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# AN OPERATOR APPROACH TO THE STRONG COUPLING TRANSMUTATION OF A YUKAWA INTERACTION*

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It is shown that the transmutation mechanism introduced in our earlier papers is both necessary and sufficient to ensure the equivalence of the Lee model and the attractive separable potential model in all sectors when the bare coupling constant of the Lee model is taken to infinity. It is demonstrated that the hamiltonian operator for the Lee model reduces to that for the separable potential. It is further demonstrated that in the same limit the finite energy eigenfunctions, and hence the scattering amplitudes and the $S$-matrix calculated from the two theories coincide in all sectors.

Ever since the possibility of a close connection between the Lee model and the separable potential model was pointed out by Houard and Jouvét [1]**, many attempts have been made to establish the equivalence of the four-point and Yukawa types of interaction***. Nambu and Jona-Lasinio demonstrated that it was possible, starting from a four-fermion interaction, to obtain a collective bosonic state that effectively coupled to the fermions via a Yukawa interaction. This idea has been extended in attempts to derive both abelian [4] and non-abelian [5] gauge theories from the four-fermion interaction.

The general proofs of equivalence can be approached in two ways. One may, starting from a four-fermion interaction, show that the existence of suitably coupled collective bosonic modes is effectively equivalent to a theory with a Yukawa type of interaction. On the other hand, one may start from a Yukawa type of interaction and show that in some limit it becomes equivalent to a four-fermion interaction. In either case, the equivalence of the two types of theories is argued by comparing the

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** For references on the Lee model, see references cited in [6].

*** For the earlier work, see e.g. the review article [2].

respective renormalized Green functions (or lagrangians), or in the case of soluble models, the corresponding wave functions. Quite apart from the validity of these arguments, there remained the following unanswered question: although the predictions of observable quantities calculated from the two clearly distinct theories coincide in the equivalence limit, how does the intrinsic difference between these theories manifest itself? For instance, the relativistic quantum field theory of the four-fermion interaction is non-renormalizable whereas the corresponding theory for the Yukawa-type interaction is renormalizable. In what sense can these two theories then be regarded as equivalent?

In our recent studies on the Lee model (Yukawa-type interaction) and the separable potential (four-point interaction) model [6], we have examined the equivalence of the two theories in the strong coupling (SC) limit, defined by introducing an ultraviolet cut-off on the momentum integrals and taking the bare coupling constant of the Yukawa theory to infinity. The cut-off may then be taken to infinity. It was found that for the cut-off theory, the spectrum of the Lee model hamiltonian contained, in addition to the spectrum of the separable potential model hamiltonian, extra contributions beyond the cut-off. These additional contributions which have no corresponding counterparts in the separable potential model, it was shown, moved to infinity in the SC limit. It was also shown by explicit computation that the spectral terms at infinity did not contribute to the finite energy wave functions in the SC limit. As a result, the finite energy scattering amplitudes and $S$-matrix elements calculated from the two theories coincided in the SC limit. We have emphasized, however, that the hamiltonians of the two models are definitely distinct, irrespective of the existence of any cut-off. Only when the spectral contributions at infinity are explicitly removed does the Lee model get transmuted to the separable potential model. In a recent paper, we have extended this result to the case where the $N$ and $\theta$ are fermions [7]. This, of course, strengthens the analogy between our model considerations and the real case of interest, i.e. the fully relativistic four-fermion interaction.

In our previous papers, we proceeded by explicitly computing the solutions for the two lowest interacting sectors and verifying the transmutation mechanism therefrom. This entailed rather tedious and complicated computation. In this paper, we present a simple proof of the fact that the conclusions arrived at in our previous studies are valid for all sectors of the theory. The present proof is not only extremely simple, but also makes the physics more transparent.

We proceed by considering the hamiltonians $H_{\mathrm{L}}$ and $H_{\mathrm{sp}}$ for the Lee model and the attractive separable potential model, respectively. We have,

$$H_{\mathrm{L}} = m_0 V^{\dagger} V + \int \mathrm{d}^3k\, k a^{\dagger}(k) a(k) + g_0 \int \mathrm{d}^3k\, f(k) \left[ V^{\dagger} N a(k) + N^{\dagger} a^{\dagger}(k) V \right], \tag{1}$$

$$H_{\mathrm{sp}} = \int \mathrm{d}^3k\, k a^{\dagger}(k) a(k) - \int \mathrm{d}^3k\, h(k) N^{\dagger} a^{\dagger}(k) \int \mathrm{d}^3l\, h(l) N a(l). \tag{2}$$

The notation used is that of ref. [6]. Also as in ref. [6], all the momentum integrals are cut-off at an upper value of momentum $|\boldsymbol{k}| = L$. Finally, in this paper, all the fields are quantized as bosons.

The solution for the $N\theta$ sector of the Lee model has been studied in detail in our previous paper [6]. We merely catalogue the necessary results here. We have,

$$|V_1\rangle\rangle = \sqrt{Z_1}\,|V\rangle + \int \mathrm{d}^3k\, F_1(k)|N\theta_k\rangle, \tag{3}$$

$$|V_2\rangle\rangle = \sqrt{Z_2}\,|V\rangle + \int \mathrm{d}^3k\, F_2(k)|N\theta_k\rangle, \tag{4}$$

$$|N\theta\rangle\rangle_\lambda = g_\lambda^*|V\rangle + \int \mathrm{d}^3k\, G_{\lambda k}|N\theta_k\rangle, \tag{5}$$

with

$$Z_i = \left(1 + g_0^2 \int \frac{f^2(k)\,\mathrm{d}^3k}{(M_i - k)^2}\right)^{-1}, \tag{6}$$

$$F_i(k) = \frac{g_0 f(k)\sqrt{Z_i}}{M_i - k}, \tag{7}$$

$$g_\lambda^* = \frac{g_0 f(\lambda)}{\alpha^+(\lambda)}, \tag{8}$$

$$G_{\lambda k} = \delta(\lambda - k) + \frac{g_0 f(k) g_\lambda^*}{\lambda - k + i\varepsilon}, \tag{9}$$

and

$$\alpha(z) = z - m_0 - g_0^2 \int \frac{f^2(k)}{z - k}\mathrm{d}^3k, \tag{10}$$

with $\alpha^\pm(\lambda) = \alpha(\lambda \pm i\varepsilon)$. $M_1$ is the discrete eigenvalue below the $N\theta$ scattering threshold and $M_2$ that above the cut-off. In the SC limit, we have [6]

$$\begin{aligned} &m_0 \sim g_0^2, \qquad M_2 \sim g_0^2, \\ &Z_1 \sim \frac{1}{g_0^2}, \qquad Z_2 \sim 1 + \mathrm{O}\left(\frac{1}{g_0^2}\right), \\ &F_1(k) \sim (g_0)^0, \qquad F_2(k) \sim 1/g_0, \\ &1 - \frac{m_0}{M_2} \sim \frac{1}{g_0^2}. \end{aligned} \tag{11}$$

We now define the operator $V_2$ as,

$$V_2 = \sqrt{Z_2}\, V + \int \mathrm{d}^3k\, F_2(k) N a(k). \tag{12}$$

Quite obviously

$$|V_2\rangle\rangle = V_2^\dagger |0\rangle. \tag{13}$$

$V_2$ satisfies the following commutation relations:

$$[V_2, V^\dagger] = \sqrt{Z_2}, \tag{14}$$

$$[V_2, N^\dagger] = \int \mathrm{d}^3k\, F_2(k) a(k), \tag{15}$$

$$[V_2, a^\dagger(k)] = F_2(k) N, \tag{16}$$

$$[V_2, V_2^\dagger] = Z_2 + \int \mathrm{d}^3k\, F_2^2(k) N N^\dagger + \int \mathrm{d}^3k\, \mathrm{d}^3l\, F_2(k) F_2(l) a^\dagger(k) a(l). \tag{17}$$

In the SC limit, the commutation relations (14)–(17) reduce to

$$[V_2, V^\dagger] \overset{s}{=} 1 + \mathrm{O}\left(\frac{1}{g_0^2}\right), \tag{14'}$$

$$[V_2, N^\dagger] \overset{s}{=} \int \mathrm{d}^3k\, \mathrm{O}\left(\frac{1}{g_0}\right) f(k) a(k), \tag{15'}$$

$$[V_2, a^\dagger(k)] \overset{s}{=} \mathrm{O}\left(\frac{1}{g_0}\right) f(k) N, \tag{16'}$$

$$[V_2, V_2^\dagger] \overset{s}{=} 1 + \mathrm{O}\left(\frac{1}{g_0^2}\right) + \int \mathrm{d}^3k\, \mathrm{O}\left(\frac{1}{g_0^2}\right) f^2(k) N N^\dagger + \int \mathrm{d}^3k\, \mathrm{d}^3l\, \mathrm{O}\left(\frac{1}{g_0^2}\right) f(k) f(l) a^\dagger(k) a(l). \tag{17'}$$

Here, $\overset{s}{=}$ denotes equality in the SC limit.

The Lee model hamiltonian can be rewritten by eliminating $V$ in favour of $V_2$. We obtain

$$H_L = H_L^1 + H_L^2, \tag{18}$$

with

$$H_L^1=\int d^3k\,ka^\dagger(k)a(k)+g_0^2\int d^3k\,d^3l\,f(k)f(l)$$

$$\times\left(\frac{m_0}{(M_2-k)(M_2-l)}-\frac{1}{M_2-k}-\frac{1}{M_2-l}\right)N^\dagger a^\dagger(k)Na(l), \tag{19}$$

$$H_L^2=\frac{m_0}{Z_2}V_2^\dagger V_2+g_0\int\frac{f(k)}{\sqrt{Z_2}}\left(1-\frac{m_0}{M_2-k}\right)\left(V_2^\dagger Na(k)+a^\dagger(k)N^\dagger V_2\right), \tag{20}$$

where $H_L^1$ is independent of the operator $V_2$ and $H_L^2$ is the remainder. It is amusing to note that $H_L^1$ can be rewritten in the form

$$H_L^1=\int d^3k\,ka^\dagger(k)a(k)+g_0^2m_0\int\frac{d^3k\,f(k)}{M_2-k}N^\dagger a^\dagger(k)\int\frac{d^3l\,f(l)}{M_2-l}Na(l)$$

$$-\tfrac{1}{2}g_0^2\int d^3k\,f(k)\left(\frac{1}{M_2-k}+1\right)N^\dagger a^\dagger(k)\int d^3l\,f(l)\left(\frac{1}{M_2-l}+1\right)Na(l)$$

$$+\tfrac{1}{2}g_0^2\int d^3k\,f(k)\left(\frac{1}{M_2-k}-1\right)N^\dagger a^\dagger(k)\int d^3l\,f(l)\left(\frac{1}{M_2-l}-1\right)Na(l).$$

$H_L^1$ is thus the hamiltonian for a system interacting via a separable potential of rank three.

By making use of the strong coupling behaviour of the various quantities listed in (11), it is simple to verify

$$H_L^1\stackrel{s}{=}\int d^3k\,ka^\dagger(k)a(k)-\int d^3k\,d^3l\,h(k)h(l)N^\dagger a^\dagger(k)Na(l)$$

$$-\int d^3k\,d^3l\,O\left(\frac{1}{g_0^2}\right)h(k)h(l)N^\dagger a^\dagger(k)Na(l)$$

$$=H_{sp}-\int d^3k\,d^3l\,O\left(\frac{1}{g_0^2}\right)h(k)h(l)N^\dagger a^\dagger(k)Na(l), \tag{21}$$

$$H_L^2\stackrel{s}{=}\frac{m_0}{Z_2}V_2^\dagger V_2+\int d^3k\,O\left(\frac{1}{g_0}\right)f(k)\left[V_2^\dagger Na(k)+N^\dagger a^\dagger(k)V_2\right], \tag{22}$$

with [6]

$$h(k)=\lim_{g_0\to\infty}\frac{g_0f(k)}{\sqrt{M_2}}. \tag{23}$$

We now come to the important point of understanding the meaning of eqs. (14′)–(17′), (21) and (22). If we naively drop terms proportional to $O(1/g_0)$ and $O(1/g_0^2)$, it seems reasonable to conclude that in the SC limit $V_2$, $N$ and $a(k)$ satisfy canonical commutation relations and that $H_L^1$ commutes with $H_L^2$. This would imply that $V_2$ is nothing but a free particle with mass $m_0/Z_2$ decoupled from the $N$ and $\theta$ particles; the hamiltonian for the Lee model would then be equivalent to that for the attractive separable potential model except for the presence of an additional uncoupled mode corresponding to the $V_2$ particle. If this is indeed the case, the Yukawa theory is equivalent to the attractive separable potential model apart from the presence of an uncoupled $V_2$ mode. The truncation of the spectrum of the Yukawa theory (in the higher sectors) discussed in our earlier papers [6, 7] would then be quite irrelevant.

The conclusion drawn above is, in fact, false. If we compute $[H_L^1, H_L^2]$ first and then take the SC limit we find that $H_L^1$ does not commute with $H_L^2$ even in the SC limit. This means that the $V_2$ particle does interact with the $N$ and $\theta$ particles*. In order to reduce the Lee model to the attractive separable potential model, we were forced to exclude all spectral contributions that moved to infinity in the SC limit. We will now show that if we carry out this procedure, the hamiltonian for the Lee model indeed reduces to that for the separable potential. Before doing so, however, we will show that those eigenfunctions of the Lee model corresponding to eigenvalues that do not diverge in the SC limit, reduce to their corresponding counterparts for the separable potential model.

We proceed by writing the equations of motion satisfied by the Källén-Pauli amplitudes, $\langle\langle\lambda|V^{a-c}N^c\theta_{k_i}^{b+c}\rangle$, in the $V^a\theta^b$ sector of the Lee model. As in ref. [6], $|\lambda\rangle\rangle$ denotes the eigenstate of the full hamiltonian, $H_L$, whereas $|V^{a-c}N^c\theta_{k_i}^{b+c}\rangle$ denote the Källén-Pauli basis vector. $k_i$ denotes the momentum of the $i$th $\theta$ particle, with $0 \leqslant i \leqslant b+c$. The equations of motion for the $(a+1)$ amplitudes corresponding to bare $V$ content $a, (a-1), \ldots 1, 0$ can be readily obtained from (1). These take the form,

$$\left[\lambda - am_0 - \sum k_i\right]\langle\langle\lambda|V^a\theta_{k_i}^b\rangle = a\int g_0 f(p)\langle\langle\lambda|V^{a-1}N\theta_{k_i}^b\theta_p\rangle\,\mathrm{d}^3p, \tag{24.1}$$

$$\left[\lambda - (a-1)m_0 - \sum k_i\right]\langle\langle\lambda|V^{a-1}N\theta_{k_i}^{b+1}\rangle = (a-1)\int \mathrm{d}^3p\, g_0 f(p)\langle\langle\lambda|V^{a-2}N^2 \times\theta_{k_i}^{b+1}\theta_p\rangle + \sum_{k_j\in(k_i)} g_0 f(k_j)\langle\langle\lambda|V^a\theta_{k_i}^b\rangle, \tag{24.2}$$

* This can also be explicitly seen from the solution for the $|V_2\theta\rangle\rangle_\lambda$ scattering state presented in refs. [6, 7]. We can easily see that $|V_2\theta\rangle\rangle \neq |V_2\rangle\rangle \otimes |\theta\rangle\rangle$ even in the strong coupling limit.

$$\vdots \quad \vdots$$

$$\left[\lambda-(a-c)m_0-\sum k_i\right]\langle\langle\lambda|V^{a-c}N^c\theta_{k_i}^{b+c}\rangle$$

$$=(a-c)\int \mathrm{d}^3p\, g_0 f(p)\langle\langle\lambda|V^{a-c-1}N^{c+1}\theta_{k_i}^{b+c}\theta_p\rangle$$

$$+\sum_{k_j\in\langle k_i\rangle} g_0 f(k_j)\langle\langle\lambda|V^{a-c+1}N^{c-1}\theta_{k_i}^{b+c-1}\rangle, \qquad (24.c+1)$$

$$\vdots \quad \vdots$$

$$\left[\lambda-m_0-\sum k_i\right]\langle\langle\lambda|VN^{a-1}\theta_{k_i}^{b+a-1}\rangle=\int \mathrm{d}^3p\, g_0 f(p)\langle\langle\lambda|N^a\theta_{k_i}^{b+a-1}\theta_p\rangle$$

$$+(a-1)\sum_{k_j\in\langle k_i\rangle} g_0 f(k_j)\langle\langle\lambda|V^2N^{a-2}\theta_{k_i}^{b+a-2}\rangle, \qquad (24.a)$$

and finally,

$$\left[\lambda-\sum k_i\right]\langle\langle\lambda|N^a\theta_{k_i}^{b+a}\rangle=a\sum_{k_j\in\langle k_i\rangle} g_0 f(k_j)\langle\langle\lambda|VN^{a-1}\theta_{k_i}^{b+a-1}\rangle. \qquad (24.a+1)$$

The sum in the brackets on the left-hand side extends over the momenta of all the $\theta$ particles in the corresponding amplitude. The symbol $k_j$ in $k_j\in\langle k_i\rangle$ in the summation on the right-hand side denotes the momentum of that $\theta$-particle that occurs in the amplitude on l.h.s. but is absent in the corresponding amplitude on the r.h.s. The sum extends over all such combinations that are possible. It can be readily seen from eqs. (24.1) and (24.2) that

$$\langle\langle\lambda|V^{a-1}N\theta_{k_i}^{b+1}\rangle\sim\frac{1}{g_0}\langle\langle\lambda|V^{a-2}N^2\theta_{k_i}^{b+2}\rangle. \qquad (25a)$$

In obtaining (25) we have used the fact that $m_0\sim g_0^2$ but $\lambda\sim(g_0)^0$. We emphasize that the conclusions we draw from our present analysis are valid only for those $\lambda$ that do not grow in the SC limit. From the other equations, it can be readily shown in the same manner in which (25a) was obtained that

$$\langle\langle\lambda|V^{a-c}N^c\theta_{k_i}^{b+c}\rangle\sim\frac{1}{g_0}\langle\langle\lambda|V^{a-c-1}N^{c+1}\theta_{k_i}^{b+c+1}\rangle, \qquad (25b)$$

*i.e. increasing the bare V-particle content by one causes the amplitude to drop by a factor of $g_0^{-1}$ in the SC limit.* We now consider eq. (24.$a$) in the same limit. It can be written as

$$[\lambda - m_0 - \sum k_i]\langle\langle\lambda|V^\dagger|N^{a-1}\theta_{k_i}^{b+a-1}\rangle = \int d^3p\, g_0 f(p)\langle\langle\lambda|N^a\theta_{k_i}^{b+a-1}\theta_p\rangle. \quad (26)$$

In terms of the operator $V_2$ introduced in eq. (12), (26) takes the form,

$$[\lambda - m_0 - \sum k_i]\langle\langle\lambda|\frac{V_2^\dagger}{\sqrt{Z_2}}|N^{a-1}\theta_{k_i}^{b+a-1}\rangle$$
$$\stackrel{s}{=} \int d^3p\left(g_0 f(p) + (\lambda - m_0 - \sum k_i)\frac{g_0 f(p)}{M_2 - p}\right)\langle\langle\lambda|N^a\theta_{k_i}^{b+a-1}\theta_p\rangle$$
$$\stackrel{s}{=} \int d^3p\, O\left(\frac{1}{g_0}\right)f(p)\langle\langle\lambda|N^a\theta_{k_i}^{b+a-1}\theta_p\rangle.$$

We are thus led to the important result*,

$$\langle\langle\lambda|V_2^\dagger|N^{a-1}\theta_{k_i}^{b+a-1}\rangle \sim \frac{1}{g_0^3}\langle\langle\lambda|N^a\theta_{k_i}^{b+a}\rangle. \quad (27)$$

As we shall see, the factor $1/g_0^3$ on the r.h.s. of eq. (27) will play a very critical role in the subsequent analysis. The last of the set of eqs. (24) can now be written in the form

$$[\lambda - \sum k_i]\langle\langle\lambda|N^a\theta_{k_i}^{b+a}\rangle = a\sum_{k_j\in\{k_i\}} g_0 f(k_j)\langle\langle\lambda|\frac{V_2^\dagger}{\sqrt{Z_2}}|N^{a-1}\theta_{k_l}^{b+a-1}\rangle$$
$$- a\sum_{k_j\in\{k_i\}}\int d^3p\, g_0^2 f(k_j)\frac{f(p)}{M_2 - p}\langle\langle\lambda|N^a\theta_{k_l}^{b+a-1}\theta_p\rangle$$
$$\stackrel{s}{=} -a\sum_{k_j\in\{k_i\}}\int d^3p\, h(k_j)h(p)\langle\langle\lambda|N^a\theta_{k_l}^{b+a-1}\theta_p\rangle, \quad (28)$$

with $h(k)$ given by (23). In obtaining this, eq. (27) was crucial to enable us to argue that the first term on the r.h.s. of (28) vanished in the SC limit. Eq. (28) is, of course, the equation of motion for an attractive separable potential. We have thus shown that for those values of $\lambda$ which do not grow in the SC limit, the equation of motion

* Although we have written the $1/g_0^3$ damping for the amplitude where the Källén-Pauli vector has no $V$-particle content, the result is true for an arbitrary $V$-particle content in the Källén-Pauli vector [see eq. (32)]. This follows in the same manner from the analysis of eq. (24.$c$ + 1).

for the one amplitude that remains finite in the SC limit, coincides with that of the separable potential model. It, therefore, follows that the finite energy wave functions and hence the scattering amplitudes and the $S$-matrix coincide in this limit.

Having shown this, we now turn our attention to the question of what is required in order for the hamiltonian of the Lee model to reduce to that of the separable potential model. In our earlier papers [6,7] we had shown that this transmutation could be realized by truncating the spectrum of the Lee model hamiltonian by projecting out all those eigenstates whose eigenvalues move to infinity in the SC limit. We write the spectral decomposition of the hamiltonian as

$$H \equiv H_{\text{low}} + H_{\text{high}} \equiv \sum \left( \lambda_{\text{low}} P_{\lambda_{\text{low}}} + \lambda_{\text{high}} P_{\lambda_{\text{high}}} \right), \tag{29}$$

with

$$P_{\lambda_{\text{low}}} \equiv |\lambda_{\text{low}}\rangle\rangle\langle\langle\lambda_{\text{low}}|, \tag{30a}$$

$$P_{\lambda_{\text{high}}} \equiv |\lambda_{\text{high}}\rangle\rangle\langle\langle\lambda_{\text{high}}|. \tag{30b}$$

In eqs. (30), $|\lambda_{\text{low}}\rangle\rangle$ $(|\lambda_{\text{high}}\rangle\rangle)$ collectively denote the eigenstates of $H_{\text{L}}$ whose eigenvalue $\lambda_{\text{low}}$ $(\lambda_{\text{high}})$ remains finite (moves to infinity) in the SC limit. The sum in eq. (29) extends over all eigenstates.

In keeping with our earlier work, we define the "transmuted" Lee model hamiltonian by

$$H_{\text{L}}^{\text{trans}} \equiv P H_{\text{L}} P, \tag{31a}$$

with

$$P \equiv \sum_{\lambda_{\text{low}}} P_{\lambda_{\text{low}}}. \tag{31b}$$

Clearly, $H_{\text{L}}^{\text{trans}}$ annihilates all the states of the form $|\lambda_{\text{high}}\rangle\rangle$. We now show that

$$H_{\text{L}}^{\text{trans}} \overset{\text{s}}{=} H_{\text{sp}}.$$

We proceed by first showing $PH_{\text{L}}^2 P \overset{\text{s}}{=} 0$, where $H_{\text{L}}^2$ is defined by (20). To this end, we consider the quantity,

$$PV_2^\dagger V_2 P = \sum |\lambda_{\text{low}}\rangle\rangle\langle\langle\lambda_{\text{low}}| V_2^\dagger |z\rangle\langle z| V_2 |\lambda'_{\text{low}}\rangle\rangle\langle\langle\lambda'_{\text{low}}|.$$

The sum runs over all the "low" eigenvalue states and over the complete set of

Källén-Pauli basis vectors, $|z\rangle$. By using eqs. (27) and (25b) together with*

$$\langle\langle\lambda|V_2^\dagger|V^{a-c-1}N^c\theta_{k_i}^{b+c}\rangle \sim \frac{1}{g_0^3}\langle\langle\lambda|V^{a-c-1}N^{c+1}\theta_{k_i}^{b+c+1}\rangle, \tag{32}$$

we conclude that

$$\langle\langle\lambda|V_2^\dagger|z\rangle \sim \frac{1}{g_0^{3+n}}\langle\langle\lambda|\text{zero } V \text{ content vector}\rangle,$$

$n$ being a non-negative integer. It is clear then that $PV_2^+ V_2 P$ damps at least as fast as $1/g_0^6$ in the SC limit. The assertion, $PH_L^2P \overset{s}{=} 0$, then readily follows from the definition of $H_L^2$ [see eq. (22)].

Also, since $P$ is a projection on to the "low" eigenvalue subspace, it acts as a unit operator on this space. It then follows from eq. (21) that

$$PH_L^1P \overset{s}{=} H_{sp}.$$

We have thus demonstrated that in all sectors of the Lee model,

$$PH_LP \overset{s}{=} H_{sp}.$$

This concludes the proof that the transmutation mechanism which had been verified by detailed computation in the two lowest interacting sectors of the Lee model indeed is true for all sectors.

To conclude, we mention that many of the corresponding proofs of equivalence for relativistic field theories proceed by a term-by-term comparison of the renormalized lagrangians in the equivalence limit, the latter being defined by the vanishing of certain renormalization constants of the Yukawa theory. It is not clear whether this would guarantee the equivalence of the two theories. We have explicitly shown that at least for the soluble models discussed in this paper, this is not the case. The Lee model and the attractive separable potential model are clearly distinct in spite of the fact that we can make the respective hamiltonian operators look the same apart from one apparently uncoupled $V_2$ mode [see eqs. (21)–(23)]. As emphasized, the theories can be made equivalent only by truncating the spectrum of the Lee model hamiltonian. It is not known to us whether a corresponding transmutation mechanism is needed for the complete equivalence of the Yukawa-type interaction and the corresponding four-fermion interaction for the case of fully relativistic quantum

* See previous footnote.

field theories. If this is indeed so, it would be of interest to see whether this transmutation mechanism is what causes a renormalizable Yukawa type interaction to alter into a non-renormalizable four-fermion interaction.

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