CAN PROTON DECAY BE ROTATED AWAY? ☆

S. NANDI, A. STERN and E.C.G. SUDARSHAN
Center for Particle Theory, The University of Texas at Austin, Austin, TX 78712, USA

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We examine the question of whether the gauge interactions leading to proton decay can be rotated away from grand unified models, via a suitable choice of values for the flavor mixing angles. Grand unified models with an arbitrary number of families and an arbitrary gauge group will be considered. In the case of the SU(5) model with three generations, it is shown that proton decay can be rotated away only if $s_3 = 0$. (s_3 is the sine of the third Kobayashi–Maskawa mixing angle.)

I is known that grand unified models [1], in general, contain more mixing angles and phases than those already present in weak-interaction physics. These new angles and phases are undetermined from the theory (except after assuming certain specific mass matrices), and there is, at present, no experimental information on their values. Yet these angles and phases play an important role in grand unified models, especially with regards to predicting the decay rates and modes for the proton. Indeed, it has been speculated that the values of the angles could be such that the interactions leading to proton (and bound neutron) decay could be eliminated entirely (e.e., "rotated away") from the theory [2]. Previously, this question has been examined within the context of some specific models. In particular, an SU(5) model where the fermion mass matrices get contributions only from a 5-dimensional (or 45-dimensional) representation of Higgs fields has been examined [3]. There it was shown that the interactions leading to proton decay could not be rotated away.

In this letter, we examine the question of wheter or not proton decay can be rotated away from a general SU(5) model. Here we make no special assumptions on the form of the fermion mass matrices. The answer to the above question is found to depend on the numbers of families n appearing in the theory. We find that for $n = 2$, the interactions leading to proton decay can never be rotated away. For the case $n = 3$, proton decay can be eliminated from the gauge sector of the theory only if $s_3 = 0$. (s_3 denotes the sine of the third Kobayashi–Maskawa [4] mixing angle). For $n \geqslant 4$, we find that it is always possible to rotate all the proton decay interactions away from the gauge sector of the theory (without putting constraints on the Kobayashi–Maskawa mixing matrix). In this letter we shall also extend our analysis to the general case where baryon decay proceeds via a four-Fermi-vector (or axial vector) exchange interaction which is invariant under the group $SU(2)_L \times U(1) \times SU(3)_C$.

We begin by examining the gauge sector of the SU(5) model [5]. The effective four-Fermi interaction for baryon-number violating processes in the SU(5) model is

$$(4G/\sqrt{2})\epsilon_{\alpha\beta\gamma}[(\bar{U}^c_\alpha\gamma^\mu U_\beta)(\bar{E}^c\gamma_\mu D_\gamma) - (\bar{U}^c_\alpha\gamma^\mu D_\beta)(\bar{E}^c\gamma_\mu U_\gamma) - (\bar{U}^c_\alpha\gamma^\mu U_\beta)(\bar{D}^c_\gamma\gamma_\mu E) + (\bar{U}^c_\alpha\gamma^\mu D_\beta)(\bar{D}^c_\gamma\gamma_\mu N)] \,, \quad (1)$$

where U_α, D_α, E and N are the n-dimensional column matrices

$$U_\alpha = \begin{pmatrix} u_\alpha \\ c_\alpha \\ t_\alpha \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}, \quad D_\alpha = \begin{pmatrix} d_\alpha \\ s_\alpha \\ b_\alpha \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}, \quad E = \begin{pmatrix} e \\ \mu \\ \tau \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}, \quad N = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}, \quad (2)$$

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and α, β, γ are color indices. G corresponds to the analoque of the Fermi constant of weak interactions. All the fields appearing in (1) are assumed to be left-handed. In expression (1), and in what follows, we assume that baryon decay proceeds predominantly via vector-boson exchange interactions. The fields appearing in (1) are current eigenstates. When the $SU(2)_L \times U(1) \times SU(3)_C$ symmetry is broken and when the fermions acquire masses, the current eigenstates of (1) must be replaced by the corresponding mass eigenstates. Since we make no assumptions on the form of the mass matrices, biunitary transformations are required in general for their diagonalization. The fields and their charge conjugates appearing in (1) must, consequently, undergo separate unitary transformations. Upon replacing the current eigenstates in (1) by the corresponding mass eigenstates, we get

$$\begin{aligned}(4G/\sqrt{2})\epsilon_{\alpha\beta\gamma}[&(\bar{U}^c_\alpha R_1\gamma^\mu U_\beta)(\bar{E}^c R_2\gamma_\mu D_\gamma)\\ &-(\bar{U}^c_\alpha R_3\gamma^\mu D_\beta)(\bar{E}^c R_4\gamma_\mu U_\gamma)\\ &-(\bar{U}^c_\alpha R_1\gamma^\mu U_\beta)(\bar{D}^c_\gamma R_5\gamma_\mu E)\\ &+(\bar{U}^c_\alpha R_4\gamma_\mu D_\beta)(\bar{D}^c_\gamma R_5\gamma_\mu N)]\ , \end{aligned} \tag{3}$$

where $R_1, R_2, ..., R_5$ denote unitary matrices, only four of which are independent [‡1]. This follows since we have the relation

$$K = R_1^\dagger R_3 = R_4^\dagger R_2\ , \tag{4}$$

where K defines the generalized Kobayashi–Maskawa mixing matrix for n generations.

In order to suppress proton (and bound neutron) decay, the following conditions must be imposed.

$$(R_1)_{11}(R_2)_{ab} + (R_3)_{1b}(R_4)_{a1} = 0\ , \tag{5a}$$

$$(R_1)_{11}(R_5)_{ab} = 0\ , \tag{5b}$$

$$(R_3)_{1a} = 0\ , \tag{5c}$$

where a and $b = 1, 2$. Eqs. (5a) and (5b) follow after setting all relevant terms involving E^c and E, respectively, to zero. Condition (5c) follows after setting the coefficients of the interactions $(\bar{u}^c\gamma_\mu d)(\bar{d}^c\gamma^\mu\nu)$ and $(\bar{u}^c\gamma_\mu s)(\bar{s}^c\gamma^\mu\nu)$ to zero and using the unitarity of R_5.

[‡1] Here for simplicity we assume that all the neutrinos are massless. Our analysis can be easily generalized to include the case of massive neutrinos. The results quoted here are the same for both cases.

Using (5c), we can replace (5a) by

$$(R_1)_{11}(R_2)_{ab} = 0\ . \tag{5a$'$}$$

Eqs. (5) imply that all relevant terms in (3) vanish. Consequently, eqs. (5) are necessary and sufficient conditions for rotating away proton decay from the SU(5) gauge interactions. Upon specializing to the case $n = 2$, eq. (5c) is inconsistent with the unitarity of R_3. Consequently, proton decay cannot be rotated away if only two generations are present in the theory. For the case $n = 3$, (5a′) [or (5b)] implies that $(R_1)_{11} = 0$, and from unitarity (5c) implies that $(R_3)_{13} = (R_3)_{23} = 0$. Then from (4), we have $K_{13} = 0$. Consequently, proton decay can be rotated away from a theory with three generations only if the sine of the third Kobayashi–Maskawa mixing angle vanishes. On the other hand, for $n \geqslant 4$ proton decay can be rotated away without placing contraints on the Kobayashi–Maskawa matrix.

The above mentioned possibility of setting $s_3 = 0$ in the case where $n = 3$ has the following phenomenological implications: (a) The bottom quark can decay directly (via gauge interactions) only to the charm quark. At present, this possibility has not been experimentally ruled out [6]. (b) *CP* violation in neutral K meson decay due to gauge interactions is proportional to $s_3 s_2 \sin\delta$, where s_2 is the sine of the second Kobayashi–Maskawa mixing angle and δ is the *CP* violating phase angle. Consequently, $s_3 = 0$ implies that the observed *CP* violation cannot be due to gauge interactions [‡2].

So far we have restricted our discussions to the gauge sector of the SU(5) model. Upon extending the above analysis to the Yakawa sector of the theory, additional terms would have to be added to the effective interaction (1). An example of such a term is

$$G'\epsilon_{\alpha\beta\gamma}(D^T_\alpha C T_1 U_\beta)(U^T_\gamma C T_2 E)\ .$$

Here C denotes charge conjugation and the fields appearing in the above interaction are current eigenstates. Unlike the corresponding gauge interactions, we must introduce new flavor matrices T_i which can-

[‡2] The observed *CP* violation could arise due to scalar exchanges. In such a case, the neutron electric dipole moment is predicted [7] to be close to the experimental upper limit of 1.6×10^{-24} cm [8]. On the other hand, the value obtained from the usual gauge interaction [9] is several orders of magnitude smaller than the above limit.

not all be set to unity (since this would lead to an unrealistic set of fermion mass matrices). The effective Yukawa interaction is defined only after we specify the T_i's, as well as the fermion mass matrices. The T_i's are determined from the Higgs structure of the gauge theory. There are no restrictions on their form (other than the requirement that they are not all unity). It follows that, it is always possible, in principle, to adjust the T_i's and the mass matrices suitably so that proton decay interactions can be rotated away from the Yukawa sector.

We now extend the preceding analysis for the SU(5) model to more general situations. Here, we no longer assume a specific grand unifying group. Rather, we insist that baryon decay proceeds via effective four-fermi interactions which are invariant under the low energy group $SU(2)_L \times U(1) \times SU(3)_C$. We shall further assume that these interactions are induced from vector-boson exchanges. The most general such interactions are given by [10]

$$\begin{aligned} & f_{ijkl}\epsilon_{\alpha\beta\gamma}[(\bar{U}^c_{i\alpha}\gamma^\mu U_{j\beta})(\bar{E}^c_k\gamma_\mu D_{l\gamma}) \\ & \quad - (\bar{U}^c_{i\alpha}\gamma^\mu D_{j\beta})(\bar{E}^c_k\gamma_\mu U_{l\gamma})] \\ & \quad + g_{ijkl}\epsilon_{\alpha\beta\gamma}[(\bar{U}^c_{i\alpha}\gamma^\mu U_{j\beta})(\bar{D}^c_{k\gamma}\gamma_\mu E_l) \\ & \quad - (\bar{U}^c_{i\alpha}\gamma^\mu D_{j\beta})(\bar{D}^c_{k\gamma}\gamma_\mu N_l)] \,, \end{aligned} \tag{6}$$

where f_{ijkl} and g_{ijkl} represent $2 \times n^4$ coupling constants and the current eigenstates U, D, E and N are given in (2). Here $i, j, k, l = 1, 2, \ldots, n$ are family indices. To reduce the number of coupling constants appearing in (6) we shall assume the "kinship hypothesis." By this we mean the requirement that all Lorentz covariant currents leading to (6) are diagonal in the generation space ‡3. This means ‡4

$$f_{ijkl} = A\delta_{ij}\delta_{kl} \,, \tag{7a}$$

$$g_{ijkl} = B\delta_{ij}\delta_{kl} \,, \quad -C\delta_{il}\delta_{jk} \,, \tag{7b}$$

Consequently, for $n > 1$ the $2 \times n^4$ coupling constants reduce to 3. Note that (7) follows automatically for single-family unification models [such as SU(5), SO(10) and E_6]. Eq. (7) was also found to be valid in the case of SO(n) family unification models [11]. Substitution of (7) into (6) leads to

$$\begin{aligned} & A\epsilon_{\alpha\beta\gamma}[(\bar{U}^c_\alpha\gamma^\mu U_\beta)(\bar{E}^c\gamma_\mu D_\gamma) - (\bar{U}^c_\alpha\gamma^\mu D_\beta)(\bar{E}^c\gamma_\mu U_\gamma)] \\ & \quad + B\epsilon_{\alpha\beta\gamma}[(\bar{U}^c_\alpha\gamma^\mu U_\beta)(\bar{D}^c_\gamma\gamma_\mu E) - (\bar{U}^c_\alpha\gamma^\mu D_\beta)(\bar{D}^c_\gamma\gamma_\mu N)] \\ & \quad + C\epsilon_{\alpha\beta\gamma}[(\bar{U}^c_\alpha\gamma^\mu E)(\bar{D}^c_\beta\gamma_\mu U_\gamma) - (\bar{U}^c_\alpha\gamma^\mu N)(\bar{D}^c_\beta\gamma_\mu D_\gamma)] \end{aligned} \tag{8}$$

In analyzing whether or not proton decay can be rotated away from (8), we must distinguish between the following 7 cases: They are:

(i) $A \neq 0, \quad B \neq 0, \quad C \neq 0,$

(ii) $A = 0, \quad B \neq 0, \quad C \neq 0,$

(iii) $A \neq 0, \quad B = 0, \quad C \neq 0,$

(iv) $A \neq 0, \quad B \neq 0, \quad C = 0,$

(v) $A = B = 0, \quad C \neq 0,$

(vi) $A = C = 0, \quad B \neq 0,$

(vii) $B = C = 0, \quad A \neq 0.$ (9)

Note from (1), case (iv) (with the additional assumption of $A = -B$) corresponds to the SU(5) model, which we have previously discussed. Both cases (i) (with $A = -B$) and (v) may be obtained from the SO(10) or E_6 models. [Of course, (iv) with $A = -B$ is also obtainable from the SO(10) or E_6 models. The question of which case (i), (iv) or (v) arises from a particular SO(10) or E_6 model depends upon how the original symmetry group is broken down to $SU(2)_L \times U(1) \times SU(3)_C$.] The remaining cases, as well, can be realized with some other grand unified models.

If one wishes to incorporate all of the cases in (9) within the framework of a single grand unified model, the appropriate group involved in the unification may have to be very large {such as SU(15) [12]}. That is, the group must be sufficiently large enough to contain 3 independent $SU(2)_L$ doublets of gauge bosons mediating proton decay. After symmetry breaking the three sets of gauge bosons acquire masses which are not necessarily related and hence can give rise to three independent constants A, B and C.

We now proceed to analyze the above cases. As before, we make no assumptions on the form of the fermion mass matrices. Upon replacing the current eigenstates in (8) by their corresponding mass eigenstates,

‡3 This differs slightly from the definition appearing in the second paper of ref. [10], where it is assumed, in addition, that the mixing induced from the fermionic mass matrices is small.

‡4 Note that setting $f_{ijkl} = A'\delta_{il}\delta_{kj}$ is equivalent to (7a) after a Fierz reordering of the fields involved.

we get

$$A\epsilon_{\alpha\beta\gamma}[(\bar{U}^c_\alpha R_1\gamma^\mu U_\beta)(\bar{E}^c R_2\gamma_\mu D_\gamma) - (\bar{U}^c_\alpha R_3\gamma^\mu D_\beta)(\bar{E}^c R_4\gamma_\mu U_\gamma)] + B\epsilon_{\alpha\beta\gamma}[(\bar{U}^c_\alpha R_1\gamma^\mu U_\beta)(\bar{D}^c_\gamma R_5\gamma_\mu E) - (\bar{U}^c_\alpha R_3\gamma^\mu D_\beta)(\bar{D}^c_\gamma R_5\gamma_\mu N)] + C\epsilon_{\alpha\beta\gamma}[(\bar{U}^c_\alpha R_6\gamma^\mu E)(\bar{D}^c_\beta R_7\gamma_\mu U_\gamma) - (\bar{U}^c_\alpha R_6\gamma^\mu N)(\bar{D}^c_\beta R_8\gamma_\mu D_\gamma)] , \quad (10)$$

where we are again assuming for simplicity that the neutrinos are massless ‡1. $R_1, R_2, ..., R_8$ are unitary matrices, $R_1, R_2, ..., R_5$ being the same as in eq. (3). Here only five R *matrices are independent since we have the relations*

$$K = R_1^+ R_3 = R_4^+ R_2 = R_7^+ R_8 , \quad (11)$$

$$R_1^+ R_6 = R_7^+ R_5 . \quad (12)$$

K once again denotes the generalized Kobayashi–Maskawa mixing matrix. Eqs. (5) are now replaced by

$$A[(R_1)_{11}(R_2)_{ab} + (R_3)_{1b}(R_4)_{a1}] = 0 , \quad (13a)$$

$$B(R_1)_{11}(R_5)_{ab} - C(R_6)_{1b}(R_7)_{a1} = 0 , \quad (13b)$$

$$B(R_3)_{1a}(R_5)_{ai} - C(R_6)_{1i}(R_8)_{aa} = 0 , \quad (13c)$$

no sum on the index a,

where $a, b = 1, 2$ and $i = 1, 2, ..., n$. As before (13a) and (13b) follow after setting all coefficients of the interaction terms involving E^c and E, respectively, equal to zero. Eq. (13c) follows after requiring that the coefficients of the interactions $(\bar{u}^c\gamma_\mu d)(\bar{d}^c\gamma^\mu\nu)$ and $(\bar{u}^c\gamma_\mu s)(\bar{s}^c\gamma^\mu\nu)$ vanish. In addition, we must require that the remaining interactions

$$(\bar{u}^c\gamma_\mu d)(\bar{s}^c\gamma^\mu\nu) \quad \text{and} \quad (\bar{u}^c\gamma_\mu s)(\bar{d}^c\gamma^\mu\nu) \quad (14)$$

are absent from the theory. This leads to the further condition

$$w_{12i} = \alpha w_{21i} ,$$

$$w_{abi} \equiv C(R_6)_{1i}(R_8)_{ab} - B(R_3)_{1b}(R_5)_{ai} , \quad (15a)$$

where α is introduced to take into account the possibility that the matrix elements computed from the two interactions in (14) are identical (up to a constant of proportionality α). α, in principle, can depend on the choice for the proton wavefunction used in computing the matrix elements. In the case where the matrix elements arising from (14) are not proportional, we must require the stronger condition:

$$w_{12i} = w_{21i} = 0 . \quad (15b)$$

We now give our results. It can be shown from eqs. (13) that when $n = 2$ and $n \geqslant 4$, our previous results for the SU(5) model generalize in fact to all other cases. That is, when only two families are present in the theory, proton decay can never be rotated away. As before, this result follows from the unitarity of the matrices R. For four or more families, we find that there are always solutions to the eqs. (13) (without placing any constraints on the Kobayashi–Maskawa mixing matrix). When $n = 3$, the situation is more complicated and the question of whether or not proton decay can be rotated away depends on the various cases (i) through (vii). The results for case (vi) are identical to those of the SU(5) model previously discussed . For case (vii), we find that there do exist solutions to eqs. (13), which do not constrain K. For both cases (iii) and (v), there are no solutions to eqs. (13) when (15b) holds. The remaining cases are difficult to analyze since they require solving a complicated system of equations.

We note the following additional result. When the fermion mass matrices are symmetric ‡5 and condition (15b) is satisfied, there are no realistic solutions to eqs. (13) for any n [with the exception of case (vii)] ‡6. This follows from the relations

$$R_1 = R_8 = 1 , \quad R_3 = R_7^+ = K ,$$

$$R_2 = R_5^+ , \quad R_6 = R_4^+ = KR_2^+ , \quad (16)$$

which is valid when the fermion mass matrices are symmetric.

‡5 This follows in the SU(5) [SO (10)] model when 5-dimensional [10- and 126- dimensional] Higgs fields are used to generate the fermion mass matrices. This also follows in SO (n) family unification models (see ref. [11]).

‡6 This result does not generalize to the case where there exist neutrinos in the theory with masses greater than the proton mass.

References

[1] For a general review, see: P. Langacker, Phys. Rep. 72 (1981) 186.

[2] C. Jarlskog, Phys. Lett. 82B (1979) 401.

[3] R.N. Mohapatra, Phys. Rev. Lett. 43 (1979) 893;
J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Phys. Lett. 88B (1979) 320.
[4] M. Kobayashi and M. Maskawa, Prog. Theor. Phys. 49 (1973) 652.
[5] H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438;
A.J. Buras, J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B135 (1978) 66.
[6] S.Stone, Cornell University preprint, CLNS 81-514.
[7] T.D. Lee, Phys. Rep. 9C (1974) 143;
S. Weinberg, Phys. Rev. Lett. 37 (1976) 657.
[8] N.F. Ramsey, Phys. Rep. 43 (1978) 409.
[9] J. Ellis, M.K. Gaillard and D.V. Nanopoulos, Nucl. Phys. B109 (1976) 213;
E.P. Shabalin, Sov. J. Nucl. Phys. 28 (1978) 75.
[10] S. Weinberg, Phys. Rev. Lett. 43 (1979) 1567;
F. Wilczek and A. Zee, Phys. Rev. Lett. 43 (1979) 1571.
[11] S. Nandi, A. Stern and E.C.G. Sudarshan, Phys. Rev. D, to be published.
[12] J.C. Pati and A. Salam, Trieste preprint IC/80/72;
J.C. Pati, A. Salam and J. Strathdee, Trieste Preprint IC/80/183.