UNIFIED FIELD THEORY*

A Search for Unity in Diversity

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The study of the universe in which we find ourselves is part of the joys of the coming of age in the Asian tradition. At the periods of twilight in the transitions between night and day and between day and night we recite the gayatri which invokes the brilliance of light to inspire one. Further, this understanding is to be a unified whole encompassing the static and the dynamic. It is therefore appropriate that when we celebrate the Asian/Pacific American Heritage that we discuss the successes of the continuing efforts at unification of particle interactions. It is an adventure filled with successes along the way but by no means a finished adventure.

Gravitation, the First Unification.

Dynamics is the description of change with a quantitative assesment of causes of changes and their influence. Though the interacation of parts was already seen in the world around us in everyday life and in the detailed description of celestial motions by Kepler's laws the beginning of modern notions of dynamics may be traced to Newton and his laws of motion. Force was what caused changes from motion along straight lines at constant speed; and force was an aspect of interaction. Falling bodies gave the simplest example of motion under constant forces.

The first unified theory of interaction took Kepler's laws and combined it with the law of falling bodies, the motion of the moon around the earth and of the tides in the ocean and tied it into a neat unified package of the Law of Universal Gravitation: Every body attracts every other body

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with a force proportional to either mass and varying inversely as the square of the distance between them. With the coming of the twentieth century and the birth of relativity theory it was recognized that Newton's unified theory was a low energy version of the relativistic theory of gravitation. The Einstein theory saw gravitation as coupled to the local flow of energy and momentum, the stress tensor. Moreover it was realized that this unified theory was best described by a curved space-time with an affine "connection". The ordinary partial derivatives were not natural objects but the fully covariant derivatives

$$D_\mu = \partial_\mu - \Gamma_\mu = \partial_\mu - \Gamma_{\mu\sigma}^{\ \rho} \chi_\rho^\sigma \ .$$

As a consequence of this approach the gravitational field is now a dynamical field with its own nonlinear interactions. After all the gravitational filed should couple with its own energy and momentum flows.

Electrodynamics, the Second Unification

The second unification came with the discovery of electricity. The charged particles behaved much like gravitating objects with the typical inverse square law but now it could be attraction or repulsion: the Coulomb law. It was followed by the discovery of magnetic forces: the Lorentz force law and the Ampere law and finally by the Faraday law. It was left to Maxwell to make the second unified theory of electromagnetism and codify them in the form in which we call them after his name. In this theory too the electromagnetic vector potential $A_\mu(x)$ is coupled to the local flow of electric charge. Just as stress is divergence free corresponding to the local conservation of energy and momentum so is the electric current divergence free corresponding to local conservation of electric charge. The quantum theory of electromagnetism is called quantum electro-

dynamics. Electrodynamic interactions cut across the entire species of particles, hadrons and leptons, stable and unstable particles and are thus universal. The conservation of electric charge leads to the Weak identies.

Chiral Weak Interactions, the Thir Unification

But gravitation and electromagnetism are not the only forces. They appear to be the most important at laboratory distances since the forces are inverse square law forces and hence "long range forces". But there are the weak interactions first discovered at the turn of the century in beta radioactivity. After several decades of study and the outlines of a theory of beta radioactivity by Fermi, it was recognized that the decays of pions and muons also belonged to the same category and that all of them shared the violation of party as discovered by Lee, Yang and Wu. It was left to Sudarshan and Marshak to discover the third unified theory, this time of weak interactions. Sudarshan and Marshak observed that all weak interactions were consistent with the coupling of left handed fermions amongst themselves in the only way they could couple: the V-A chiral coupling. The significance of the vectorial nature of the interaction was not lost on its discoverers who recognized that it was like the electromagnetic interaction in that left handed fermions were coupled to left handed fermions only; but unlike in weak interactions right handed fermions coupled to right handed fermions also participated in electromagnetic interactions. The two interactions could be both characterized by interaction currents: charged left handed currents for weak interactions, neutral left and right handed currents for electromagnetism. The electromagnetic interaction was a coupling of the vector (potential) field with the current while the weak interaction looked like a current-current direct

coupling. The two could be made to look more similar if a charged intermediate vector boson could be used to mediate the weak interaction and the apparent direct coupling be the consequence of a large mass (several time the proton mass, the mass of a light nucleus).

The Standard Model of Electroweak Interactions

The fourth unification completed these observations on the similarity of electromagnetic and weak interactions and built a unified electroweak theory. Various components of this unification were developed by Glashow, Salam, t'Hooft, Ward and Weinberg. The theory involves four vector fields which could be seen in terms of a triplet consisting of two charged and one neutral field coupled to left handed fermions and a neutral field coupled to the right handed fermions. The electromagnetic field is a superposition of the two neutral fields and has strictly zero mass; but there is another heavy neutral vector boson called the Z^0 which leads to neutral current weak interactions. A numerical prediction could be made for the mass of the charged vector bosons $W\pm$ and the neutral boson Z^0 based on one unknown angle θ and the known values of the weak coupling constant and the fine structure constant.

Color Forces, More of the Same

The strong nuclear interactions are still not accounted for by these; there is no easy way to discover any current-current structure in them. Yet an indirect way of arriving at such a structure has emerged based on the structure of the low lying hadrons, the strongly interacting bosons and fermions. There is a nuclear proliferation; and we would like to see the basic object to be not so many. This is based on a picture of the low-lying baryons and mesons being considered as composites, the elementary entities being a set of as yet unobserved particles called quarks. The

quarks must carry the distinctions which distinguish the proton from the neutron or the strange baryons; this distinction is called the "flavor". But in addition yet another distinction must exist between them to bring the ideas of composite nucleons in accordance with the general principle of spin-statistics relation: this distinction is called "color". The multiplicity associated with equivalent colors imply a symmetry group and some new vector fields which are coupled to them. These fields are called gluon fields and the corresponding theory now called quantum chromodynamics. The gluon fields act between the quarks; and they are expected to keep the quarks from ever being free. The quarks are "confined". The observed strong interactions are the residual forces very much like the electric interaction between atoms is the residual effect of the electrostatic binding between electrons and stomic nuclei. There should be a least three colors; but we may have even more.

The Grand Unification

The fifth unification tries to combine the electroweak interactions with the strong color interactions. The electroweak interactions of the hadrons can be comprehended as belonging to the quarks. We should now attempt to devise a theory with quarks and leptons taken on an equal footing. Such theories have been constructed by Pati, Salam, Georgi and Glashow and are called Grand Unified Theories. Fermions participate with left-handed and right handed fields coupled differently. Strong, electromagnetic and weak interactions are all treated as aspects of the same interaction. Just as weak interactions were mediated by heavy bosons we can now have transitions between quarks and leptions mediated by still heavier bosons. By suitably choosing the couplings these theories can lead to the decay of protons into positrons (plus pions). Since matter as found around us

appears stable the lifetime for such decays must be very large. Based on some (renormalization group) considerations on the apparent difference in the strong and electric interactions and their presumed identity of form , the mass of the heavy intermediate bosons could be estimated leading to a prediction of the proton lifetime. These estimates yield a lifetime of $10^{31\pm1}$ years, many orders larger than the generally accepted age of the universe. Still since there are lots of protons around us, there is the possibility of detection of these decays; and they are being actively sought by several groups of experimenters. One can go even beyond this and calculate the relative abundance of protons to light quanta in the universe. Yet other models of grand unified theories can lead to neutrons oscillating to antineutrons and back again. Cosmology and the world of particles are now intimately related.

Gauge Field Theories

Crucial to all these theories is the notion of fields coupled to flows. In a quantum theoy of fields this could be associated with the need to make the theory reflect the freedom to make symmetry transformations on the dynamical fields independently at different localities. The field equations are differential equations; and derivatives change when local symmetry transformations are made. The fields coupled to flows must then transform in a special manner and be coupled in a special manner to leave the system invariant under such transformations. Such couplings are called "gauge invariant" and the fields are called gauge fields. In terms of the five unified theories we have gauge fields for all of them.

The reason why gauge theory of all interactions took so long to come was the fact that some gauge fields acquired masses and consequently lost their long range character. A mechanism had to be discovered to have

massive gauge fields. Such a mechanism is provided by the spontaneous breakdown of the gauge symmetry arising out of the ground state of the system (the "vacuum") not being invariant under the symmetry. The possibility that the grand unified gauge theory first breaks into the color gauge theory and the electroweak theory and the electroweak theory then breaks into weak and electromagnetic interactions point out that there are a number of stages of symmetry breaking, a gauge heirarchy. There is yet another problem of families of leptons and quarks; for leptons the muon and the tau family and for quarks the strange and charmed family and possibly the top and bottom family. Perhaps here are "horizontal" gauge fields connecting the families.

But there are still two big areas of unification seeking expression. One is to combine gravitation with all these color and electroweak interactions. And the other is to combine the notion of a particle by itself and the notion of interaction: the unification of Kinematics and Dynamics. To do these we must go beyond grand unification! I would like to outline such a theory that Ward and I have developed recently.

Beyond Grand Unification

Yang and Ward have developed the geometric aspects of the theory of general gauge fields in terms of connections over a Riemannian manifold. Sudarshan and Ward start from such a starting point reminiscent of Einstein's formulation of general relativistic theory of gravitation. Start with a four-dimensional manifold with points labelled by x and a primary field $\psi(x)$ defined over it. The differential equations should follow from an action

$$A = \int d^4x \sqrt{-\det g(x)}\, \mathcal{L}(x)$$

where the Lagrangian density $\mathcal{L}(x)$ is a function of $\psi(x)$ and its covariant derivitives. The covariant derivitives involve connections both Riemannian and "internal" (flavor + color + spin + ...). The lowest invariant is

$$g^{\mu\nu}(x)\ D_\mu\ \overline{\psi}(x)\ .\ D_\nu\ \psi(x)$$

but this reduces in the "flat limit" to a quadratic system not suitable for describing fermions. So we seek other alternatives. It turns out that the fourth order invariants

$$g^{\mu\lambda}\ g^{\nu\rho}\ D_\mu\ D_\lambda\ D_\nu\ D_\rho$$

$$g^{\mu\lambda}\ g^{\nu\rho}\ D_\mu\ D_\nu\ D_\rho\ D_\lambda$$

$$g^{\mu\lambda}\ g^{\nu\rho}\ D_\mu\ D_\nu\ D_\lambda\ D_\rho$$

are all invariants and the difference of any two is no more than linear in the derivatives. So we choose $\mathcal{L}(x)$ as a linear combination involving the derivative operators

$$g^{\mu\lambda}\ g^{\nu\rho}\ [D_\mu,\ [D_\lambda,\ D_\nu]]\ D_\rho = \Delta_1$$

and

$$g^{\mu\lambda}\ g^{\nu\rho}\ [D_\mu,\ D_\nu]\,[D_\lambda,\ D_\rho] = \Delta_2$$

in the form

$$\mathcal{L}(x) = \overline{\psi}(x)\ (a_1\Delta_1 + a_2\ \Delta_2)\ \psi(x)\ .$$

It is interesting to note that if all the gauge couplings and hence all connections vanish the action itself would vanish: no Kinematics without Dynamics!

We can now see qualitatively how the various interactions are contained

in this action. If we take a limiting case with the vacuum expectation value of the scalar $\bar{\psi}\psi$ vanishing but nonvanishing for the tensor $D_\mu\bar{\psi} \cdot D_\nu\psi$ according to

$$< D_\mu\bar{\psi} \cdot D_\mu\psi > = \Lambda g_{\mu\nu}$$

due to relativistic invariance, we have the effective Einstein Lagrangian

$$- \int d^4x \ \sqrt{-\det g(x)} \ (a_1+a_2) \ \Lambda \ g^{\nu\rho} \ R_{\lambda\nu\rho}{}^{\lambda}$$

where $R_{\lambda\nu\rho}$ is the Riemann tensor.

If we take the limit of the spinor connection

$$C_\mu = <i\theta(x)g_\mu^\alpha \ \gamma_\alpha> \ = <\theta>g_\mu^\alpha\gamma_\alpha$$

we get the Dirac equation for $\psi(x)$ corresponding to freely moving particles by a suitable rescaling of the fields. But if the spinor connection is treated as spatially variable we get the Higgs limit with a scalar field $\theta(x)$. The mass of the fermion is given by the vacuum expectation value of the Higgs field. It is interesting to note that the Higgs field is neither more nor less elementary than the other fields in the problem but is not arbitrarily introduced. It is a part of the general connection.

When the internal symmetries are introduced the color, electroweak and proton decay fields all appear automatically. If the spinor connection and the internal symmetries are to be integrated into a single unified connection the ultra-grand unified symmetry should be a spinor realization of an orthogonal group (like the O(10) of Georgi and Glashow) or some group which has that as a subgroup. Family unification using orthogonal groups is also a possibility. Clearly the details of the theory are to be worked out both with regard to the gauge group and the mechanisms of its breaking

into a gauge hierarchy.

Unity in Diversity

We now return to the theme of unity in diversity. The gauge theory idea is on the one hand to have the freedom to have independent local symmetry transformations; and on the other to tying the fields at adjacent points in terms of the action function and hence the differential equation. Even the particle itself is a consequence of symmetry breakdown: creation by destruction. I remind you that this unity in diversity is the secret of wave motion itself: disturbance at one locale influences the next and hence nothing can be isolated. It is amazing that equipped with the gauge principle this same simple mechanism has been instrumental in the various stages of unified field theory. These developments evoke the sentiments of the stanza I mentioned earlier:

akhanda mandalākārām
vyāptam yena carācaram
tat padam dar'sitam yena
tasmai srī qurave namah

(Salutations to the Principle of Knowledge who shows things as they are through the unity in the diversity of all that is static and dynamic.) The Asian tradition incorporates the search for unity in diversity. It has been my privilege to outline the history of the search in our times.