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RENORMALIZATION GROUP AND COMPOSITENESS IN QUANTUM-FIELD THEORY

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By making use of the renormalization group technique, we show that the compositeness conditions required for the equivalence of quantum chromodynamics and the corresponding four-fermion theory may be demonstrated for a large class of gauges. It is also pointed out from a study of soluble models that the equivalence in the sense that the wave functions are the same for finite energy does not guarantee the total equivalence of the Yukawa-type theory to the four-fermion theory.

A reduction in the number of fundamental fields in interaction naturally leads to a unification of interparticle interactions. This has motivated several authors[1–5]) to attempt to obtain the intermediate boson of a Yukawa-type theory as a composite of a single, quartically[6]) self-coupled spinor field. Of particular interest is the case when the resulting Yukawa theory admits either an Abelian or a non-Abelian local gauge symmetry. The gauge bosons are then regarded as composites of the fermions.

It has recently been shown that for certain types of non-Abelian gauge theories, the compositeness conditions are automatically satisfied, e.g., in the case of quantum chromodynamics with ten to sixteen flavours[7,8]). We argue, however, that satisfying the compositeness conditions may not in itself be sufficient to ensure the equivalence of quantum chromodynamics with the corresponding four-fermion theory.

We proceed, following ref. 8, by first deriving the compositeness conditions and then showing that they are automatically satisfied. To this end, we rewrite the four-fermion Lagrangian

$$\mathscr{L}_{\mathrm{f}} = \bar{\psi}_{\mathrm{b}}(\mathrm{i}\partial\!\!\!/ - m_{\mathrm{b}})\psi_{\mathrm{b}} - \tfrac{1}{2}G_{\mathrm{b}}\left(\bar{\psi}_{\mathrm{b}}\gamma_\mu \frac{\boldsymbol{\lambda}}{2}\psi_{\mathrm{b}}\right)^2, \tag{1}$$

by introducing auxiliary fields $\mathbf{A}_\mu$ as

$$\mathscr{L}'_{\mathrm{f}} = \bar{\psi}_{\mathrm{b}}(\mathrm{i}\partial\!\!\!/ - m_{\mathrm{b}})\psi_{\mathrm{b}} - g_{\mathrm{b}}\bar{\psi}_{\mathrm{b}}\gamma_\mu \frac{\boldsymbol{\lambda}}{2}\psi_{\mathrm{b}} \cdot \mathbf{A}_{\mathrm{b}} + \tfrac{1}{2}\delta\mu^2 \mathbf{A}_{\mathrm{b}\mu} \cdot \mathbf{A}_{\mathrm{b}}^{\mu}, \tag{2}$$

with $G_{\mathrm{b}} = g_{\mathrm{b}}^2/\delta\mu^2$. $\mathscr{L}'_{\mathrm{f}}$ can then be written formally in terms of the renor-

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malized quantities (the subscript, b, in (1) and (2) indicating bare quantities) defined by $\sqrt{Z_2}\psi = \psi_b$, $\sqrt{Z_3}\mathbf{A}_\mu = \mathbf{A}_{b\mu}$, $Z_3^{-3/2}Z_1 g = g_b$, as*

$$\mathcal{L}_{fR} = \bar\psi(i\partial\!\!\!/ - m)\psi - g\bar\psi\gamma_\mu \frac{\boldsymbol\lambda}{2}\psi\cdot\mathbf{A}^\mu - \tfrac14 \mathbf{F}_{\mu\nu}\cdot\mathbf{F}^{\mu\nu}$$
$$+\Big[(Z_2-1)\bar\psi(i\partial\!\!\!/ - m)\psi + Z_2(m-m_b)\bar\psi\psi - (Z_{1f}-1)g\bar\psi\gamma_\mu\frac{\boldsymbol\lambda}{2}\psi\cdot\mathbf{A}^\mu$$
$$+\tfrac14(\partial_\mu\mathbf{A}_\nu - \partial_\nu\mathbf{A}_\mu)^2$$
$$+\tfrac12 g(\partial_\mu\mathbf{A}_\nu - \partial_\nu\mathbf{A}_\mu)\cdot(\mathbf{A}^\mu\times\mathbf{A}^\nu) + \tfrac14 g^2(\mathbf{A}^\mu\times\mathbf{A}^\nu)^2 + \tfrac12 Z_3\delta\mu^2\mathbf{A}_\mu\cdot\mathbf{A}^\mu\Big], \tag{3}$$

where $Z_{1f} = Z_3^{-1}Z_1Z_2$ and $\mathbf{F}_{\mu\nu} = (\partial_\mu\mathbf{A}_\nu - \partial_\nu\mathbf{A}_\mu) + g\mathbf{A}_\mu\times\mathbf{A}_\nu$.

Following the same steps, the quantum chromodynamic Lagrangian

$$\mathcal{L}_Q = \bar\psi_b(i\partial\!\!\!/ - m_b)\psi_b - g\bar\psi_b\gamma_\mu\frac{\boldsymbol\lambda}{2}\psi_b\cdot\mathbf{A}^\mu - \tfrac14\mathbf{F}_{b\mu\nu}\cdot\mathbf{F}_b^{\mu\nu}, \tag{4}$$

can also be written in terms of the renormalized quantities as

$$\mathcal{L}_{QR} = \bar\psi(i\partial\!\!\!/ - m)\psi - g\bar\psi\gamma_\mu\frac{\boldsymbol\lambda}{2}\psi\cdot\mathbf{A}^\mu - \tfrac14\mathbf{F}_{\mu\nu}\cdot\mathbf{F}^{\mu\nu} + \Big[(Z_2-1)\bar\psi(i\partial\!\!\!/ - m)\psi$$
$$+ Z_2(m-m_b)\bar\psi\psi - (Z_{1f}-1)g\bar\psi\gamma_\mu\frac{\boldsymbol\lambda}{2}\psi\cdot\mathbf{A}^\mu - \tfrac14(Z_3-1)(\partial_\mu\mathbf{A}_\nu - \partial_\nu\mathbf{A}_\mu)^2$$
$$-\tfrac12(Z_1-1)g(\partial_\mu\mathbf{A}_\nu - \partial_\nu\mathbf{A}_\mu)\cdot(\mathbf{A}_\mu\times\mathbf{A}_\nu) - \tfrac14(Z_4-1)g^2(\mathbf{A}_\mu\times\mathbf{A}_\nu)^2\Big], \tag{5}$$

with $Z_4 = Z_1^2/Z_3$. The separation in (5) is to be understood in the sense that the terms in the square brackets are counter-terms to cancel the divergent parts of the radiative corrections which occur in perturbation theory based on the renormalized quantum chromodynamic Lagrangian,

$$\mathcal{L}'_{QR} = \bar\psi(i\partial\!\!\!/ - m)\psi - g\bar\psi\gamma_\mu\frac{\boldsymbol\lambda}{2}\psi\cdot\mathbf{A}^\mu - \tfrac14\mathbf{F}_{\mu\nu}\cdot\mathbf{F}^{\mu\nu}. \tag{6}$$

The conditions for the form equivalence of Lagrangians (3) and (5) can be easily seen to be

$$Z_3 = 0,\quad Z_1 = 0 \quad\text{and}\quad Z_4 = 0. \tag{7}$$

We now use the renormalization-group method to show that eqs. (7) are satisfied for quantum chromodynamics. The renormalization constants Z_i that relate bare quantities to the corresponding quantities renormalized at the

* The mass $\delta\mu$ of the field $\mathbf{A}_{b\mu}$ is chosen so as to cancel that which arises in the calculation of the self-energy of the $\mathbf{A}_\mu$ field. We may regard this, following Eguchi[4]), as a regularization prescription.

Euclidean point, μ, have the functional dependence $Z_i = Z_i(\Lambda/\mu, g_b, \alpha_b)$, where Λ is the ultraviolet cut-off and g_b and α_b are the bare coupling constant and gauge-fixing parameter, respectively. We carefully distinguish[8]) the Z_i's from the constants

$$Z_i^R(\mu/\mu_0, g(\mu_0), \alpha(\mu_0)) = \frac{Z_i(\Lambda/\mu, g_b, \alpha_b)}{Z_i(\Lambda/\mu_0, g_b, \alpha_b)}$$

which relate quantities renormalized at two different points μ and μ_0. The quantities $g(\mu_0)$ and $\alpha(\mu_0)$ are the coupling constants and gauge-fixing parameters, renormalized at μ_0.

We define, as usual, the quantity γ_i as

$$\gamma_i \equiv \frac{-\mu}{Z_i}\frac{\partial Z_i}{\partial\mu} = \frac{-\mu}{Z_i^R}\frac{\partial Z_i^R}{\partial\mu}. \tag{8}$$

By rather straightforward manipulations, the integral form of (8) can be written as

$$\int_{Z_i(\mu_0)}^{Z_i(\mu)} \frac{1}{Z_i}\mathrm{d}Z_i = -\int_{g(\mu_0)}^{g(\mu)} \frac{\gamma_i(g(\mu'), \alpha(\mu'))}{\beta(g(\mu'))}\mathrm{d}g(\mu'), \tag{9}$$

where $\beta \equiv \mu(\partial g/\partial\mu)$. Expanding γ_i and β as power series in g, we have

$$\gamma_i = \tilde{\gamma}_i(\alpha)g^2 + \cdots \quad \text{and} \quad \beta = -bg^3, \tag{10}$$

where [9])

$$\begin{gathered} b = \frac{1}{16\pi^2}\left[\frac{11}{3}N_c - \frac{2}{3}N_f\right], \quad \tilde{\gamma}_3(\alpha) = \frac{1}{16\pi^2}\left[\left(\frac{13}{6} - \frac{\alpha}{2}\right)N_c - \frac{2}{3}N_f\right], \\ \tilde{\gamma}_1(\alpha) = -b + \frac{3}{2}\tilde{\gamma}_3(\alpha), \quad \gamma_2(\alpha) = \frac{-1}{16\pi^2}\alpha N_c, \quad \tilde{\gamma}_4(\alpha) = 2\tilde{\gamma}_1(\alpha) - \tilde{\gamma}_3(\alpha), \end{gathered} \tag{11}$$

where N_c and N_f are the number of colours and flavours, respectively.

In order to obtain the solution to (9), we have first to solve the renormalization group equation

$$\frac{\partial\alpha}{\partial \ln\mu} = \gamma_3(\alpha)\alpha(\mu) \tag{12}$$

for $\alpha(\mu)$. The solution to (12) can be readily written as[10])

$$\alpha(\mu) = \frac{[g(\mu_0)/g(\mu)]^k\left(\frac{13}{6}N_c - \frac{2}{3}N_f\right)\alpha(\mu_0)}{\frac{13}{6}N_c - \frac{2}{3}N_f - \frac{N_c}{2}\alpha(\mu_0) + [g(\mu_0)/g(\mu)]^k\frac{N_c}{2}\alpha(\mu_0)} \tag{13}$$

with

$$k \equiv \frac{\tilde{\gamma}_3(0)}{b} = \left(\frac{13}{6}N_c - \frac{2}{3}N_f\right) \Big/ \left(\frac{11}{3}N_c - \frac{2}{3}N_f\right).$$

We will restrict ourselves to asymptotically-free theories with $\tilde{\gamma}_3(0) < 0$ since it is only then that the compositeness conditions can be satisfied. Also, as pointed out in ref. 8, the validity of eq. (13) is restricted to $\alpha > \alpha_c = 32\pi^2\tilde{\gamma}_3(0)/N_c$. Hence we restrict the forthcoming discussion only to such choices of gauge.

The effect of the renormalization constants on the renormalization point can be readily obtained using eqs. (9) and (13). We find,

$$Z_3(\Lambda/\mu) = Z_3(\Lambda/\mu_0)\left\{\frac{1+\Delta}{1+\Delta[g(\mu_0)/g(\mu)]^k}\right\}, \tag{14a}$$

$$Z_1(\Lambda/\mu) = Z_1(\Lambda/\mu_0)\left[\frac{Z_3(\Lambda/\mu)}{Z_3(\Lambda/\mu_0)}\right]^{3/2} g(\mu_0)/g(\mu), \tag{14b}$$

$$Z_4(\Lambda/\mu) = Z_4(\Lambda/\mu_0)\left[\frac{Z_3(\Lambda/\mu)}{Z_3(\Lambda/\mu_0)}\right]^2 [g(\mu_0)/g(\mu)]^2, \tag{14c}$$

with $\Delta \equiv 32\pi^2\tilde{\gamma}_3(\alpha)/N_c\alpha$. We see from eqs. (14) that if the renormalization constants can be independently argued to be finite*, the only solutions to these equations for asymptotically-free theories with $N_f > (13/4)N_c$ (so that $k < 0$) are null solutions[8]), so that the compositeness conditions (7) are satisfied.

At this point, we note that our study[11]) of the corresponding situation for the equivalence of the soluble Lee model (Yukawa theory) and the separable potential model (four-point interaction) is an explicit demonstration of a theory wherein the compositeness conditions are satisfied without the two theories being completely equivalent. We have further shown that in order to transmute the Yukawa interaction into the four-point interaction, the spectrum of the Yukawa theory has to be truncated. If a similar scenario is prevalent for the case of fully relativistic field theories, the present proofs of equivalence of quantum chromodynamics with the corresponding four-fermion theories will need to be re-examined.

* Heuristic arguments for this have been presented in the footnote before. Admittedly these arguments are dependent on the particular procedure for taking limits in the computations.

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