.

In such a unified theory, we still have a relation between G_F, e and M (the mass of the intermediate vector bosons), which is essentially the same as that in the Weinberg model. The observed dissimilarities between weak and electromagnetic interactions are attributed to the mass difference between $W_\mu^\pm$ field and the photon field A_μ. Admittedly, we do not understand why M is so massive while the photon mass is zero. In this sense, the unified theory is obtained by simply putting $W_\mu^\pm$ and A_μ together in the same Lagrangian with the same coupling constant and with suitable coupling terms. On the other hand, in the usual unified theories, M is expressed in terms of another unknown parameter, which is also put in the Lagrangian "by hand", at least in calculations carried out so far, rather than obtained as the result of a dynamical calculation. Therefore, unless one has a way to calculate M from "first principles", the usual unified theories with a relation between G_F, e and M do not seem to help our understanding of nature to any greater extent. However, if a vector meson with the predicted mass is found, it would be intriguing. And the present model allows the presence of massive charged vector mesons, yet still retaining the renormalizability of the model.

2. A Renormalizable Unified Model Without Isospin Gauge Symmetry

We shall first consider a simple model to illustrate how to construct a renormalizable model for the massive vector bosons coupled to a nonconserved current (and not appealing to gauge symmetry and its spontaneous breakdown). For simplicity, we shall consider a physical system involving three vector bosons and two fermions. The Lagrangian for the vector bosons is "similar" to that in the Georgi-Glashow model with the weak intermediate boson mass put in at the beginning (and without any scalar Higgs field). A model of η^{abc} leptons is given in sec. 5.

The Lagrangian of the model is

$$\mathscr{L} = \mathscr{L}_B + \mathscr{L}_\psi , \tag{1}$$

$$\mathscr{L}_B = -\frac{1}{4}\vec{B}_{\mu\nu}\cdot\vec{B}^{\mu\nu}+\frac{1}{2}M^2\vec{B}_{\mu t}\cdot\vec{B}^{\mu}_{t} +M\vec{\chi}\cdot(\partial_\mu\vec{B}^\mu-e(\vec{B}^\mu x\vec{\eta})\vec{B}_\mu\cdot\vec{\eta})+\frac{\alpha}{2}(M\chi_3)^2+\frac{1}{2}(M\vec{\chi}_t)^2 ,$$

$$\mathscr{L}_\psi = -\bar{\psi}\gamma^\mu(-i\partial_\mu-\frac{1}{2}e\vec{\tau}\cdot\vec{B}_\mu)\psi-m\bar{\psi}\psi-\Delta\bar{\psi}\tau^3\psi ,$$

where $\vec{B}_{\mu\nu}=\partial_\mu\vec{B}_\nu-\partial_\nu\vec{B}_\mu+e\vec{B}_\mu x\vec{B}_\nu$,

$$\vec{B}^\mu =\vec{B}_t+\vec{\eta}B^\mu_3,\ \vec{B}^\mu_t\cdot\vec{\eta}=0,\ \vec{\eta}^2=1, \tag{2}$$

and $\vec{\chi}$ is the Lagrange multiplier field, and M, m, e and α are real. The charged boson field $W^\pm_\mu$ with mass M is defined by

$$W^{\mp}_\mu \equiv \frac{1}{\sqrt{2}}\ (B^{1}_\mu \pm iB^2_\mu) , \tag{3}$$

and the massless photon field is $B_3^\mu = A^\mu$. We note that the Lagrangian $\mathcal{L}$ is <u>not</u> invariant under the isospin gauge transformation because of the presence of mass terms and the terms involving the Lagrange multiplier fields. Yet, it is still Abelian-gauge invariant. (cf. equations (12), (13), and (14) below).

The Lagrangian (1) leads to the equations:

$$-\partial_\mu \vec{B}^{\mu\nu} + M\partial^\nu \vec{\chi} + e\vec{B}^{\mu\nu} \times \vec{B}_\mu - M^2 \vec{B}_t^\nu - \frac{1}{2} e\bar{\psi}\gamma^\nu \vec{\tau}\psi + Me\vec{\chi}\cdot(\vec{B}^\nu \times \vec{\eta})\vec{\eta} - Me(\vec{\chi}\times\vec{\eta})A^\nu = 0, \quad (4)$$

$$\partial_\mu \vec{B}_t^\mu + M\vec{\chi}_t - e(\vec{B}^\mu \times \vec{\eta})A_\mu = 0, \quad (5)$$

$$\partial_\mu B_3^\mu + \alpha M\chi_0 = 0 \quad , \qquad (\chi_0 \equiv \chi_3). \quad (6)$$

$$\gamma^\mu(-i\partial_\mu - \frac{1}{2} e\vec{\tau}\cdot\vec{B}_\mu)\psi + m\psi + \Delta\tau^3\psi = 0. \quad (7)$$

From eq. (7) and its adjoint equation, we have

$$\partial_\nu(\bar{\psi}\gamma^\nu \tau^a \psi) = e\varepsilon^{abc}\bar{\psi}\gamma^\nu \tau^b \psi B_\nu^c + 2\Delta\varepsilon^{ca3}\bar{\psi}\tau^c\psi. \quad (8)$$

The divergence of (4) together with (5), (6) and (8) gives

$$(\Box + M^2)\vec{\chi}_t - 2eA_\nu(\partial^\nu \vec{\chi}_t \times \vec{\eta}) - e^2 A_\mu A^\mu \vec{\chi}_t - e^2[\vec{\chi}\cdot(\vec{B}^\mu \times \vec{\eta})]\vec{\eta}\times\vec{B}_\mu - e\Delta\vec{\eta}\times\bar{\psi}\vec{\tau}\psi = 0, \quad (9)$$

$$\Box\chi_0 = 0, \quad (10)$$

where we have set $\partial_\nu \chi_0 = \chi_0 = 0$ in (9), which is consistent with (10).

We note that, just as in quantum electrodynamics, the "longitudinal" component $\partial_\mu A^\mu \propto \chi_0$ in this model is a free field.

The interactions of the Lagrange multiplier field $\vec{\chi}_t$ in the intermediate states will contribute an extra amplitude and upset unitarity unless we redefine the physical amplitude by a suitable modification. The equation (9) completely determines the interactions of the Lagrange multiplier field $\vec{\chi}_t$. For unitarity, the amplitudes due to the source terms in the equation (9) must be removed from the theory.[2,3] We may remark that, although the Lagrange multiplier fields $\vec{\chi}$ do not have kinetic terms in (1), the masses and the interactions of $\vec{\chi}$ can be seen from their equations of motion (9), and (10).

The Lagrangian (1) can be quantized according the usual quantization procedure, just like the massive Yang-Mills field with a Lagrange multiplier field.[3] For our purpose here, we may mention that after quantization $\vec{\chi}_t$ is a negative metric field while $\chi_0 \equiv \chi_3$ is a massless field with zero norm. Consequently, the unphysical particles corresponding to these unphysical fields should not be allowed, by definition, to appear in the initial and final physical states.

In terms of W^+_μ, W^-_μ and A_μ, the Lagrangian (1) is

$$\mathscr{L}=\mathscr{L}_{W\gamma} + \mathscr{L}_\psi + \mathscr{L}_L, \tag{11}$$

$$\begin{aligned}\mathscr{L}_{W\gamma} &= -\frac{1}{2}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu + ie(W^+_\mu A_\nu - W^+_\nu A_\mu)|^2 + M^2 W^+_\mu W^{-\mu} \\ &\quad -\frac{1}{4}|\partial_\mu A_\nu - \partial_\nu A_\mu - ie(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu)|^2,\end{aligned} \tag{12}$$

$$\mathcal{L}_L = M\chi_0\partial^\mu A_\mu+\frac{\alpha}{2}M^2\chi_0^2+M\chi^+(\partial_\mu+ieA_\mu)W^{-\mu}+M\chi^-(\partial_\mu-ieA_\mu)W^{+\mu}+M^2\chi^+\chi^-. \tag{13}$$

It is clear that the Lagrangians $\mathcal{L}_{W\gamma}$, $\mathcal{L}_\psi$, and $\mathcal{L}_L$ are invariant under the restricted gauge transformation

$$W_\mu^+\to W_\mu^+e^{-ie\Lambda(x)},\quad W_\mu^-\to W_\mu^-e^{+ie\Lambda(x)},\quad A_\mu\to A_\mu-\partial_\mu\Lambda(x);\ \Box\Lambda(x)=0 \tag{14}$$

$$\chi^+\to\chi^+e^{-ie\Lambda(x)},\quad \chi^-\to\chi^-e^{+ie\Lambda(x)},\quad \chi_0\to\chi_0;\quad \psi\to\psi\exp(-\tfrac{i}{2}e\tau_3\Lambda(x)).$$

Equation (9) can be written as

$$(\Box+M^2)\chi^-+2ieA_\nu\partial^\nu\chi^--e^2A_\mu A^\mu\chi^-+e^2W^{-\mu}(\chi^-W_\mu^+-\chi^+W_\mu^-)-2ie\Delta\bar{\psi}\tau^+\psi=0 \tag{15}$$

and

$$(\Box+M^2)\chi^+-2ieA_\nu\partial^\nu\chi^+-e^2A_\mu A^\mu\chi^++e^2W^{+\mu}(\chi^+W_\mu^--\chi^-W_\mu^+)+2ie\Delta\bar{\psi}\tau^-\psi=0 \tag{16}$$

We observe that these two equations could be derived from an effective Lagrangian for the negative metric fields $\chi^\pm$:

$$\begin{aligned}\mathcal{L}(\chi^+,\chi^-) = &-\partial_\mu\chi^+\partial^\mu\chi^-+M^2\chi^+\chi^-+2ieA_\nu\chi^+\partial^\nu\chi^- \\ &-e^2A_\mu A^\mu\chi^+\chi^--\tfrac{1}{2}e^2(W_\mu^-\chi^+-W_\mu^+\chi^-)^2+2ie\Delta\bar{\psi}\tau^+\psi\chi^+-2ie\Delta\bar{\psi}\tau^-\psi\chi^-.\end{aligned} \tag{17}$$

The Lagrangian $\mathcal{L}(\chi)$ completely determines the interactions of χ^+ and χ^-, which upset unitarity of the theory. We note that the couplings in (17) are of renormalizable type.

3. The Unitary Transition Amplitude

We are only interested in the amplitude with external physical particles. The primitive transition amplitude can be expressed as the path integral

$$A = \int \exp\{i\int d^4x(\mathscr{L}+\mathscr{L}_s)\}d[W^+_\lambda,W^-_\rho,A_\mu,\psi,\bar{\psi},\chi^+,\chi^-,\chi_0]$$

$$= \int \exp\{i\int d^4x(\mathscr{L}_{W\gamma}-\frac{1}{2\alpha}(\partial_\mu A^\mu)^2-|(\partial^\mu_\mu+ieA^\mu)W^-_\mu|^2+\mathscr{L}_\psi+\mathscr{L}_s)\}$$

$$\times d[W^+_\lambda,W^-_\rho,A_\mu,\psi,\bar{\psi}] \quad , \tag{18}$$

$\mathscr{L}_s$ = external source term.

This amplitude A is not unitary because the unphysical degrees of freedom in the 4-vector fields $W^\pm_\mu$ are coupled to the three physical degrees of freedom. The extra amplitude X due to such coupling must be removed from (18) in order to obtain a unitary theory. With the help of the Lagrange multiplier fields, the unwanted extra amplitude can be expressed in the form

$$X = \int \exp\{i\int d^4x \;\mathscr{L}(\chi^+,\chi^-)\}\; d[\chi^+,\chi^-] \quad , \tag{19}$$

where $\mathscr{L}(\chi^+,\chi^-)$ is given by (17); it is a function of $W^\pm_\mu$, A_μ and ψ and can be transcribed into an easily computed expression using fictitious fields. If the terms involving ψ are omitted we get

$$X(\psi=0) = \{\det R\}^{-\frac{1}{2}} \quad , \tag{20}$$

$$R = R_+R_- - \frac{1}{4}\vec{Q}\;\vec{Q} \quad ,$$

$$R_{\pm} = -\frac{1}{2}e^2 W_\mu^{\pm} W^{\mu\pm} ,$$

$$Q = \Box + M^2 + 2ieA_\nu \partial^\nu - e^2 A_\nu A^\nu + e^2 W_\mu^- W^{\mu+} .$$

In any case, to remove the extra amplitude from (18) we multiply the integrand of the path integral by X^{-1} to define the physical transition amplitude which is unitary. We obtain

$$A_u = \int X^{-1} \exp\{i\int d^4x(\mathscr{L}_{WY} - \frac{1}{2\alpha}(\partial_\mu A^\mu)^2 - |(\partial^\mu + ieA^\mu)W_\mu^-|^2 + \mathscr{L}_\psi + \mathscr{L}_S)\}$$

$$d[W_\lambda^+, W_\rho^-, A_\mu, \psi, \bar{\psi}] , \tag{21}$$

where the external particles are physical. In the general case X^{-1} can be expressed in terms of a Lagrangian density involving fictitious fields:

$$X^{-1} = \{\det R\}Y, \tag{22}$$

$$Y = \int \exp\{i\int d^4x \mathscr{L}(F^+, F^-)\} d[F^+, F^-] ,$$

$$\mathscr{L}(F^+, F^-) = \partial_\mu F^+ \partial^\mu F^- - M^2 F^+ F^- - 2ieA_\nu F^+ \partial^\nu F^- + e^2 A_\mu A^\mu F^+ F^- + \frac{1}{2}e^2 |W_\mu^- F^+ - W_\mu^+ F^-|^2$$

$$-2ie\Delta\bar{\psi}\tau^+\psi F^+ + 2ie\Delta\bar{\psi}\tau^-\psi F^- . \tag{23}$$

We note that Y is computed in terms of fictitious scalar fields $F^{\pm}(x)$ which obey Bose statistics using simple Feynman rules; the interactions of the $F^{\pm}$ fields are renormalizable.

It is useful to note that the amplitude Y is related simply to det R. We get

$$\int \exp\{i\int d^4x \mathscr{L}(F^+, F^-)|_{\psi=\bar{\psi}=0}\} d(F^+, F^-) = Y(\psi=0) = (\det R)^{-\frac{1}{2}} = X(\psi=0) . \tag{24}$$

Since the contributions come from $\ln X$ and $\ln Y$, the contribution from the factor (det R) is twice the contribution from $Y(\psi=0)$ and of the opposite sign. Thus we may compute the contribution from the unitarizing factor X^{-1} in terms of two sets of fictitious scalar particles $D^{\pm}$ obeying Fermi statistics and one set of fictitious scalar particles $F^{\pm}$ obeying Bose statistics. The same thing happens also in the case of massive Yang-Mills field as discussed in the second paper of Ref. 4.

4. Rules for the Feynman Diagrams

Since the fermion Lagrangian $\mathscr{L}_\psi$ is in any case, not realistic for the unified theory, we shall ignore the fermion ψ in this section. The physical amplitude (21) completely defines the theory. The rules for the Feynman diagrams can be derived from the path integral (21). The contributions to the amplitudes due to the fictitious scalar fermion $D^\pm$ which come from det R cancel those due to the fictitious scalar boson $F^\pm$ which come from $(\det R)^{-\frac{1}{2}}$. Effectively, we are left with the amplitudes corresponding to $(\det R)^{+\frac{1}{2}}$. The rules are given in Fig. 1. One can see from these rules that the theory is explicitly renormalizable by power-counting. It can be shown that the theory is unitary by explicit calculations of the nontrivial lowest order processes.[4] We may remark that the renormalizability remains intact when the fermions are included according to the prescription. Also, the fictitious $D^\pm$ are, by definition, not allowed to appear either in the initial or in the final physical states. A more realistic model of leptons is given in section 5.

5. Discussion

The renormalizable Lagrangian is constructed by starting with the Yang-Mills Lagrangian and then we break it by adding the mass terms. In other words the kinematic and the interaction terms have a high degree of symmetry which is destroyed by the mass terms. This situation is reminiscent of the structure of the chiral V-A four-fermion interaction in beta decay.[5] Another way of constructing a renormalizable model is as follows. Suppose one tries a model for the interaction of massive charged vector boson field $W^{\pm}_{\mu}(x)$ and the photon field. One introduces the Lagrange multiplier fields χ^{+}, χ^{-}, and χ_0, and one writes down all possible couplings consistent with the gauge transformation (14); etc.,:

$$\begin{aligned}\mathscr{L}^{\xi}_{WA} = & -\frac{1}{2}G^{+}_{\mu\nu}G^{-\mu\nu}+M^2W^{+}_{\mu}W^{-\mu}-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & +ie\kappa F^{\mu\nu}W^{+}_{\mu}W^{-}_{\nu}+(ie)^2\lambda(W^{+}_{\mu}W^{-}_{\nu}-W^{-}_{\mu}W^{+}_{\nu})W^{+\mu}W^{-\nu} \\ & +M\chi^{-}(\partial^{\mu}-ieA^{\mu})W^{+}_{\mu}+M\chi^{+}(\partial^{\mu}+ieA^{\mu})W^{-}_{\mu}+\xi^{-1}M^2\chi^{+}\chi^{-} \\ & +M\partial^{\mu}A_{\mu}\chi_0 + \alpha M^2\chi_0^2/2 \quad , \end{aligned} \tag{25}$$

$$G^{+}_{\mu\nu} = (\partial_{\mu}-ieA_{\mu})W^{+}_{\nu}-(\partial_{\nu}-ieA_{\nu})W^{+}_{\mu},$$

$$F_{\mu\nu} = \partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}.$$

The problem of ξ independence will be discussed below. By appeal to the variational principle, one obtains the field equations for $W_\mu^\pm$ and A_μ and the constraints for $\chi^\pm$ and χ_0. One may derive, for example, the field equation for χ^+ by taking the divergence of the field equation for W_μ^+ and using the constraint $(\partial^\mu - ieA^\mu)W_\mu^+ + M\chi^+ = 0$ for χ^+:

$$\begin{aligned}(\Box+\xi^{-1}M^2)\chi^+ = {} & 2ieA_\nu\partial^\nu\chi^+ + e^2A_\mu A^\mu\chi^+ - e^2W^{+\mu}(\chi^+W_\mu^- - \chi^-W_\mu^+)\\ & -\tfrac{1}{2}ie\xi^{-1}(\kappa-1)F^{\mu\nu}G_{\mu\nu}^+\\ & +(ie)\xi^{-1}(\kappa+2\lambda)W^{\nu+}W^{\mu-}G_{\mu\nu}^+\\ & +(ie)^2\xi^{-1}(\kappa^2+2\lambda)W_\nu^+\partial^\mu(W^{\mu-}W_\nu^+)\\ & -(ie)^2\xi^{-1}(\kappa^2+2\lambda)W_\nu^+\partial^\mu(W^{\nu-}W_\mu^+);\ \xi\neq 0. \end{aligned} \tag{26}$$

Because of the parameter ξ in (3), the propagators for the vector fields $W_\mu^\pm$ and the fictitious field $D^\pm$ are respectively:

$$-i(g_{\mu\nu}-(1-\xi^{-1})k_\mu k_\nu/(k^2-\xi^{-1}M^2))(k^2-M^2)^{-1} \tag{27}$$

and

$$i(k^2-\xi^{-1}M^2)^{-1}. \tag{28}$$

Thus, the source terms $2ieA_\nu\partial^\nu\chi^+$, $e^2A_\mu A^\mu\chi^+$ and $e^2W^{+\mu}(\chi^+W_\mu^- - \chi^-W_\mu^+)$ correspond to renormalizable vertices, while all other source terms in (26) correspond to nonrenormalizable vertices. However, if one chooses $\kappa = -2\lambda = 1$ all nonrenormalizable vertices vanish and the theory becomes renormalizable as seen by standard power-counting. With this choice of κ, λ and $\xi = 1$, the Lagrangian (25) is identical to $\mathcal{L}_{W\gamma}$ and $\mathcal{L}_L$ given by (12) and (13) which is obtained from symmetry considerations.

We observe that the parameter ξ enters into the Lagrangian (25) through the Lagrange multiplier field $\chi^{\pm}$ only. It changes the mass of $\chi^{\pm}$-quanta as shown in (26). Since the extra unwanted amplitudes due to the interactions of $\chi^{\pm}$ are to be removed from the theory by the general method discussed in sections 2 and 3, the resultant theory must be independent of ξ. If one starts from (26), then the propagators for $W_{\mu}^{\pm}$ and $D^{\pm}$ in Fig. 1 are respectively replaced by (27) and (28). Also, the rules for the γWW and $\gamma\gamma WW$ vertices in Fig. 1 are replaced respectively by

$$ie\{(p+p')_{\mu}g_{\alpha\beta}+[2q_{\beta}+(\xi-1)p'_{\beta}]g_{\alpha\mu}-[2q_{\alpha}-(\xi-1)p_{\alpha}]g_{\beta\mu}\}. \qquad (29)$$

and

$$-ie^{2}[2g_{\alpha\beta}g_{\mu\nu}-(1-\xi)g_{\alpha\mu}g_{\beta\nu}-(1-\xi)g_{\alpha\nu}g_{\beta\mu}]. \qquad (30)$$

All others in Fig. 1 are unchanged. It is gratifying to have the physical transition amplitudes turn out to be independent of the parameter ξ especially since the theory is not isospin gauge invariant. Explicit calculations of the fourth order processes $WW \rightarrow WW$ and $W\gamma \rightarrow W\gamma$ show that the physical amplitudes are indeed independent of ξ and, therefore, unitary. If this is generally true (and we believe that this is so), then one may take the limit $\xi \to 0$, in which the masses of the unphysical particles become infinity. In this limit, there can be no unphysical particles in either the initial state or the final state. Thus, the S matrix is manifestly unitary. The situation is similar to that discussed in Ref. 8. We may remark that one must be careful in taking the limit $\xi \to 0$ because it may interfere with the large-momentum contributions to the loop integrations; apparently one has to carry out integration in the amplitudes and then let $\xi \to 0$.

The Lagrangian (1) is not realistic because $\mathscr{L}_\psi$ does not correctly describe the interactions of known fermions. For a more realistic model we may, for example, replace $\mathscr{L}_\psi$ by $\mathscr{L}_\ell$:[6]

$$\begin{aligned}\mathscr{L}_\ell = {} & \bar{y}^+(i\gamma^\mu\partial_\mu - m_{y+})y^+ + \bar{y}^0(i\gamma^\mu\partial_\mu - m_{y0})y^0 \\ & + \bar{\mu}(i\gamma^\mu\partial_\mu - m_\mu)\mu + \bar{\nu}_L i\gamma^\mu\partial_\mu\nu_L \\ & + e\bar{L}\gamma^\mu(\vec{T}\cdot\vec{B})_\mu L + e\bar{R}\gamma^\mu(\vec{T}\cdot\vec{B})_\mu R ,\end{aligned} \tag{29}$$

where

$$L \equiv \begin{pmatrix} y_L^+ \\ \cos\theta\, y_L^0 + \sin\theta\, \nu_L \\ \mu_L \end{pmatrix} , \qquad R \equiv \begin{pmatrix} y_R^+ \\ y_R^0 \\ \mu_R \end{pmatrix} ,$$

$$e^2\sin^2\theta/(4M^2) = G_F 2^{-\frac{1}{2}}$$

$$(\vec{T}\cdot\vec{B})_\mu = \begin{pmatrix} A_\mu & -W^+_\mu & 0 \\ -W^-_\mu & 0 & W^+_\mu \\ 0 & W^-_\mu & -A_\mu \end{pmatrix} ,$$

and μ_R, μ_L are the right and left handed components. The particles y^+ and y^0 are hypothetical heavy leptons.[7] The physical particles involved in this model are almost the same as those in Ref. (6). However, the structure of the present model is much simpler. The leptons only couple to $W^\pm_\mu$ and A_μ in a highly symmetric form. The couplings between charged particles and the photon are explicitly gauge invariant. Co quently, equation (10) still holds. The form of equation (15) is essentially the same. Namely, the source terms involving bosons are precisely the same and the term $-2ie\Delta\bar{\psi}\tau^+\psi$ is replaced by $-ie\sin\theta(m_\mu/\mu)\bar{\mu}_R\nu_L$ and other terms with the same form. The resultant rules for the diagrams involving leptons are essentially the same as that in the Georgi-Glashow model.[7] An important difference of the Lagrangian (27) from that in the Georgi-Glashow model is that the lepton masses in (27) are free parameters and that there is no relationship among them.

6. Remarks and Conclusion

The introduction of a Lagrange multiplier field is essential for the covariance and the unitarity of the theory. It makes it possible to treat all four components of vector fields as independent canonical variables and it leads to covariant rules for the Feynman diagrams.[8] Its equation of motion shows how the unphysical components of the four-vector fields interact and upset unitarity.[2,3] With its help, we can define the unitary amplitudes of the theory.

The Lagrangian $\mathscr{L}^{\xi}_{wA}$ in (25) with $\lambda = 0$ is the same as that considered in Ref. 8 with arbitrary ξ. In this case, the interactions of χ^{+} as shown in (26) are badly divergent and non-renormalizable.[9] The condition $\kappa = -2\lambda = 1$ is essential for renormalizability as can be seen from (26).

In our opinion, the crucial step in constructing a renormalizable theory of massive vector mesons is to arrange the coupling terms such that the source terms in the field equation for the Lagrange multiplier correspond to renormalizable vertices. Equivalently, a renormalizable unified theory should have the Yang-Mills type Lagrangian[10] with additional mass terms as the only terms which break the isospin gauge symmetry. In this way, one can dispense with the concept of spontaneously broken gauge symmetry in constructing a renormalizable theory.

What we have achieved here is the formulation of a renormalizable unified theory of weak and electromagnetic interactions without Higgs mesons and without spontaneously broken gauge symmetry. Using this approach, the resultant theory is much simpler than the conventional unified gauge theory. This simplicity leads, as a by-product, to the result that the theory is asymptotically free.[4] Furthermore, the general method indicates an interesting possibility that one can formulate a class of equivalent theories with different values of the parameter ξ. We have verified unitarity[4] and ξ independence of several nontrivial lowest order processes. This indicates that the general method of removing the unwanted amplitudes is indeed working in the processes we have considered. In fact, this method has also been successfully applied to obtain unitary amplitudes in quantum electrodynamics with a nonlinear gauge condition, in conventional unified gauge theories and in the theory of massive and massless Yang-Mills fields.[2,3] The most interesting point of this method is that it sheds light on a new way for constructing massive and asymptotically free unified theories which includes all the strong, weak and electromagnetic interactions.[4]

Asymptotically free quantum field theories have been sought as a possible scheme in which Bjorken scaling is essentially valid. Such theories imply[11] that the effective coupling constant for large (space-like) momenta tend to vanish; and they would reproduce the naive "light-cone" or "parton model" relations. But it has been shown[12] that field theories with Yukawa, quartic scalar or Abelian vector field couplings only are not asymptotically free and that unless a theory contains non-Abelian gauge fields it cannot be asymptotically free. There is also the conjecture that the asymptotic stability of the Yang-Mills type theories result from their severe infra-red singularities.[11] Fermions can be incorporated into such a gauge field theory and even some scalar mesons. But the number of scalar fields is severely restricted and it appears difficult to include a sufficient number of them to break the gauge symmetry completely and provide masses for all the vector mesons relying on Higgs mesons.

Our work[4] shows that asymptotically free theories can be constructed involving self-coupled vector mesons with the coupling term restricted to a highly symmetric form. But the vector mesons need not have zero mass, nor do the mesons need have all the same mass. Unless we <u>choose</u> some of the masses to be zero there is no question of infrared problems. The asymptotic freedom is therefore <u>not</u> a consequence of the infrared singularities. We can introduce fermions and scalar mesons in moderation. In subsequent papers of this series we intend to present more realistic models of particle interactions; but we stress that the general theoretical framework we have developed in these papers is capable of generating strong interactions without the embarrassment of zero mass gluons and without destroying asymptotic freedom.

When we calculate the renormalization group parameters we see[4] that the asymptotic freedom can be traced to the predominant "anomalous magnetic" coupling of the vector mesons implicit in the symmetric vector meson self-coupling. It is known from elementary electromagnetism that while like changes repel, like magnets attract each other. This feature rather than any drastic shielding resulting from severe infrared singularities is the probable reason for the asymptotic freedom of our theories: and perhaps for all gauge-like theories.

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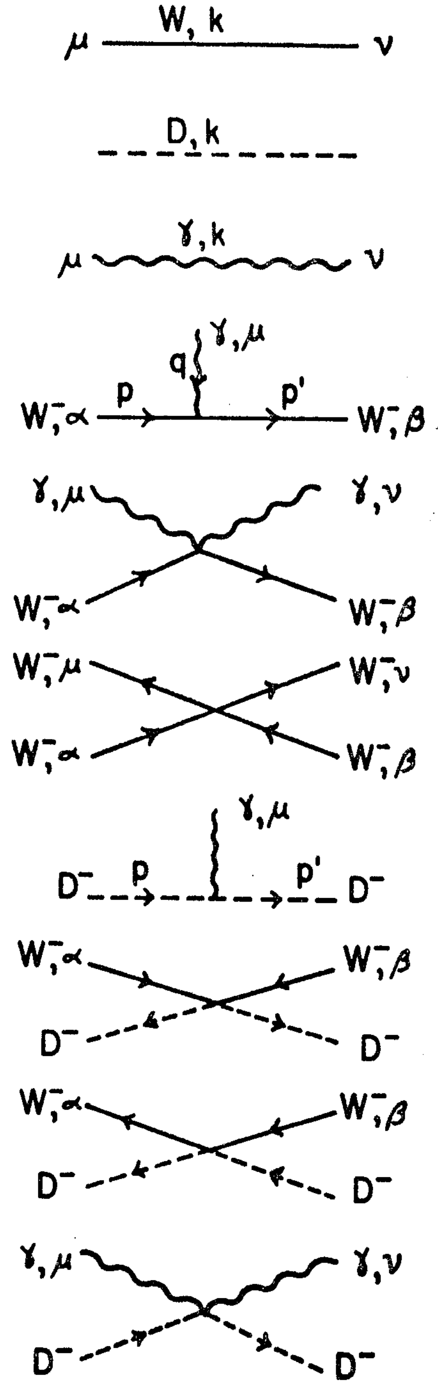
Figure Caption

Fig. 1. The rules for the Feynman diagrams involving only the bosons. The fictitious D-loop carries an additional factor -1.



$$-ig_{\mu\nu}/(k^2-M^2)$$

$$i/(k^2-M^2)$$

$$-i\left[g_{\mu\nu}-(1-\alpha)k_\mu k_\nu/k^2\right]/k^2$$

$$ie\left[(p+p')_\mu g_{\alpha\beta}+2q_\beta g_{\mu\alpha}-2q_\alpha g_{\mu\beta}\right]$$

$$-2ie^2 g_{\mu\nu}g_{\alpha\beta}$$

$$-ie^2(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}-2g_{\mu\nu}g_{\alpha\beta})$$

$$-ie(p+p')_\mu$$

$$2ie^2 g_{\alpha\beta}$$

$$-ie^2 g_{\alpha\beta}$$

$$2ie^2 g_{\mu\nu}$$

Fig. 1