Phenomenological implications of the existence of conjugate families on the $V-A$ structure of weak interactions

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We investigate the possibility of mixing occurring between the families and the conjugate families of SO(n) grand unified theories. Such a mixing alters the $V-A$ structure of the usual charged weak currents. By comparing with the data on muon and pion decays, we set limits on the corresponding mixing angles. We consider the separate cases corresponding to the conjugate neutrinos being either light or heavy.

There have been many proposals recently[1,2] for SO(n), $n>10$, grand unified models which can incorporate many families into a single irreducible spinor representation of the group. For such models, it is known that for every family contained in the representation, there exists a corresponding conjugate family. Conjugate families are identical to ordinary families except that they have $V+A$ weak interactions.[1–3] For example, in the SO(11) or SO(12) models, there exists one family and one conjugate family in a 32-dimensional spinor representation. So in addition to the leptons e and ν_e, we have E and N_e (having identical electric charges as e and ν_e, respectively) which are members of the conjugate family. The corresponding charged weak current has the form

$$J_\alpha=\overline{e^0}\gamma_\alpha(1-\gamma_5)\nu_e^0+\overline{E^0}\gamma_\alpha(1+\gamma_5)N_e^0 \ , \tag{1}$$

where the superscript 0 denotes "current eigenstates" in terms of which the currents assume the simplest form. Upon enlarging the group to SO(13) or SO(14), one obtains two families, which we can identify as the electron and muon families, and two conjugate families. In this case, analogous muon currents (and conjugate muon currents) must be added to (1).

The charged fermions of the conjugate families are assumed to be heavy to be in agreement with observation. However, the neutral leptons (e.g., N_e) can be light or heavy. In this paper, we study the effects on the low-energy weak-interaction phenomena due to the presence of conjugate families (more precisely, due to the mixing of conjugate families with ordinary families). We shall consider the separate cases in which the neutral leptons (in the weak doublet) belonging to the conjugate families are either light or heavy.

After spontaneous symmetry breaking, the fermions in these theories acquire masses via Yukawa interactions, and, in general, the "mass eigenstates" need not be identical to the "current eigenstates." Consequently, a mixing can occur amongst particles of the same charge.[4] Assuming, for simplicity, a real mass matrix, we have for the case of the SO(11) or SO(12) models

$$\begin{aligned} e^0&=e\cos\theta_e+E\sin\theta_e \ ,\\ E^0&-e\sin\theta_e+E\cos\theta_e \ ,\\ \nu_e^0&=\nu_e\cos\phi_e+N_e\sin\phi_e \ ,\\ N_e^0&=-\nu_e\sin\phi_e+N_e\cos\phi_e \ , \end{aligned} \tag{2}$$

where no superscript denotes "mass eigenstates." Substituting (2) into (1) we find

$$\begin{aligned} J_\alpha=&J_\alpha(e\nu_e)+J_\alpha(eN_e)\\ &+J_\alpha(E\nu_e)+J_\alpha(EN_e) \ , \end{aligned} \tag{3}$$

where, for example,

$$\begin{aligned} J_\alpha(e\nu_e)=&\cos(\theta_e-\phi_e)\bar{e}\gamma_\alpha\nu_e\\ &-\cos(\theta_e+\phi_e)\bar{e}\gamma_\alpha\gamma_5\nu_e \ ,\\ J_\alpha(eN_e)=&-\sin(\theta_e-\phi_e)\bar{e}\gamma_\alpha N_e\\ &-\sin(\theta_e+\phi_e)\bar{e}\gamma_\alpha\gamma_5 N_e \ . \end{aligned} \tag{4}$$

Analogous expressions for the muon family and its conjugate family must be included in (3) upon considering groups such as SO(13) or SO(14). For the purposes of this paper, we shall ignore the Cabibbo-type mixing among the families and only consider the mixing between a family and its corresponding conjugate family.

Upon setting all the angles in (4) equal to zero, we recover the left chiral $V-A$ charged weak current.[5,6] In general, however, the angles need not all vanish. We obtain below constraints on the angles θ_e,ϕ_e, and θ_μ,ϕ_μ (the corresponding muon and muon neutrino mixing angles) by examining the data on (A) muon decay and (B) pion decay.

(A) Muon decay. The differential decay distribution for muon decay is given by [7]

$$d\Gamma=\frac{m_\mu \sin\theta\, d\theta\, p_e E_e dE_e}{48\pi^3}K\left\{3(W-E_e)+2\rho\left[\tfrac{4}{3}E_e-W-\frac{1}{3}\frac{m_e^2}{E_e}\right]+3\frac{m_e}{E_e}\eta(W-E_e)\right.$$
$$\left.-\frac{p_e}{E_e}\xi\cos\theta\left[(W-E_e)+2\delta\left[\tfrac{4}{3}E_e-W-\frac{1}{3}\frac{m_e^2}{m_\mu}\right]\right]\right\}, \qquad (5)$$

where θ is the angle between the electron momentum $\vec{p}_e$ and the muon spin direction, E_e is the electron energy, $p_e=|\vec{p}_e|$, K is defined in terms of the Fermi coupling constant, and

$$W=(m_\mu^2+m_e^2)/2m_\mu .$$

In the derivation of (5) it has been assumed that the neutrino mass is negligible. The parameters ρ, ξ, and δ will be given below in terms of the mixing angles θ_e, θ_μ, ϕ_e, and ϕ_μ. For our purposes, $\eta=0$.

(B) Pion decay. The ratio of the widths $\Gamma(\pi\to e)/\Gamma(\pi\to\mu)$ is given by

$$\frac{\Gamma(\pi\to e)}{\Gamma(\pi\to\mu)}=\left[\frac{m_e}{m_\mu}\right]^2\left[\frac{(m_\pi^2-m_e^2)}{(m_\pi^2-m_\mu^2)}\right]^2(1+r)\kappa , \qquad (6)$$

where the radiative correction r to lowest order is[8,9]

$$r\approx(3\alpha/\pi)\ln(m_e/m_\mu) ,$$

and κ will be given below in terms of the mixing angles θ_e, θ_μ, ϕ_e and ϕ_μ.

In computing the parameters for muon and pion decays we shall consider the following three possibilities for the neutrino masses: (i) $m_{N_e},m_{N_\mu} \ll m_e$, (ii) $m_{N_e}\ll m_e$, $m_{N_\mu}>m_\mu$, and (iii) $m_{N_e},m_{N_\mu}>m_\mu$.

(i) $m_{N_e},m_{N_\mu}\ll m_e$. Here the effective four-fermion interaction Hamiltonian for muon decay is

$$\frac{G}{\sqrt{2}}[J^\alpha(e\nu_e)+J^\alpha(eN_e)][J_\alpha^\dagger(\mu\nu_\mu)+J_\alpha^\dagger(\mu N_\mu)] , \qquad (7)$$

where the currents involving the electron are given in (4). From (7) we find

$$\rho=\tfrac{3}{8}(1+\cos2\theta_e\cos2\theta_\mu),\xi=2\cos2\theta_e-\cos2\theta_\mu ,$$
$$\delta=\tfrac{3}{8}(\cos2\theta_e+\cos2\theta_\mu)/(2\cos2\theta_e-\cos2\theta_\mu) . \qquad (8)$$

The interaction Hamiltonian relevant for pion decay is

$$\frac{G}{\sqrt{2}}J_\alpha^h[J^{\alpha^\dagger}(e\nu_e)+J^{\alpha^\dagger}(eN_e)+J^{\alpha^\dagger}(\mu\nu_\mu)+J^{\alpha^\dagger}(\mu N_\mu)] , \qquad (9)$$

where J_α^h is the hadronic weak interaction current. From (9) we find $\kappa=1$.

The prediction for the chiral $V-A$ theory is recovered when $\theta_e=\theta_\mu=0$. *We note that the neutrino mixing angles ϕ_e and ϕ_μ do not appear in (8). Therefore, they may be arbitrarily large and still consistent with the data on muon and pion decay.* Note also that $0\le\rho\le\frac{3}{4}$.

Good agreement of the data (cf. Table I)[10] with the values given by $V-A$ theory restrict the angles θ_e and θ_μ to be small. Upon expanding (8) around $\theta_e=\theta_\mu=0$ up to second order, we find,

$$\rho=\tfrac{3}{4}[1-(\theta_e^2+\theta_\mu^2)],\ \ \xi=1-4\theta_e^2+2\theta_\mu^2 ,$$

and

$$\delta=\tfrac{3}{4}[1+3(\theta_e^2-\theta_\mu^2] .$$

Allowing up to two standard deviations in the data

TABLE I. Experimental data for the muon decay parameters.

ρ	0.7517±0.0026
ξ	0.972 ±0.013
δ	0.7551±0.0085
κ	1.03 ±0.02

for ρ, ξ, and δ, we find $|\theta_e|<0.07$, and $|\theta_\mu|<0.06$.

(ii) $m_{N_e} << m_e$, $m_{N_\mu} > m_\mu$. Here the current $J_\alpha(\mu N_\mu)$ no longer plays an important role in the muon and pion decay processes. Upon removing this current from the expressions (7) and (9), we now obtain

$$\rho=\tfrac{3}{8}(1+2\sigma_\mu\cos2\theta_e)\ ,$$
$$\xi=2(\cos2\theta_e-\sigma_\mu)\ ,$$
$$\delta=\frac{3}{16}\left[\frac{\cos2\theta_e+2\sigma_\mu}{\cos2\theta_e-\sigma_\mu}\right]\ , \tag{10}$$
$$\kappa=2/[\cos^2(\theta_\mu-\phi_\mu)+\cos^2(\theta_\mu+\phi_\mu)]\ ,$$

where

$$\sigma_i=\frac{\cos(\theta_i+\phi_i)\cos(\theta_i-\phi_i)}{\cos^2(\theta_i+\phi_i)+\cos^2(\theta_i-\phi_i)}\ ,$$
$$i=e,\mu\ .$$

The prediction for the $V-A$ theory is recovered when $\theta_e=\theta_\mu=\phi_\mu=0$. *Since ϕ_e does not appear in the expression (10) it may be arbitrarily large and still consistent with the data on muon and pion decay.* Here $-\tfrac{3}{4}\le\rho\le\tfrac{9}{8}$ and $\kappa\ge1$.

Upon expanding (10) around $\theta_e=\theta_\mu=\phi_\mu=0$ up to second order,

$$\rho=\tfrac{3}{4}(1-\theta_e^{\,2}),\quad \xi=1-4\theta_e^{\,2}\ ,$$
$$\delta=\tfrac{3}{4}(1+3\theta_e^{\,2})\ ,$$

and

$$\kappa=1+\theta_\mu^{\,2}+\phi_\mu^{\,2}\ .$$

Allowing up to two standard deviations in the data for ρ, ξ, δ, and κ we find $|\theta_e|<0.07$, and $|\theta_\mu|$, $|\phi_\mu|<0.27$.

(iii) $m_{N_e}, m_{N_\mu} > m_\mu$. Now $J_\alpha(\mu N_\mu)$ and $J_\alpha(eN_e)$ no longer play an important role in the decay processes and must be removed from (7) and (9). This yields

$$\rho=\tfrac{3}{8}(1+4\sigma_e\sigma_\mu),\quad \xi=2(2\sigma_e-\sigma_\mu)\ ,$$
$$\delta=\tfrac{3}{8}[(\sigma_e+\sigma_\mu)/(2\sigma_e-\sigma_\mu)]\ , \tag{11}$$
$$\kappa=\frac{\cos^2(\theta_e-\phi_e)+\cos^2(\theta_e+\phi_e)}{\cos^2(\theta_\mu-\phi_\mu)+\cos^2(\theta_\mu+\phi_\mu)}\ .$$

From (11), the prediction for the $V-A$ theory is recovered when $\sigma_e=\sigma_\mu=\tfrac{1}{2}$. This relation along with $\kappa=1$ leads to the following possibilities: (a) $\theta_\mu=\theta_e=0$, $|\phi_e|=|\phi_\mu|=\alpha$, (b) $\phi_e=\phi_\mu=0$, $|\theta_e|=|\theta_\mu|=\alpha$, (c) $\theta_\mu=\phi_e=0$, $|\phi_\mu|=|\theta_e|=\alpha$, and (d) $\phi_\mu=\theta_e=0$, $|\theta_\mu|=|\phi_e|=\alpha$, where α is an arbitrary (positive) angle. As in case (i), we again have $0\le\rho\le\tfrac{3}{4}$.

Good agreement of the data (cf. Table I) with the values given by $V-A$ theory restrict at least two of the angles, specifically, (θ_μ,θ_e), (ϕ_μ,ϕ_e), (θ_μ,ϕ_e), or (ϕ_μ,θ_e) to be small. Upon expanding (11) around (a), i.e., $\theta_e=\theta_\mu=0$ with $|\phi_e|=|\phi_\mu|=\alpha$ held fixed, up to second order, we find

$$\rho=\tfrac{3}{4}[1-(\theta_e^{\,2}+\theta_\mu^{\,2})\tan^2\alpha]\ ,$$
$$\xi=1-2(2\theta_e^{\,2}-\theta_\mu^{\,2})\tan^2\alpha\ ,$$
$$\delta=\tfrac{3}{4}[1+3(\theta_e^{\,2}-\theta_\mu^{\,2})\tan^2\alpha]\ ,$$
$$\kappa=1-(\theta_e^{\,2}-\theta_\mu^{\,2})(1-\tan^2\alpha)\ ,$$

$\alpha\neq\pm\pi/2$. Allowing up to two standard deviations in the data for ρ, ξ, δ, and κ, we find the following inequalities for θ_e, θ_μ, and α:

$$|\theta_e|\,|\tan\alpha|\le0.07\ ,$$
$$|\theta_\mu|\,|\tan\alpha|\le0.06\ ,$$
$$-0.02\le\theta_\mu^{\,2}-\theta_e^{\,2}\le0.07\ .$$

The analogous expressions corresponding to expanding around (b), (c), and (d) are obtained by interchanging θ_μ, ϕ_μ, θ_e, and ϕ_e appropriately in the above inequalities.

In conclusion, we have pointed out that mixing between the particles of a family with those of a conjugate family induces a $V+A$ admixture into low-energy weak interactions.[11] In applying this to pion and muon decays, we find that although the data is in very good agreement with the predictions of the chiral $V-A$ theory, a certain amount of mixing can be tolerated (cf. Table II). In fact

TABLE II. Estimated values for the upper bounds on the mixing angle θ_e, θ_μ, ϕ_e, and ϕ_μ (in radians) for case (i) and (ii). No simple bounds on the mixing angle emerge in case (iii).

	Case (i)	Case (ii)
$\|\theta_e\|$	<0.07	<0.07
$\|\theta_\mu\|$	<0.06	<0.27
$\|\phi_e\|$	undetermined	undetermined
$\|\phi_\mu\|$	undetermined	<0.27

some of the angles may be arbitrarily large. We have investigated three separate cases arising from whether the conjugate neutrinos are either light or heavy. Accurate measurements of the parameters ρ and κ may help to distinguish the three cases.

Finally, in some specific models[12] mixing angles are given by $(m_i/M_i)^{1/2}$, where m_i corresponds to the mass of a particle in an ordinary family and M_i corresponds to the mass of the particle in the conjugate family with which it mixes. The bounds in Table II can then be used to give lower limits on the masses of some of the leptons in the conjugate families. For example, using $|\theta_\mu| \leq 0.06$ [cf. Table II (case i)] and $m_\mu = 0.105$ GeV, we obtain 30 GeV as a lower bound on the mass of the conjugate muon. Consequently, searching for heavy leptons with $V+A$ weak interaction may prove worthwhile.

Note added. While this work was being completed we received a paper by K. Enqvist, K. Mursula, J. Maalampi, and M. Roos [University of Helsinki report (unpublished)], where a similar analysis is carried out. Unlike us, they restrict their discussions to the case where both of the conjugate neutrinos are heavy [case (iii) in our paper].

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[1]M. Gell-Mann, P. Ramond, and R. Slansky, Rev. Mod. Phys. 50, 721 (1978).

[2]F. Wilczek and A. Zee, Princeton report, 1979 (unpublished).

[3]R. N. Mohapatra and B. Sakita, Phys. Rev. D 21, 1062 (1980).

[4]Within the context of the SO(12) model, a mixing between the family and the conjugate family is obtained by giving nonzero vacuum expectation values to the neutral, colorless components of a 495-plet (and a singlet) representation of Higgs particles. Examples of such components are $\phi_{11,12,9,10}$, $\phi_{11,12,7,8}$, $\phi_{9,10,7,8}$, and $\phi_{i,j,1,2} + \phi_{i,j,3,4} + \phi_{i,j,5,6}$, where $(i,j) = (11,12)$, (9, 10), (7, 8), and we are using the conventions of S. Nandi, A. Stern, and E. C. G. Sudarshan, Phys. Rev. D 26, 1653 (1982).

[5]E. C. G. Sudarshan and R. E. Marshak, in Proceedings of the Padua-Venice Conference on Mesons and Newly Discovered Particles, 1957 (unpublished) and Phys. Rev. 109, 1860 (1958); reprinted in P. K. Kabir, *Development of Weak Interaction Theory* (Gordon and Breach, New York, 1963), pp. 118–128.

[6]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

[7]See for example, R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions* (Wiley-Interscience, New York, 1969).

[8]W. J. Marciano and A. Sirlin, Phys. Rev. Lett. 36, 1425 (1976).

[9]T. Goldman and W. J. Wilson, Phys. Rev. D 15, 709 (1977).

[10]Particle Data Group, Rev. Mod. Phys. 52, Sl (1980).

[11]We note that there exist alternative schemes for introducing a $V+A$ admixture into the low-energy weak interactions. A detailed analysis of the consequences of such an admixture arising in the context of an $SU(2)_L \times SU(2)_R \times U(1)$ model has been carried out by M. A. B. Bég, R. V. Budny, R. Mohapatra, and A. Sirlin, Phys. Rev. Lett. 38, 1252 (1977).

[12]In the context of Cabibbo mixing see, e.g., H. Fritzsch and P. Minkowski, Phys. Rep. 73, 68 (1981).