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TACHYONS AND COSMOLOGY

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SUMMARY

The propagation of tachyons in an expanding universe is discussed. It is shown that a primordial tachyon in the big-bang universe cannot survive unless it had very large energy initially. In an indefinitely expanding universe the tachyon trajectory turns back in time. This time barrier is found to exist even in the quantum mechanical discussion of tachyons.

This property is used to set limits on the mass of a tachyon. The possible astronomical checks on the hypothesis that neutrinos or photons may be tachyonic are also discussed.

1. INTRODUCTION

Tachyons are particles travelling faster than light. Contrary to the general belief their existence does not violate the theory of relativity, although their detection may require a modification of certain established notions of causality in physics. Indeed, a considerable part of research on tachyons has been devoted to resolving different conceptual paradoxes (*cf.* for example, Bilaniuk, Deshpande & Sudarshan 1962; Sudarshan 1970). From the experimental point of view, however, it is necessary to know how tachyons can be produced and how they can be detected through their interaction with ordinary matter. Some particle physicists have discussed the quantum mechanical properties of faster than light particles, which would be relevant to their production and detection (Sudarshan 1968; Feinberg 1967; Dhar & Sudarshan 1968). So far attempts to produce and detect such particles in a laboratory have yielded null results.

So far as production is concerned we need not confine our attempts to the terrestrial laboratory alone. Astronomical discoveries have shown us time and again that the Universe contains phenomena on a much grander scale than could ever be produced in the laboratory. Cosmic rays, pulsars, supernovae, quasars and the exploding galactic nuclei are some of the examples of such astronomical phenomena. One can get round the production problem by arguing that tachyons may somehow be produced in some such extreme event in the Universe. And, the most extreme of such events is the big bang itself, provided the Universe originated in a big bang.

In this paper we will assume that such a big-bang event did exist and that the tachyons were produced at or just after this event along with the elementary particles of other matter. In that case, would such primordial tachyons exist up to the present epoch to provide a tachyonic background, analogous, say, to the microwave background? In attempting to answer this question we will also assume that the tachyons produced in the early stages of the Universe have very little interaction with matter so that their survival depends only on their propagation through the expanding Universe.

We begin by recapitulating briefly the properties of tachyons in flat space-time.

2. SOME BASIC PROPERTIES OF TACHYONS

Throughout this paper we shall take the velocity of light and the Planck constant divided by 2π as units. In Minkowski space with the line element

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 \tag{1}$$

we define the 3-velocity of a tachyon by

$$\mathbf{v} = \left[\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt}\right] \tag{2}$$

with

$$v^2 = |\mathbf{v}|^2 > 1. \tag{3}$$

The energy and momentum $[\mathbf{E}, P]$ of the tachyon are given by

$$E = \frac{M}{\sqrt{v^2 - 1}} = M\Gamma, \qquad P = \frac{M\mathbf{v}}{\sqrt{v^2 - 1}}, \tag{4}$$

where M is a real constant, and Γ is the relativistic factor

$$\Gamma = \frac{1}{\sqrt{v^2 - 1}}, \tag{5}$$

analogous to $\gamma = (1 - v^2)^{-1/2}$ for particles with $v^2 < 1$. It is convenient to define another parameter

$$\beta = \frac{|\mathbf{v}|}{\sqrt{v^2 - 1}} \tag{6}$$

so that

$$P = |\mathbf{P}| = \beta M. \tag{7}$$

A tachyon with $v \to 1$ has infinite energy while a tachyon of infinite velocity has zero energy and momentum $P = M$. Like the rest mass for ordinary matter, M may be identified with the magnitude of tachyon momentum in the frame of reference in which it has infinite velocity.

Quantum mechanically, the spin of the tachyon may be determined from the considerations of unitary irreducible representations of the inhomogeneous Lorentz group (the Poincaré group). It turns out that the tachyonic representations are either spinless or infinite dimensional. This is because the group of transformations which keep a particular energy momentum vector of a tachyon invariant the so-called little group, is the Lorentz group in $2+1$ dimensions. All the unitary representations of this group are infinite dimensional except for the trivial one-dimensional representation. The infinite dimensional representations fall into two mutually conjugate discrete series for which the helicities have a fixed sign, an exceptional series which is continuous and a principal series which is also continuous. All these could correspond to integral or half integral spin, e.g. for a photon or a neutrino. In fact all these representations correspond to the existence of a infinite number of polarization states. We will return to the question of tachyon spin in Section 5.

Taking the simplest, spinless, case we may describe the quantum tachyon by a scalar wavefunction $\phi(x, y, z, t)$ satisfying the Klein–Gordon equation

$$(\Box - M^2)\,\phi \equiv \frac{\partial^2\phi}{\partial t^2} - \nabla^2\phi - M^2\phi = 0. \tag{8}$$

These plane wave solutions of this are of the form

$$\phi \sim \exp(-iEt + i\mathbf{r}.\mathbf{P})$$

where $\mathbf{r} = (x, y, z)$ and

$$E^2 = P^2 - M^2. \tag{9}$$

The quantization problems connected with (8) have been discussed by Arons & Sudarshan (1968) and by Dhar & Sudarshan (1968).

3. TACHYON PROPAGATION IN THE EXPANDING UNIVERSE

In this section we assume that the tachyon, created in the early stages of a big-bang universe, obeys the relativistic law of gravitation, but is otherwise free from any other interaction. The Universe itself we assume to be given by the Robertson–Walker line element

$$ds^2 = dt^2 - S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]. \tag{10}$$

Here (r, θ, ϕ) are the constant coordinates of a Weyl-type fundamental observer, t is the cosmic time and k the parameter with possible values $-1, 0, +1$ specifying the sign of curvature of the spaces t = constant. The function $S(t)$ represents the expansion factor. The discussion given here could apply to any $S(t)$; but we will restrict specific examples to the classical Friedmann models.

3.1 *Classical considerations*

The classical tachyon under the above assumptions, will describe a spacelike geodesic. Without loss of generality we will assume that it was created at the point $r = 0$, at the epoch $t = t_0$ and was moving in the direction $\theta = \theta_0$, $\phi = \phi_0$. The geodesic equations immediately lead to the result that this direction is unchanged throughout the motion. The r, t equations respectively become

$$\frac{d^2r}{ds^2} + 2\frac{\dot{S}}{S}\frac{dr}{ds}\frac{dt}{ds} - \frac{kr}{1-kr^2}\left(\frac{dr}{ds}\right)^2 = 0, \tag{11}$$

and

$$\frac{d^2t}{ds^2} + \frac{\dot{S}}{S}\frac{1}{1-kr^2}\left(\frac{dr}{ds}\right)^2 = 0. \tag{12}$$

The equation (11) integrates to

$$\frac{S^2}{\sqrt{1-kr^2}}\frac{dr}{ds} = \text{constant}. \tag{13}$$

For a tachyon we anticipate that the constant on the right-hand side will be imaginary, because ds is imaginary. Writing

$$ds = i\, d\sigma \tag{14}$$

we rewrite (13) as

$$\frac{S^2}{\sqrt{1-kr^2}}\frac{dr}{d\sigma} = S_{\mathrm{m}} \tag{15}$$

where S_{m} is a real constant. Using (15) and (10) we get the first integral of (12):

$$S^2\left(\frac{dt}{d\sigma}\right)^2 = S_{\mathrm{m}}^2 - S^2. \tag{16}$$

Finally, we relate (15) and (16) to the tachyon 3-velocity in the rest of frame of the fundamental observer instantaneously coinciding with the tachyon at r, t. This velocity is given by

$$v(t) = \frac{S}{\sqrt{1-kr^2}}\frac{dr}{dt}. \tag{17}$$

We get from (15) to (17) the simple relation

$$\frac{S(t)\, v(t)}{\sqrt{v^2-1}} \equiv S(t)\,\beta(t) = S_{\mathrm{m}}. \tag{18}$$

The momentum P of the tachyon therefore steadily decreases as S increases, according to the relation

$$P(t) = M\frac{S_{\mathrm{m}}}{S(t)}. \tag{19}$$

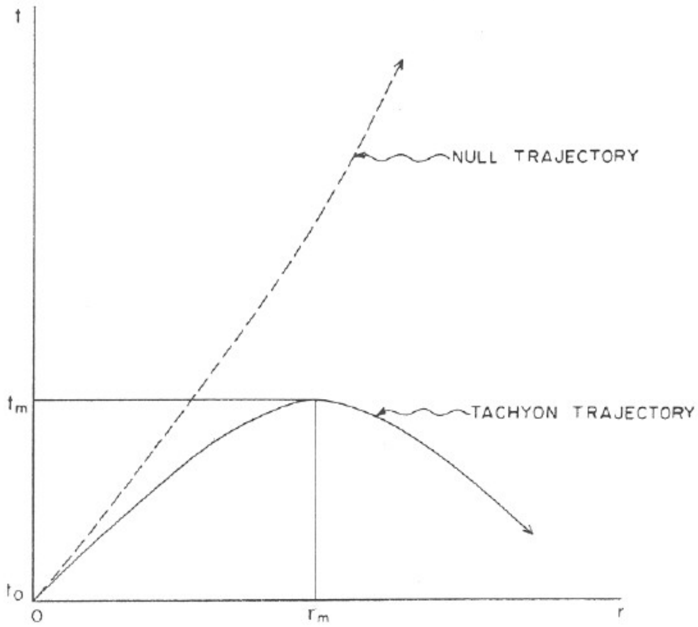

FIG. 1. *The space-time diagram shows the difference between a null trajectory and a tachyon trajectory in an expanding Robertson–Walker universe with $k = 0$. The trajectory for the tachyon bends back at the epoch t_m and is symmetrical about the line $r = r_m$.*

This is the cosmological redshift as applied to tachyons. It is interesting to note that the momentum of ordinary massive particles as well as zero rest-mass particles also decrease inversely as S.

However, there is an important difference in one respect. For tachyons $P(t) \geqslant M$ and hence $S(t) \leqslant S_{\mathrm{m}}$. This is clearly impossible if a tachyon continues to propagate forward in time in an indefinitely expanding universe. What happens when $S(t)$ finally becomes equal to S_{m}, say at $t = t_{\mathrm{m}}$? The answer is provided by the equations (16) and (12). At $t = t_{\mathrm{m}}$, $dt/d\sigma = 0$ but $d^2t/d\sigma^2 < 0$. In other words, t_{m} is a maximum of the tachyon trajectory!

The situation is illustrated in Fig. 1, which is a space–time diagram. The dotted line represents the null line from $r = 0$, $t = t_0$. The solid line, representing the tachyon motion, is always below the null line and steadily decreases in slope until it is flat at $t = t_{\mathrm{m}}$, where $r = r_{\mathrm{m}}$ (say). For $r > r_{\mathrm{m}}$ the trajectory bends back in time and heads towards the singular epoch $t = 0$. At $t = t_{\mathrm{m}}$ the energy of the tachyon is zero, and its momentum equal to M. The backward in time trajectory may be interpreted as an antitachyon moving forward in time which just annihilates the tachyon at $t = t_{\mathrm{m}}$. Thus nothing is left for later epochs $t > t_{\mathrm{m}}$.

A simple calculation shows that the energy of the tachyon, as it approaches the time barrier at t_{m} falls to zero at the limiting rate

$$E \approx M\sqrt{2H_{\mathrm{m}}(t_{\mathrm{m}}-t)}, \tag{20}$$

where $H_{\mathrm{m}} = [\dot{S}/S]_{t\mathrm{m}}$ is the Hubble constant at the epoch t_{m}. The tachyon–antitachyon annihilation at t_{m} is therefore a gentle process unlike the annihilation of an electron positron pair.

3.2 *Quantum considerations*

We now consider the corresponding problem for a quantum tachyon. For simplicity we shall discuss the spinless case mentioned in Section 2. Before we do so it is necessary to generalize the Klein–Gordon equation (8) to curved space-time. This may be done in a variety of ways; but we choose the one which is a conformally invariant transform of the wave operator:

$$(\Box + \tfrac{1}{6}R)\,\phi - M^2\phi = 0. \tag{21}$$

Here R is the scalar curvature. Apart from the convenience in the present problem there are reasons, discussed elsewhere (Hoyle & Narlikar 1974) why physical theories should be conformally invariant.

The convenience arises from the fact that the cosmological space-time in the present case, is conformally flat (Infeld & Schild 1945). The demonstration of this is obvious in the case $k = 0$ which will be considered below.

The coordinate transformation

$$\tau = \int \frac{dt}{S(t)} \tag{22}$$

transforms (10) to the conformally flat form

$$ds^2 = \Omega^2(\tau)[d\tau^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2)], \quad \Omega(\tau) = S(t). \tag{23}$$

The equation (21) is transformed to

$$\frac{\partial^2\bar{\phi}}{\partial\tau^2} - \nabla^2\bar{\phi} - M^2\Omega^2(\tau)\,\bar{\phi} = 0 \tag{24}$$

where

$$\bar{\phi} = \Omega(\tau)\,\phi. \tag{25}$$

Thus (24) is the flat-space Klein–Gordon equation but with an epoch-dependent mass term.

Corresponding to the classical free tachyon we now consider the planewave solutions of (24) of the form

$$\bar{\phi} = f(\tau)\exp(ipx) \tag{26}$$

where p is constant and x is a specified θ, ϕ direction. The function $f(\tau)$ satisfies the equation

$$\frac{d^2f}{d\tau^2}+[p^2-M^2\Omega^2(\tau)]\,f = 0. \tag{27}$$

In principle, given a specific cosmological model, f can be solved. However, since the atomic time scales corresponding to M^{-1} are small compared to the cosmological time scales, the following approximation procedure will be adopted. Over a time interval comparable to a few multiples of M^{-1} around a given epoch τ_1, we will approximate $\Omega(\tau)$ by the constant $\Omega(\tau_1)$. There is an exceptional epoch, where greater care is needed and which will be discussed separately later. Writing $f \propto \exp(i\omega\tau)$ we get

$$\omega = \sqrt{p^2-M^2\Omega^2(\tau_1)}. \tag{28}$$

In the proper time of a fundamental observer, the 'energy' of the tachyon is given by

$$E_1 = \omega\,\frac{d\tau}{dt} = \left(\frac{p^2}{\Omega_1{}^2}-M^2\right)^{1/2}, \tag{29}$$

where $\Omega_1 = \Omega(\tau_1)$. Therefore, we interpret the momentum of the tachyon at the epoch τ_1, as given by

$$P_1 = \frac{p}{\Omega_1}. \tag{30}$$

This is the quantum analogue of the classical relation (19).

The exceptional epoch is the one given by

$$\Omega(\tau_{\mathrm{m}}) = \frac{p}{M} = \Omega_{\mathrm{m}}\ (\text{say}), \tag{31}$$

and corresponds to $t = t_{\mathrm{m}}$ on the t-coordinate scale. For $\tau<\tau_{\mathrm{m}}$, ω is real and we have an oscillatory wavefunction. For $\tau>\tau_{\mathrm{m}}$ the wavefunction is expected to be damped. The change-over at τ_{m} is therefore worth a more careful examination.

Near $\tau = \tau_{\mathrm{m}}$ use the following approximation

$$\Omega(\tau) \cong \Omega(\tau_{\mathrm{m}})+(\tau-\tau_{\mathrm{m}})\,H_{\mathrm{m}}\Omega_{\mathrm{m}}{}^2 \tag{32}$$

and define

$$\xi = (2\Omega_{\mathrm{m}}{}^3M^2H_{\mathrm{m}})^{1/3}\,(\tau-\tau_{\mathrm{m}}). \tag{33}$$

The equation (27) is then approximated by the Airy equation

$$\frac{d^2f}{d\xi^2}-\xi f = 0. \tag{34}$$

For $\xi \gg 1$, the damped solution of this is given by the asymptotic form

$$f = Ai(\xi) \sim \frac{1}{2\sqrt{\pi}} \xi^{-1/4} \exp\left(-\tfrac{2}{3}\xi^{3/2}\right). \tag{35}$$

This solution when continued through $\xi = 0$ on to the negative values of ξ, has the asymptotic form for $-\xi \gg 1$,

$$f = Ai(\xi) \sim \frac{1}{\sqrt{\pi}}(-\xi)^{-1/4} \sin\left[\frac{2}{3}(-\xi)^{3/2} + \frac{\pi}{4}\right]. \tag{36}$$

The details of this may be found, for example, in Jeffreys & Jeffreys (1946). We first note that these asymptotic forms are justified under the given assumptions. The large value of ξ does not imply a large enough value of $|\tau - \tau_m|$ for (32) to become invalid. For, (32) becomes inapplicable when $\Omega_m(\tau - \tau_m) H_m \sim 1$. Since

$$\frac{\xi}{\Omega_m(\tau - \tau_m) H_m} = \left(\frac{2M^2}{H_m{}^2}\right)^{1/3} \ll 1, \tag{37}$$

we can have $\xi \gg 1$ but $\Omega_m(\tau - \tau_m) H_m \ll 1$.

Within such permissible limits, it is easy to see that (36) represents an incoming and an outgoing wave. The magnitude of the energy associated with either wave is given by

$$E = \left| \frac{d}{dt}\frac{2}{3}(-\xi)^{3/2} \right| = (2M^2 H_m)^{1/2}(t_m - t)^{1/2}. \tag{38}$$

This is the exact analogue of (20). As in the classical case we can look upon (36) as representing a tachyon and an antitachyon each with energy E tending to zero as the epoch t_m or τ_m is approached.

The solution for $\tau > \tau_m$ represents a damped wave. The characteristic time scale for damping is given by $(M^2 H_m)^{-1/3}$. If H is the Hubble constant at the present epoch, we have

$$(M^2 H_m)^{-1/3} \sim \left(\frac{m_e}{M}\right)^{2/3}\left(\frac{H}{H_m}\right)^{1/3} . 10^{-8}\,\mathrm{s}, \tag{39}$$

where m_e is the electron mass. Thus the time scale of survival for a tachyon whose time barrier happens to be at the present epoch, and whose mass is the same as that of the electron is about 10 ns.

The choice of the particular Airy function and the existence of the time barrier implies that the boundary conditions cannot be set at random but must be chosen to be symmetric between tachyons and antitachyons (if they are distinct).

4. THE COSMOLOGICAL LIFE-TIME OF A TACHYON

We now ask the following question: ' Under what conditions can a primordial tachyon survive to the present epoch? ' The answer is provided by the formula (18). If at some early epoch to the tachyon had $\beta = \beta_0$ we must have

$$S_0 \beta_0 > S_p \tag{40}$$

where S_p is the value of $S(t)$ for the present epoch $t = t_p$. Similarly $S_0 = S(t_0)$.

As a typical primordial epoch we take $t_0 \sim 1$ s to represent the early nucleosynthesis of deuterium and helium. The present epoch t_p cannot be determined without specifying the cosmological model. We shall obtain numerical estimates for two well-known models: (i) the Einstein–de Sitter model with $k = 0$ and $S \propto t^{2/3}$, and (ii) the empty Friedmann model with $k = -1$ and $S \propto t$. The two models will be denoted briefly by E–dS and E–F respectively. Setting the present value of Hubble's constant $H_p \sim 1{\cdot}5 \times 10^{-18}\,\mathrm{s}^{-1}$ we get the following limits on β_0.

$$\begin{aligned} \beta_0 &> \sim 5{\cdot}5 \times 10^{11} \quad (\text{E–dS}), \\ \beta_0 &> \sim 6 \times 10^{17} \quad (\text{E–F}). \end{aligned} \tag{41}$$

Since, for $v \approx 1$, $\beta_0 \approx \Gamma_0$, the same limits are also applicable to Γ_0, and they imply that only the very energetic tachyons will survive to the present epoch. The values given here are high but not too high in the context of a hot big bang. Even in the present relatively quiet epoch we encounter cosmic ray protons with γ-values up to 10^{12}.

The β_0 or Γ_0 values can be related to the mass M by an equipartition argument. Suppose at t_0 there existed an equipartition of energy per particle for particles of all species. Then we may write

$$M\Gamma_0 \sim m_e \gamma_e \tag{42}$$

where γ_e was the γ-value of the electrons. At the time of element synthesis γ_e is estimated to be ~ 1–10. Taking $\gamma_e \sim 10$ we get from (41) and (42)

$$\frac{M}{m_e} < \begin{cases} \sim 1{\cdot}8 \times 10^{-11} & (\text{E–dS}) \\ \sim 1{\cdot}6 \times 10^{-16} & (\text{E–F}). \end{cases} \tag{43}$$

Thus primordial tachyons have to be considerably less massive than electrons if they have to survive to the present epoch. If, we further require, that the primoridal tachyons not only survive to the present epoch, but do so with a considerable energy, i.e. with $\beta(t_p) \gg 1$, then the right-hand sides of (43) will have to be lowered further.

5. ARE PHOTONS OR NEUTRINOS TACHYONS?

If tachyonic masses are very low, much lower than indicated by the limits in (43), the question may be posed: whether the so-called zero rest mass particles like the photon and the neutrinos might not in fact be tachyonic. In this section we review the possible checks that can be made on this provocative statement—both theoretically and observationally.

In Section 2 we have remarked on the infinite dimensional unitary representations of the $2+1$ Lorentz group, and how these lead to infinite number of polarization states. These states are likely to be important, through their mutual interference, when the tachyon has low energy ($\Gamma \lesssim 1$). At high energies ($\Gamma \gg 1$), however, the various polarization states virtually decouple and in the limit $\Gamma \to \infty$ they become independent. So in the case of a high energy tachyon, the higher modes of spin will be negligible and one may approximate the exact situation closely by the state of spin 1 for a photon and spin $\frac{1}{2}$ for a neutrino.

Consider first the possibility that the photons in the microwave background are tachyonic. If the background is primordial in origin, such a statement already implies that the limit (40) operates. However, in view of what has been said about the spin states of a tachyon, we must have the present Γ-value of these tachyonic photons also large compared to unity. This is because the microwave background appears to conform closely to a blackbody distribution and this implies that only two polarization states contribute to the numerical magnitude of the Stefan–Boltzmann constant.

Therefore for the microwave background photons the energy must be large compared to the mass M. For a photon of wavelength λ this relation is equivalent to

$$\frac{M}{m_e} \ll \frac{\lambda_e}{\lambda}, \tag{44}$$

where λ_e is the Compton-wavelength of the electron. At the peak wavelength of $\lambda \sim 1$ mm for this background, (44) gives $M/m_e \ll \sim 10^{-1}$:. However, if we go down to radio photons of a few MHz frequency this limit will have to be lowered by two or three orders of magnitude. These limits are in the same range as those given by (43) using the cosmological survival and equipartition arguments.

There have been numerous attempts to set upper limits to the photon rest-mass μ (see Goldhaber & Nieto (1971) for an excellent review) and these have resulted in upper limits on the ratio of μ to the electron mass of the order of

$$\frac{\mu}{m_e} < \sim 10^{-21} - 10^{-20}. \tag{45}$$

It is not immediately clear whether these experiments also imply a similar limit (or a lower one) to M/m_e. We briefly discuss the astronomical technique using the Crab Pulsar NP0532. This was suggested by Feinberg (1969) as a method for setting an upper limit on μ. We will consider the case of a tachyonic photon ($M \neq 0$).

In this case a photon of frequency ν_1 arrives *later* than a photon of frequency $\nu_2\,(<\nu_1)$ emitted simultaneously from a source located at distance D from the receiver, with a time gap of

$$\begin{aligned}\Delta t &= \frac{D}{c}\left[\left(1+\frac{M^2c^4}{h^2\nu_1{}^2}\right)^{-1/2} - \left(1+\frac{M^2c^4}{h^2\nu_2{}^2}\right)^{-1/2}\right] \\ &\cong \frac{DM^2c^3}{2h^2}\left\{\frac{1}{\nu_2{}^2}-\frac{1}{\nu_1{}^2}\right\}\end{aligned} \tag{46}$$

in the limit $Mc^2 \ll h\nu_2$. For D measured in kpc and ν_1, ν_2 in MHz, (46) may be written in the form

$$[\Delta t]_{\text{sec}} \cong 10^{39}\left(\frac{M}{m_e}\right)^2 D_{\text{kpc}}[\nu_2{}^{-2} - \nu_1{}^{-2}]_{\text{MHz}}. \tag{47}$$

Note that this effect is the reverse of that discussed for $M = 0$, $\mu \neq 0$. In the case of $\mu \neq 0$, the short wavelength photon arrives first; an effect similar to that produced by interstellar dispersion. In the present case the interstellar dispersion runs counter to the above effect. The time delay produced by a plasma density N is given by

$$\Delta t \cong -\frac{DNe^2}{2\pi m_e c}\left\{\frac{1}{\nu_2{}^2}-\frac{1}{\nu_1{}^2}\right\},$$

i.e.

$$[\Delta t]_{\text{sec}} \cong -4.10^6\, D_{\text{kpc}}[\nu_2{}^{-2} - \nu_1{}^{-2}]_{\text{MHz}}[N]_{\text{cm}^{-3}}. \tag{48}$$

For $N \simeq 2{\cdot}5 \times 10^{33}\,(M/m_e)^2$ cm^{-3} these effects will exactly cancel. An interstellar electron density of 1 cm^{-3} will more than counteract any tachyonic mass effect for $M/m_e < \sim 2 \times 10^{-17}$. Caution must therefore be exercised in interpreting Δt.

With regard to the possible tachyonic nature of the neutrino the experimental limits are much less incisive. In this case we have no genuine information about the time of flight of neutrinos of various energies and we must rely entirely on the energy momentum relation and the present cosmological considerations. The allowed shape of the beta spectra of nuclei, in particular the behaviour of the distribution function at the high electron energy end are consistent with a zero neutrino mass (Marmier & Sheldon 1969). There is, however, considerable uncertainty at the high electron energy end to permit a neutrino rest mass

$$m_\nu < \sim 5\times 10^{-4}\, m_e. \tag{49}$$

A similar limit would be permitted on a tachyonic mass of the neutrino on the basis of the experimental Kurie plot (Marmier & Sheldon 1969). It is interesting that the limits set by (43) are much lower. Thus we can argue that primordial neutrinos cannot survive to the present day as tachyons if their tachyonic mass exceeds the limit (43).

Weinberg (1962, 1972) has discussed the nature of the neutrino (and antineutrino) background to be expected in the expanding Universe. Because neutrinos are fermions their degeneracy, complete or partial will affect experiments involving their emission or absorption, e.g. the beta decay experiment. Weinberg (1972) also discusses the effect of a neutrino and antineutrino background on the survival of very high energy cosmic ray protons. In the present context, because the tachyonic neutrinos do not survive for ever, the above type of degeneracy arguments are not likely to be significant in the observed phenomena about neutrinos.

The nature of spin mentioned earlier in this section, does raise the important question: how are tachyonic neutrinos to be coupled with ordinary elementary particles like n, p, e, π, etc., described by finite dimensional spin representations? Clearly the coupling cannot be linear in such cases, unless we are willing to re-examine the various assumptions that determine the coupling. For instance, we may consider describing hadrons also by infinite component objects as has been tried by Nambu, Barut, Böhm and others. It will then be possible to describe the semi-leptonic weak interactions by the familiar hadron–neutrino–lepton coupling.

Alternatively, we can have spin half tachyonic neutrinos if we choose a non-unitary representation. For $\Gamma \gg 1$, the extent of 'non-unitarity' is small. So far no experiments have been done to test the unitarity of processes involving neutrinos to any degree of precision.

6. TACHYONS NEAR THE BIG BANG

Regardless of the question discussed in the previous section, the tachyons, if they exist, are likely to influence the big-bang cosmology in a significant way. We describe two aspects in which tachyons would have played an important role near the big bang.

Consider first the coordinate distance covered by the tachyon following the trajectory described in Section 3. We illustrate here the simple $k = 0$ case, for the E–dS cosmology. Write

$$S = (t/t_p)^{2/3} \tag{50}$$

and define $v_0 = \operatorname{cosec}\phi_0$. Then from (18) we have

$$S_m = S_0 \sec\phi_0, \quad t_m = t_0(\sec\phi_0)^{3/2}. \tag{51}$$

The distance r_m is given by the integral

$$r_m = \frac{3t_p}{2}\sqrt{S_m}\int_0^{\phi_0}\frac{d\phi}{\sqrt{\sin\phi}}. \qquad (52)$$

For very energetic tachyons $\phi_0 \approx \pi/2$ and we can approximate (52) by

$$r_m \cong \frac{3t_p}{2}\sqrt{S_m}\int_0^{\pi/2}\frac{d\phi}{\sqrt{\sin\phi}} \cong 3\cdot 9 t_p\sqrt{S_m}. \qquad (53)$$

The proper distance at the epoch t_m, from the starting point, is $r_m S_m \cong 3\cdot 9\ t_m$. However, it is the coordinate distance which is of interest here. As seen in Fig. 1, the tachyon turns back in time for $r > r_m$ and follows a symmetric trajectory about $r = r_m$. Thus when it reaches close to the big-bang epoch it will have covered a coordinate distance $\sim 2r_m$. In other words, the tachyon at $r = 0$, $t = t_0$ is correlated in its motion with an antitachyon at $r = 2r_m$, $t = t_0$. A tachyon–antitachyon pair therefore provides an efficient way of establishing correlations over distant parts of the Universe in the early stages. Such a contact is the first step towards understanding why the Universe has been so homogeneous and isotropic right from the very early stages. Ordinary matter with $v \leqslant 1$ is bound by short particle horizons which prevent such large-scale communication in the early stages of a big-bang universe.

The second effect of tachyon is expected to be on the dynamics of expansion of the Universe. A tachyon fluid has large pressure to density ratio so that its energy momentum tensor is likely to violate the energy conditions of the established singularity theorems (Hawking & Ellis 1973). The situation, in principle, is similar to the C-field of Hoyle & Narlikar (1964). When applied to cosmology the C-field yields non-singular but exploding models of the Universe (Narlikar 1974). In the same way the presence of an appreciable number of tachyons in the early stages may remove the space-time singularity associated with the big bang. It is proposed to investigate this problem for the Universe as well as for compact objects, in a future paper.

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