SPONTANEOUSLY BROKEN SUPERSYMMETRY AND POINCARÉ INVARIANCE

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It is argued that the spontaneous breakdown of global supersymmetry is consistent with unbroken Poincaré invariance if and only if the supersymmetry algebra "$\mathcal{A} = 0$" is understood to mean the invariance of the dynamical variables ϕ under the transformations generated by the algebra, i.e. $[\mathcal{A}, \phi] = 0$ rather than as an operator equation. It is further argued that this "weakening" of the algebra does not alter any of the conclusions about supersymmetry quantum field theories that have been obtained using the original (stronger) form of the algebra.

In recent years, there has been a lot of interest in global supersymmetry [1]. One motivation for studying this is that it is the only known symmetry that protects scalar particles from obtaining large masses via radiative corrections. It has, therefore, been suggested that supersymmetry may enable us to introduce small mass scales into a theory which also contains a large mass scale, without the need for adjusting the parameters in the lagrangian to an uncanny accuracy, order by order in perturbation theory. Unfortunately, however, exact and unbroken supersymmetry leads to a spectrum of particles in which there are (mass) degenerate pairs of bosons and fermions. Since these are not observed in nature, we are led to conclude that supersymmetry (if it exists) is either explicitly or spontaneously broken. The purpose of this letter is to clarify some of the features of spontaneously broken supersymmetry.

In Poincaré invariant $N = 1$ supersymmetric theories, in addition to the Poincaré group generators P_μ and $J_{\mu\nu}$, we have an additional fermionic generator that commutes with space-time translations, transforms as a Majorana spinor under Lorentz transformations and further satisfies

$$\{Q_\alpha, \bar{Q}_\beta\} = -2\mathrm{i}(P^\mu \gamma_\mu)_{\alpha\beta}. \qquad (1)$$

From eq. (1) it easily follows that

$$\sum_\alpha \{Q_\alpha, Q_\alpha^\dagger\} = 4P^0, \qquad (2)$$

where P^0 is identified with the hamiltonian, H. Since the left-hand side of eq. (2) is a non-negative operator (we assume that the state-space metric is positive definite), we are led to the conclusion that supersymmetry is unbroken if and only if the vacuum expectation value of the hamiltonian vanishes. At this point, we simply remark that the above conclusion is valid only when the action of Q on the vacuum is defined, i.e. leads to a normalizable state. The study of the circumstances for which this is not the case is the main point of this paper.

On the other hand, in a Poincaré invariant theory in which Poincaré invariance is not broken, we have,

$$\langle 0|P^\mu|0\rangle = 0. \qquad (3)$$

Eqs. (2) and (3) taken together imply that if Poincaré invariance is unbroken, so also is supersymmetry [2] [‡1].

[‡1] Domokos and Kovesi-Domokos argue that although translation invariance is broken, it may be possible to leave the Lorentz subgroup of the Poincaré group unbroken. See ref. [3].

The catch in this simple-minded argument is that the action of Q may not be defined.

We now recall [4] that it is precisely when the action of any (formally) conserved charge on the vacuum leads to a non-normalizable state, is the symmetry corresponding to that particular charge spontaneously broken. Under these circumstances, the original vacuum and the new state obtained by the said transformation are not unitarily related.

The situation for the case of supersymmetry with the spinorial charge Q or alternatively with the "bosonic charge" $\xi^\alpha Q_\alpha$ (with ξ^α being an anticommuting constant) being undefined is exactly the same. The action, $\exp(\mathrm{i}\xi^\alpha Q_\alpha)$, of the bosonic charge $\xi^\alpha Q_\alpha$ is not unitarily implemented. Furthermore, eq. (2) ceases to be meaningful in the form in which it has been written. We now attempt to modify the above discussion so that it does not lead to seemingly paradoxical situations.

To this end, we first point out that a discussion of any symmetry entails the specification of the action of the symmetry transformations on the dynamical variables of the system. For quantum field theory, this means that the symmetries can be discussed by specifying the action of the transformations on field operators. The infinitesimal action of the supersymmetry and Poincaré transformations can all be specified by specifying the commutator of the bosonic generators $P_\mu, J_{\mu\nu}$ and $\xi^\alpha Q_\alpha$ with the field operators. Furthermore, the supersymmetry algebra may be weakened [‡2] by requiring that it be satisfied only when (in fact, it may only then be well defined) acting on field operators. For instance, eq. (2) would be replaced by [‡3]

$$\left[\sum_\alpha \{Q_\alpha, Q_\alpha^\dagger\},\ \phi(\bar{x},t)\right] = [4P^0, \phi(\bar{x},t)]\,, \tag{4}$$

where $\phi(\bar{x},t)$ is a generic field operator. We emphasize that it is only in the weak form (4), that the supersymmetry algebra is relevant for the computation of physical effects in lagrangian quantum field theory.

‡2 See also ref. [5], appendix A.

‡3 The generators P^μ are thus defined only up to a neutral element. It is amusing to observe that in $J_{\mu\nu} = \int \mathrm{d}^3x\,[x_\mu T_{\nu 4} - x_\nu T_{\mu 4}]$ ($T_{\mu\nu}$ is the symmetric energy momentum tensor) the dependence on the neutral element is absent. The commutator between $J_{\mu\nu}$ and P_σ is then necessarily to be viewed as a "weak" relation.

All the results that have been proven would continue to hold even with the "weak form" (4) of the algebra, e.q. supersymmetry is spontaneously broken when and only when auxiliary fields develop vacuum expectation values or that supersymmetry breaking yields a massless goldstino. The proof of these results entailed a knowledge of only the *commutator* of the supersymmetry generators with the field operators. The supersymmetry algebra as a set of operator equations [as in eq. (2)] need never be used.

The recognition that the weak form of the algebra alone is relevant enables us to break supersymmetry without arriving at the unacceptable conclusion that Poincaré invariance is simultaneously broken. This is so because eq. (4) is satisfied by

$$\sum_\alpha \{Q_\alpha, Q_\alpha^\dagger\} = 4P^0 + N, \tag{5}$$

where N is a neutral element. We can then argue based on the Lorentz invariance of the theory that N should be chosen to be a neutral element that commutes with all field operators and all the generators of the Poincaré group; hence we may take it to be a numerical constant such that eq. (3) is satisfied. In other words, the vacuum expectation value of the hamiltonian can be zero (and should be zero if Poincaré invariance is unbroken) even when supersymmetry is broken.

To clarify the conditions for a spontaneous breakdown of supersymmetry, we consider the anticommutator of the spinorial supersymmetry current $S^\nu(x,0)$ with the supersymmetry generator. We then have

$$\{Q_\alpha, S^\nu_\beta(x,0)\} = -2\mathrm{i}T^{\mu\nu}(x,0)(\gamma_\mu)_{\alpha\beta}, \tag{6}$$

where $T^{\mu\nu}$ is the energy momentum tensor. Integration of eq. (6) leads to eq. (1) when $\nu = 0$. It is quite true that the $T^{\mu\nu}$ can develop a vacuum expectation value in a Lorentz invariant fashion as

$$\langle 0|T^{\mu\nu}|0\rangle = \rho\eta^{\mu\nu}.$$

ρ is position independent by virtue of the translation invariance of the vacuum. A non-zero value of ρ is a signal for the spontaneous breakdown of supersymmetry since if Q annihilated the vacuum we would, by eq. (6), have $\rho = 0$. We note, however, that when this happens, the generators

$$P^\mu = \int T^{\mu 0}(\boldsymbol{x},0)\, d^3x$$

are undefined.

In this paper, we have suggested that the lagrangian density be altered by a numerical constant, say $+\rho$ (and hence the hamiltonian density by $-\rho$), such that eq. (3) is satisfied. This changes the energy momentum tensor by an amount $-\rho\eta^{\mu\nu}$ but the supercurrents derived from the two hamiltonian densities $\mathcal{H}$ and $\mathcal{H}' = \mathcal{H} - \rho$ are the same. For the primed system, we can write eq. (6) as

$$\{Q'_\alpha, S^\nu_\beta(\boldsymbol{x},0)\} = -2\mathrm{i}[T'^{\mu\nu}(\boldsymbol{x},0) + \rho\eta^{\mu\nu}](\gamma_\mu)_{\alpha\beta}, \quad (7)$$

with $\langle 0|T'^{\mu\nu}|0\rangle = 0$. The spontaneous breakdown of supersymmetry is still signalled by a non-zero value of ρ. We remark that the "weak forms" of eqs. (6) and (7) are the same, viz.

$$[\{Q_\alpha, S^\nu_\beta(\boldsymbol{x},0)\}, \phi(y,t)]$$
$$= 2\mathrm{i}[(T^{\mu\nu}(\boldsymbol{x},0)\gamma_\mu)_{\alpha\beta}, \phi(y,t)]\,.$$

In view of the above discussion, we see that unbroken Lorentz invariance always implies the existence of a zero energy vacuum state provided one is dealing with the appropriate hamiltonian ($\mathcal{H}'$ in this case). Then the role of Witten's index criterion [6] for a spontaneous breakdown of supersymmetry needs clarification. His arguments go through exactly as before, with the following stipulation: wherever the term "energy" is used, it should be replaced by "the eigenvalue of the operator $H = \int \mathcal{H}\, d^3x$". Thus supersymmetry is unbroken if $n_b - n_f \neq 0$ [n_b (n_f) are the number of bosonic (fermionic) states with a zero eigenvalue of H]. Instead, the energy of the system is to be identified with $H' = \int \mathcal{H}'\, d^3x$ [‡4].

In summary, we note that by considering a weaker form of the supersymmetry algebra, we are able to spontaneously break supersymmetry and yet not be led to conclude that $E_0 = \langle 0|\text{hamiltonian}|0\rangle \neq 0$. The conclusion $E_0 \neq 0$ is not acceptable since Poincaré invariance is unbroken. Furthermore, a non-zero vacuum energy would lead to a cosmological constant when the supersymmetric theory is coupled to gravity [‡5]. The price paid is that the conditions for the spontaneous breakdown of supersymmetry are decoupled from the spectrum of the hamiltonian. It is emphasized though that all the results obtained for supersymmetric lagrangian field theories continue to hold since they can all be derived from the weak form of the algebra, the only difference being that the quantity generally regarded as the energy is that up to a constant which can be adjusted so that $E_0 = 0$. We remark that the spirit of this present approach is probably implicit in much of the literature, but it seems worthwhile to have made it explicit in that it may serve as a basis for further discussion.

[‡4] If supersymmetry is unbroken, $\rho = 0$ and $H = H'$ is indeed the energy of the system.

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[‡5] It would be of great interest to see whether this method can be consistently extended when the supersymmetric theory is coupled to gravity, thereby eliminating the problem of the cosmological constant.

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