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The Superfluid Vacuum State, Time-Varying Cosmological Constant, and Nonsingular Cosmological Models

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Some cosmological consequences of the superfluid vacuum state developed previously by the authors are discussed, particularly with regard to the initial stages of the universe. The transition temperature of the hadronic superfluid (superfluid during the hadron era) is estimated to be 10^{13} K, which is the same as the Hagedorn temperature, giving a physical basis of the thermodynamic bootstrap model.

1. INTRODUCTION

In a recent paper,[1] the vacuum state, normally undefined in field theories (or defined only up to an arbitrary constant), was postulated to be a superfluid of particle–antiparticle (fermion) pairs. The global effect of such a superfluid vacuum we sought to describe through the Einstein field equations by retaining the cosmological term $\Lambda g_{\mu\nu}$ and identifying $g_{\mu\nu}\Lambda/\kappa$ as the energy–momentum tensor $(T_{\mu\nu})_{\text{vac}}$ of this vacuum.

The introduction of such a term would also be consistent with current ideas of spontaneous symmetry breaking in gauge field theories, which have recently led to unified models of weak and electromagnetic interactions and seem to provide a promising basis for understanding strong interactions also. With a view to accommodating the gravitational interactions as well in this framework, some authors have recently made the suggestion that a relationship exists between spontaneous broken gauge theories and the cosmological constant. Indeed, Linde[2] has pointed out that by disregarding the Λ term in Einstein's gravitational equations we would encounter a conflict with modern ideas of elementary particle theory, that is, the ideas of gauge-invariant

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Lagrangians and spontaneous symmetry breaking. Spontaneous symmetry breaking results in a change in the temperature of the medium which decreases with increase of this energy and vanishes at a certain critical temperature. When applied to the vacuum energy given by the cosmological term this implies a dependence of Λ on the temperature of the medium. For the evolving hot Robertson–Walker universes this in turn implies a dependence of Λ on time. One can consider the nature of the dependence of Λ on temperature (or time) in a particular model, say the Salam[3]–Weinberg[4] model. In fact Linde concludes that the gauge theories of elementary particles with spontaneous symmetry breaking introduce into the gravitational field equations a cosmological term that varies with time and that the change in this term during the evolution of the universe amounts to at least 49 orders of magnitude (in the context of the Salam–Weinberg model) from the experimentally observed value of the constant.

We shall examine below the consequences of such a rapidly changing Λ term for the early universe, especially the possibility of averting singularities in cosmological models with a Λ term. Dreitlein[5] has considered the case of a cosmological Higgs[6] field which, because of the upper experimental limit on Λ, implies a scalar boson with mass $m_\phi \approx 2.35 \times 10^{-27}$ MeV, the range of the associated force thus being $\sim 10^{16}$ cm, i.e.. it is macroscopic. Much earlier,[7–9] we had considered the Λ term in the context of hadronic gravitational interactions, Λ being connected with the mass of the f-meson mediating a strong, short-range gravitational interaction. The role of the strong gravity in averting cosmological singularities was also studied.[10,11]

2. GRAVITATIONAL FIELD AS A GAUGE FIELD

It is not often realized that one can easily formulate Einstein's general relativity theory in the language of gauge invariance and spontaneous symmetry breaking. To begin with, Utiyama[12] showed that Einstein's gravitational field is a kind of non-Abelian gauge field (a Yang–Mills[13] field) for the general coordinate transformations. A similar conclusion was reached for the case of the massive, strong short-range gravitational field mediated by spin-2 f-mesons.[9] This equivalence between the Yang–Mills field and gravitation is especially transparent if we write Einstein's Lagrangian in Weyl's Vierbein formalism, i.e. (see Refs. 14 and 15 for details),

$$\mathscr{L}_{\text{grav}} = -i \operatorname{Tr}[V^\mu, V^\nu] \Gamma_{\mu\nu} \tag{1}$$

where the 'Vierbein' quantities V^μ and $\Gamma_{\mu\nu}$ are related to the usual geometric quantities such as the metric tensor $g_{\mu\nu}$ and the Riemann tensor as[14]

$$g^{\mu\nu} = \tfrac{1}{4} \operatorname{Tr}(V^\mu V^\nu); \qquad \Gamma_{\mu\nu} = -\tfrac{1}{8} R_{\alpha\beta\mu\nu} V^\alpha V^\beta; \ldots \tag{2}$$

Further, $\Gamma_{\mu\nu} = \partial_\nu \Gamma_\mu - \partial_\mu \Gamma_\nu + [\Gamma_\nu, \Gamma_\mu]$, where $\Gamma_\mu = \Gamma_\mu^{ab} \gamma^a \gamma^b$; Γ_{ab}^μ is the so-called Weyl connection introduced to ensure that covariant derivatives of spinors transform like spinors, and the γ's are the usual Dirac matrices. Under tetrad rotations the fields $\Gamma^{\mu\nu}$ and γ^μ transform as[14]

$$\gamma^\mu \to S\gamma^\mu S^{-1}, \qquad \Gamma_{\mu\nu} \to S\Gamma_{\mu\nu} S^{-1}$$

i.e., they exhibit invariance under the noncompact symmetry group $SL(2, C)$ i.e., they transform like the Yang–Mills field, which is invariant under local isotopic rotations, i.e., $SU(2)$. The above Lagrangian is thus invariant under $SL(2, C)$ and the 'covariant curl' $\Gamma_{\mu\nu}$ transforms like the Yang–Mills field $B_{\mu\nu}$. Thus the expression $-i \operatorname{Tr}[V^\mu, V^\nu] \Gamma_{\mu\nu}$ transforms like a scalar for the general coordinate transformations which are the analog of the gauge group in electrodynamics. It is easily shown that the Lagrangian given by Eq. (1) does lead to Einstein's field equations. Again it must be remarked that the ideas of spontaneous symmetry breaking and nonzero expectation values for tensor fields (i.e., $\langle g^{\mu\nu} \rangle = \lambda \eta^{\mu\nu}$) were inadvertently introduced for the first time by Einstein when he sought to linearize his field equations by postulating

$$V^{\mu a}(x) = \eta^{\mu a} + \kappa \phi^{\mu a}(x) \tag{3}$$

where κ is the Einstein gravitational constant and $\phi^{\mu a}(x)$ is the quantized field variable with zero expectation value, i.e., Eq. (3) implies spontaneous symmetry breaking, i.e., $\langle V^\mu \rangle = \gamma^\mu$, namely that we have a 16-component field V^μ with a nonzero expectation value. It is well known that by substitution of Eq. (3) into the Lagrangian given by Eq. (1), one can recover the linearized field equations, i.e., wave equations for a massless field ϕ describing the graviton. In other words, we can recover a set of bilinear terms corresponding to a massless particle spectrum. Thus the gravitons are described in this approach to gravitation as the Goldstone bosons corresponding to the breakdown of the Lorentz symmetry of the vacuum as implied by the Einstein postulate given by Eq. (3). The addition of further $SL(2, C)$ gauge-invariant terms to the above Lagrangian, for instance, a cosmological term, can then generate the particle mass (corresponding to massive spin-2 particle) through the intrinsic symmetry-breaking mechanism.

3. TIME-VARYING COSMOLOGICAL CONSTANT AND NONSINGULAR COSMOLOGICAL MODELS

Following Linde,[2] one can consider the nature of the dependence of Λ on temperature in a particular model, say the Salam–Weinberg model. Consider the Lagrangian

$$\hat{L} = \hat{L}_\sigma + \tfrac{1}{2}\mu^2\sigma^2 - \tfrac{1}{4}\lambda\sigma^4 \tag{4}$$

σ is the symmetry-breaking parameter, $\sigma = \sigma_{(T)}$, and $\mu^2\sigma^2 - (\lambda\sigma^4/4)C$ is a numerical term corresponding to the change in the energy density of the vacuum. $\sigma_{(T)}$ decreases with increasing T, vanishing at $T > T_c$, where $T_c \sim \sigma_{(0)}$. This gives for the vacuum energy density a temperature dependence of the form

$$\epsilon_{(T)} = \epsilon_{(0)} + \tfrac{1}{4}\lambda\kappa(\sigma_{(0)}^2 - \sigma_{(T)}^2)^2 \tag{5}$$

giving for the cosmological constant a temperature dependence of the type

$$\Lambda_{(T)} = \Lambda_{(0)} + \tfrac{1}{4}\lambda\kappa(\sigma_{(0)}^2 - \sigma_{(T)}^2)^2 \tag{6}$$

The usual temperature–time dependence in evolving Robertson–Walker (RW) universes then gives

$$\Lambda_{(t)} = \Lambda_{(0)} + \frac{\lambda\kappa}{4}\left(\frac{1}{t_0} - \frac{1}{t}\right)^2 \tag{7}$$

yielding the differential equation

$$\dot{R}^2 = \frac{2GM}{R} + \frac{2}{3}\Lambda_{(0)}\, C^2R^2 + \frac{2}{3}\, C^2R^2\Lambda\left(\frac{1}{t_0} - \frac{1}{t}\right) \tag{8}$$

It is interesting to note that among the plethora of cosmological models in general relativity there is one Robertson–Walker model which has no singularity to begin with. This is the model with $k = +1$ (k being the curvature of the 3-space in the RW metric) and a nonzero cosmological constant $\Lambda = \Lambda_c$, where Λ_c (in a pressure-free model) determines the initial minimum radius to which the model collapses. The initial state is a static Einstein universe with radius given by

$$R = R_{\min} = 1/\Lambda_c^{1/2} = (c^2/8\pi G\rho_0)^{1/2} \tag{9}$$

where ρ_0 is the corresponding density at the static phase. The static initial condition is unstable with respect to perturbations and explodes after reaching the minimum value of R given by $R_{\min}$. The reason why this model was not considered earlier is that it gives too low an initial density (i.e., a large initial radius) $\sim 10^{-28}$ gm/cm^3, in as much as the observational upper limit on Λ at the current epoch is 10^{-55} cm^{-2}. However, the large value of Λ at the initial stage of the expansion as deduced by Linde on the basis of the Salam–Weinberg model and earlier by Sivaram and Sinha[7] on the basis of strong gravity mediated by the f-meson should now revive this particular model, as the initial density would now be large enough to be consistent with the observation of blackbody 3K microwave radiation which is a remnant of this hot, dense initial phase of the universe. Consider first the case of

the Salam–Weinberg model discussed by Linde. For this the value at the critical 'temperature' where the weak interactions become long range, i.e., at the initial phase, $\Lambda_{(T)_c} \gg \Lambda_{(0)} \approx 10^{-6}$. The corresponding density is $\rho_{\max} \approx 10^{20}$ gm/cm^3. This initial value for Λ_c would give a minimum radius (and hence absence of singularity) $R_{\min} \approx 10^3$ cm. More reasonable estimates were obtained by Sivaram *et al.* earlier[10,11] on the basis of f-gravity for which $\Lambda_f \approx (m_f c/\hbar)^2$, m_f being the mass of the f-meson mediating the strong gravitational field. This will be elaborated subsequently.

4. WEAK INTERACTIONS AND NONSINGULAR COSMOLOGICAL MODELS

Now, in a superconductor (as implied by London's modification of Maxwell's equations) the photon can be interpreted as having an effective rest mass given by $\kappa \sim |\psi|$, with $\psi \sim \langle a_{\mathbf{p}}{}^{+} a_{-\mathbf{p}}^{+} \rangle$, where $a_{\mathbf{p}}{}^{+}$ is the creation operator for an electron with momentum $\mathbf{p}$. Thus the Maxwell equations for electrodynamics (where the photon has zero rest mass) have to be replaced inside a superconductor by the London equations; thus the field has a short range and this implies an effective rest mass for the photon. As the temperature increases, this effective mass decreases and vanishes at $T > T_c$ (T_c being the superconducting critical temperature), when the usual Maxwell equations for massless photons hold and the symmetry that is broken at lower temperatures [i.e., $a_{\mathbf{p}} \to a_{\mathbf{p}} \exp(i\alpha)$] is restored). All this is well described by the Ginzburg–Landau Lagrangian:

$$L = L_0 + \frac{H^2}{8\pi} + \frac{|(\nabla - 2ieA)\,\psi|^2}{2m} - a\,|\psi|^2 + b\,|\psi|^4 \qquad (10)$$

It is remarkable that this Lagrangian looks identical to the scalar part of the Higgs–Weinberg Lagrangian, which forms the basis for the gauge unification of weak and electromagnetic interactions, i.e.,

$$Z = -\tfrac{1}{2}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 + |(\partial_\mu - 2igB_\mu)\phi|^2 - \mu\,|\phi|^2 - \lambda\,|\phi|^4 \quad (11)$$

(However, for the intrinsic symmetry breaking approach see Hsu and Sudarshan.[17])

The mass of the scalar field arises because $\langle \phi \rangle \neq 0$; we have a condensation of the scalar Bose particles, just as the photon is not massless inside a superconductor because of the condensation of the Cooper (Bose) pairs. The analogy between Eqs. (10) and (11) shows that the intermediate boson which mediates the weak interaction becomes massless at energies above 300 BeV, corresponding to a transition temperature $\sim 10^{15}$ K, which

gives a critical density of $\sim 10^{20}$ gm/cm^3, corresponding to a value of $\Lambda \approx 10^{-6}$ cm^{-2}. The weak interaction forces become long range above this temperature and as these forces are repulsive, they can avert the final singularity. To see this, consider Eq. (8). As we are considering the initial phase near the critical temperature we have just $\Lambda_c \approx 10^{-6}$ cm^{-2} and since $\Lambda_c \gg \Lambda_{(0)}$ (the change occurring as a phase transition), we can ignore the time evolution and consider just this initial value of Λ, i.e., Λ_c. Since the repulsive weak interactions are dominant, we can write[18] for the gravitational constant in Eq. (8)

$$G \to G_F(c/\hbar)^2 \tag{12}$$

where G_F is the universal Fermi constant. Since this is repulsive, it will have an opposite sign to the conventional G. Thus for a positive Λ and negative G, the normal solution[8] of Eq. (8), i.e.,

$$R = \frac{3/G/M}{\Lambda c^2} \cosh^{3/2}\left(t \frac{(3\Lambda c^2)^{1/2}}{2}\right) \tag{13}$$

has a minimum at [using Eq. (12)]

$$R_{\min} = \left(\frac{3/G_F/M}{\Lambda_c \hbar^2}\right)^{1/3} \tag{14}$$

With the above value for $\Lambda_c \approx 10^{-6}$ cm^{-2} and a value of G_F of 1.5×10^{-49} erg cm^3, the value of $R_{\min}$ is (M in the above relations is the mass of the universe)

$$R_{\min} = 10^{22} \text{ cm} \tag{15}$$

Although this value for $R_{\min}$ is much lower than the present value of $R \approx 10^{28}$ cm, it is still not low enough for a hot, dense initial phase.

In this context, it must be remarked that consideration of f-gravity and the hadron era gives reasonable estimates, as will be indicated below.

5. STRONG GRAVITY AND NONSINGULAR COSMOLOGY

Einstein's general relativity field equations, when carried over into hadronic physics to describe the strong gravitational interactions of hadrons, have to be supplemented with a term that takes into account the finite rest mass of the mediating f-meson and also the usual Newtonian constant has to be replaced by the strong gravity constant G_f. The easiest way to achieve the former is to reinterpret the cosmological constant as $\Lambda_f \approx (m_f c/\hbar)^2$, which makes the hadron a 'de Sitter' world of radius $\sim 10^{-14}$ cm. It was

shown recently that the field equations of the strong, short-range massive gravity (with cosmological constant Λ_f) give rise to a massive scalar–tensor theory,[9,19] where the ratio of the masses of the mediating particles is $m_0(\text{scalar})/m_2(\text{tensor}) = \sqrt{3}$, indicating that the scalar has a shorter range. The tensor field was found to be attractive (as in ordinary general relativity) but the scalar is a repulsive potential dominating at still shorter distances and higher densities. This repulsive interaction can avert the cosmological collapse. Thus with G_f negative, Eq. (13) has a minimum at ($G_f \approx 6.6 \times 10^{30}$ cgs units)[20]

$$R_{\min} = \left(\frac{3/G_f/M}{\Lambda_f c^2}\right)^{1/3} \approx 10^{13} \text{ cm} \tag{16}$$

This corresponds to a density of

$$\rho_{\max} \approx \Lambda_f c^2/8\pi G_f \sim 10^{17} \text{ gm/cm}^3 \tag{17}$$

Thus these values are more reasonable than the Linde–Weinberg case.[2]

In this context it must be noted that recently Bekenstein[21] has pointed out that the usual Hawking–Penrose energy condition,[10] which results in the inevitability of gravitational singularities, that is,

$$(T_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}T)\, t^\mu t^\nu \geqslant 0 \tag{18}$$

can be circumvented if one considers the energy–momentum tensor of a massive scalar field, i.e., the interaction mediated by a massive scalar meson, i.e.,

$$T_{\mu\nu} = (1/4\pi)[\phi_{,\mu}\phi_{,\nu} - \tfrac{1}{2}g_{\mu\nu}(\phi_{,\alpha}\phi^{,\alpha} - \mu^2\phi^2)] \tag{19}$$

We have seen above how the repulsive massive scalar component of strong gravity averts the singularity. This can be understood in the sense that inclusion of the effects of the energy–momentum tensor of the massive scalar field can cause a breakdown in the Hawking–Penrose energy condition, thus averting the singularity.

6. THE SUPERFLUID VACUUM IN THE HADRON ERA

As noted above, in the hadron era of the universe, the energy density of the superfluid vacuum would be given by[8]

$$\rho_h \approx \Lambda_f c^2/8\pi G_f = 10^{17} \text{ gm/cm}^3 \tag{20}$$

The type of relation implied by Eq. (20) also holds inside a typical hadron (the space in the interior being of the de Sitter type[7,9,19]) and can be looked upon as the energy density of the hadronic fluid.

From the analogy given in Section 4 with phase transitions, we could expect the superfluid in the hadron era (i.e., the hadronic superfluid) to undergo a Bose–Einstein condensation. The Bose–Einstein condensation temperature is given by (for the nonrelativistic case)

$$T_c = (N_h/2.612)^{2/3}\,\hbar^2/2\pi m_h K_B \tag{21}$$

where K_B is the Boltzmann constant, N_h is the density of hadronic matter expressed as a number density, i.e., $N_h \approx 10^{17}/10^{-24} \approx 10^{41}$ per cm^3, and m_h is the mass of a typical hadron (say proton) $\sim 10^{-24}$ gm. Substitution of these parameters gives the critical transition temperature of the hadronic superfluid as

$$T_c \approx 10^{13}\text{K} \tag{22}$$

It is remarkable that the condensation temperature for the hadronic superfluid is the same as the Hagedorn temperature occurring in the thermodynamic statistical bootstrap model of hadrons.[8] The pecularity of the behavior of hadronic matter as manifested in high-energy hadron collisions provides strong experimental evidence for the existence of the Hagedorn temperature. This can perhaps be understood in terms of a phase transition, the Hagedorn temperature being the Bose–Einstein condensation temperature for the hadronic superfluid. The hadronic forces become short range (analogous to that of a superconductor), probably supplying masses for pions and other mesons, via the symmetry-breaking mechanism, which mediate short-range strong interactions. The Bose–Einstein condensation temperature can also be worked out for an ultrarelativistic hadron fluid. For the ultrarelativistic case, $m_h \ll |\,p\,|_{\text{av}}$, the hadronic number density, is given by

$$\begin{aligned} N_h &= (4\pi/3)\sum_{j=1}^{\infty} e^{j\mu/K_BT}\int_0^{\infty} e^{-jpc/K_BT}p^2\,dp \\ &= 8(K_BT/\hbar c)^3\sum_{j=1}^{\infty}(e^{j\mu/K_BT}/j^3) \end{aligned} \tag{23}$$

This gives a Bose–Einstein condensation temperature of

$$T_c = (N_h/1.2)^{1/3}\,\hbar c/2K_B \approx 10^{13}\text{K} \tag{24}$$

the same as the Hagedorn temperature. This calculation is more consistent with the ultrarelativistic conditions in the hadron era. Further, since the density of the hadronic superfluid does not exceed 10^{17} gm/cm^3, in the hadron era, the Hagedorn temperature is the upper limit to the temperature of the hadronic matter, providing a kind of cutoff in the transverse momenta of colliding hadrons as indicated in high-energy experiments. At temperatures

exceeding the Hagedorn temperature hadron–antihadron pairs may spontaneously manifest themselves from the superfluid hadronic vacuum. One can equate the pressure exerted by the hadronic superfluid inside a hadron, i.e., $\Lambda_f c^4/8\pi G_f$, to the energy density of the strong interaction (with coupling constant $g^2/\hbar c$) close to hadronic matter, i.e., $g^2/8\pi r^4$, r being a typical hadron radius $r \approx 2 \times 10^{-14}$ cm ($r \approx \hbar/m_h c$). This gives for the strong coupling constant

$$g^2/\hbar c = \Lambda_f \hbar^3/G_f m_h{}^4 c \approx 15 \tag{25}$$

Hence, we obtain an interesting relation between the strong coupling constant and the Hagedorn temperature T_H, using Eq. (24), as

$$g^2/\hbar c = (76.8\pi/c^6)(K_B T_H/m_h)^3 \approx 15 \tag{26}$$

7. CONCLUDING REMARK

In the foregoing sections it has been shown that in the early stage of the evolution of the universe, the hadron fluid underwent a phase transition to a superfluid state. The 'cosmological' constant Λ_f in the framework of strong gravity[7–11] can be interpreted as the symmetry-breaking parameter. This averts the cosmological singularity. The transition temperature T_c is estimated to be of the order of 10^{13} K, the same as the Hagedorn temperature. The strong interaction coupling constant between nucleons is related to this temperature and turns out to be close to the observed value.

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