NATURAL LAW

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Growing up in the Asian Tradition

The study of the universe in which we find ourselves is part of the joys of the coming of age in the Asian tradition. At the periods of twilight in the transitions between night and day and between day and night we recite the gāyatri which invokes the brilliance or light to inspire one. Further, this understanding is to be a unified whole ecompassing the static and the dynamic.

Law is often viewed as normative, as upholding a higher order than the unconstrained functioning of individual human action according to their inclinations. Whether it be the contemporary notion of sovereignity residing ultimately in the «people» or in the medieval notion of the «divine right of kingship» law derives from human minds. In ancient India we had the notion of a level of functioning beyond kingship, that of the «ćakravarti» (the Maintainer of the Wheel) who was the embodiment of the supremacy of the functioning. At this level all external norms are absent and the ćakravarti functions in accordance with natural law and evolves the norms from within.

The Laws of Motion

Our present understanding of the physical universe owes a great deal to the way of approaching dynamics as set out by Galileo and Newton. The latter enumerated a set of «laws of motion» which introduce the notions of inertia and impressed forces, the quantitative relation between change of momentum (quantity of motion) and the impressed force and enumerated

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the law of conservation of the total momentum of a system whose parts are in interaction. We now known that these three laws of motion are all of different categories. The First Law («A body continues in its state of rest or of uniform motion unless acted upon by external force»), comprising in itself a bold declaration about the nature of inertia (that uniform motion needed no cause!), defines the free particle. The Second Law («The rate of change of momentum is equal to the impressed force») is both a definition of the concept of the force and a quantitative measure of the same. The Third law («Action and reaction are equal and opposite») is a relationship between forces exerted by parts of a system on each other and says that the total momentum of an isolated system is conserved notwithstanding any mutual interaction; we recognize it as the forerunner of a series of conservation laws which derive from invariance – in this case, that all locations in space are equivalent for an isolated system.

These laws set a pattern for dynamics. The inertia of a «particle» (the prototype isolated system) is an intrinsic quality of the particle. When there is a collection of mutually interacting particles the simpler systems can be characterized by a potential energy function from which the mutual forces can be derived. In this Langragian generalization the concept of motion is liberalized: instead of looking at a point (or collection of points) moving in time we may have «generalized coordinates» evolving in time. Such an evolution produces the trajectory of the system. Physics is thus poised for describing general dynamical evolution of systems richer than moving particles.

The next generalization is Hamiltonian dynamics which treats position and momentum (or more generally, configuration and flow) on par to describe the «phase» of the system. Dynamics then describes the evolution of the phase, the phase space trajectory. The dynamical content is still the same, the force or potential now being generalized into a Hamiltonian. The realization that uniform motion is an aspect of inertia is however realized in treating position and momentum on the same footing. Within the Hamiltonian scheme an entirely independent view of dynamics becomes most transparent. There are invariance requirements that we wish to impose on the dynamics of an isolated system. The irrelevance of the orientation of the coordinate axes implies the constancy of the angular momentum; and the irrelevance of the absolute origin of time implies the constancy of the energy. Thus the invariance properties are related to conservation laws: irrelevance of some quantities implies the immutability of some (conjugate) quantities!

One can go one step beyond this to classify and even crystallize dynmical laws which admit of certain invariance principles. Apart from the invariances with respect to coordinate origin (space translation), coordinate orientation (rotation) and choice of time zero (time translation) we need one more invariance principle: that of the irrelevance of uniform translatory motion (boost). The First Law refers only to uniform motion; and observers moving uniformly with respect to each other find the same dynamical laws. Given an «inertial observer» any one related by space or time translation, space rotation or a boost is also an inertial observer. These transformations constitute a group of invariance transformations on dynamics: it is the relativity group. With usual ideas of how a boost is to function, we get the Galilei group. With a revised notion of time as seen by a moving observer (incorporating the constancy of the speed of light) we get the Poincaré group of special relatively. In either case we get an invariance group whice should be obeyed by the dynamics.

Particle as a Realization of Symmetry

We might go further and ask for the simplest realizations of the invariance group. To our pleasant surprise we see the (Newtonian) particle emerge in this analysis as the simplest possible dynamical system: the irreducible realization. The mass appears an essential label of the realization. So the roles of the particle and of the invariance group are now interchanged: the particle becomes the crystallization of the invariance properties and is implicitly contained in it rather than the particle with its dinamical potentialities exhibiting the invariance properties. Newton's First Law in a higher and richer setting provides the particle as its realization.

We remind ourselves that a great advantage of this method of deriving a particle from the symmetry principles is that the method is equally applicable to quantum theory as for classical theory. Because of the noncommutative nature of the dynamical attributes of any quantum system we can no longer observe the trajectory of a system: the position and momentum cannot be simultaneously observed. Yet we remember that the First Law applies as well as the general invariance principles. The quantum particle can be recognized as the irreducible representation (quantum realization) of the invariance group. The (quantum) particle is the simplest realization of the invariance principles of dynamics. Dynamical systems can all be built up from such irreducible components. The «word becomes flesh»: śabda entails dravya. (In this context one is reminded of the stanza[1]:

> «ṛṣaya pitaro deva sarva bhūtāni dhātava
> jangamājangamam ćedam jagannārayanodbhavam»).

One may also note that in this view of the genesis of dynamical systems are no causes and effects, no agents, no agencies. There are not even intrinsic differences between various states of motion: all differences are differences entailed by frame choices. What is fixed in one frame is moving in another. Uniform motion is uncaused. Time is the vehicle for unfolding: the anterior and posterior configurations are related but there is a symmetric relation between them, not a directed cause-effect relation. Collections of particles constitute the physical universe.

From Force to Geometry

Can such ideas be relevant for interacting systems? Can we derive the forces and the changes of motion from a few principles of symmetry? Is there the possibility that there is a grand unified picture of dynamics? On other words can we perceive dynamical law as natural law?

Sir Isaac Newton saw in the falling apple the operation of the force of gravity; and recognized gravity as an aspect of the law of universal gravitation which manifested itself in such diverse phenomena as the motion of tides, the orbit of the moon around the earth and of the planets around the sun: Many systems, many motions, but one law.

But one law is still a law: there is now a cause for motion changes, namely gravitation. Who or what imposed this force, this cause for changes of momentum? Have we broken out of the «causeless evolution» characterizing collections of free particles? Not necessarily, if we can ascend to a more natural method of describing the natural evolution of a system. Since the gravitational force according to Newton is proportional to the mass of

[1] From the Hymn of a Thousand Names of Viṣṇu. Freely translated it says: The scientists and progenitors, all the statistic and dynamics world in all its diversity emerges from the cognizer of motion.

the particle on which it acts, it follows that the acceleration caused by the gravitational force is independent of the mass: consequently every particle which is subject to gravitation has an acceleration which depends only on the location: we have a modification of space itself. Albert Einstein used this as the fundamental starting point: under the influence of gravitation space itself undergoes a change in which uniform rectilinear motion is impossible. Space becomes curved. Gravitation is the same as curvature of space. By demanding further that the manner in which the curvature comes about must (1) lead to the Newtonian gravitational equation for weak gravitational fields; (2) it must incorporate the principle of relativity; and (3) the description must be unchanged by the specific choice of coordinates in a curved space, we are led to a theory of gravitation in terms of the curvature of space-time, the curvature itself being related to the energy and momentum distribution in space.

Even in planetary motion and stellar astronomy the new Einsteinian predictions had small but computable deviations from the old Newtonian predictions, the former being in better agreement with measurements. The more general applications including the evolution of space-time curvature itself led to an understanding of the «expansion of the universe» and more generally serves to make cosmology a realm of physical science.

In a uniform space we would expect to define measures of lengths which are «the same» at each point: we would expect that if the unit of lenght is defined at one location we can «translate» that unit of lenght. In a curved space this is given by specifying a «connection», a rule to specify how «sameness» is to be realized. Such a rule is necessary since any method of assigning rectilinear coordinates in a curved space is valid only over a limited region (a «patch»). The curved space is mathematically specified by these coordinate patches which collectively cover the entire space-time and the connections. While the coordinate labels and the numerical quantities that specify the connections depend on the scales and orientations of the rectilinear coordinate systems in each patch (the «gauge») there are quantities that describe the space which are invariant under such changes. These «gauge-invariant» quantities should be used to describe the physics of gravitation: they are the curvature tensors. Einstein found that Newtonian gravitation could be subsumed and generalized by postulating a relationship between curvature of space and the distribution of energy and momentum in space. Gravitation is thus seen as a natural law–it is made to inhere in the fabric of space-time.

An immediate bonus of such an approach is the ability to do physical cosmology: to ask questions about this very universe, at least in its large-scale properties. The nonstatic solutions of the Einstein equations for cosmology produce for us a model of an expanding universe, quite in accordance with the discovery of red-shifts of galaxies and a mutual recession of all galaxies with a speed proportional to their mutual separation. The evolution of the existing universe itself is subject to a natural law.

Electrodynamics, the Second Unification

The second unification came with the discovery of electricity. The charged particles behaved much like gravitating objects with the typical inverse square law but now it could be attraction or repulsion: the Coulomb law. It was followed by the discovery of magnetic forces: the Lorentz force law and the Ampere law and finally by the Faraday law. It was left to Maxwell to make the second unified theory of electromagnetism and codify them in the form in which we call them after his name. In this theory too the electromagnetic vector potential $A_\mu(x)$ is coupled to the local flow of electric charge. Just as stress is divergence free corresponding to the local conservation of energy and momentum so is the electric current divergence free corresponding to local conservation of electric charge. The quantum theory of electromagnetism is called quantum electrodynamics. Electrodynamic interactions cut across the entire species of particles, hadrons and leptons, stable and unstable particles and are thus universal. The conservation of electric charge leads to the Ward identities.

Chiral Weak Interactions, the Third Unification

But gravitation and electromagnetism are not the only forces. They appear to be the most important at laboratory distances since the forces are inverse square law forces and hence «long range forces». But there are the weak interactions first discovered at the turn of the century in beta radioactivity. After several decades of study and the outlines of a theory of beta radioactivity by Fermi, it was recognized that the decays of pions and muons also belonged to the same category and that all of them shared the violation of parity as discovered by Lee, Yang and Wu. It was left to

Sudarshan and Marshak to discover the third unified theory, this time of weak interactions. Sudarshan and Marshak observed that all weak interactions were consistent with the coupling of left handed fermions amongst themselves in the only they could couple: the V-A chiral coupling. The significance of the vectorial nature of the interaction was not lost on its discoverers who recognized that it was like the electromagnetic interaction in that left handed fermions were coupled to left handed fermions only; but unlike in weak interactions right handed fermions coupled to right handed fermions also participated in electromagnetic interactions. The two interactions could be both characterized by interaction currents: charged left handed currents for weak interactions, neutral left and right handed currents for electromagnetism. The electromagnetic interaction was a coupling of the vector (potential) field with the current while the weak interaction looked like a current-current direct coupling. The two could be made to look more similar if a charged intermediate vector boson could be used to mediate the weak interaction and the apparent direct coupling be the consequence of a large mass (several time the proton mass, the mass of a light nucleus).

Gauge Theories and Local Symmetries

The gauge principle which states that «interaction» is a consequence of the requirement of local symmetry transformations is of much wider applicability than gravitation. In a «field theory» the dynamical configurations are specified by the assignment of a collection of fields at each space-time point. Field equations describing the evolution connect the space-time derivatives and the field, and are usually differential equations. Symmetries of the fields imply a multiplet of fields at each point which can be replaced by linear combinations without changing the equations of motion. Just as the equations of motion of Newtonian particles in empty space are unchanged by change of coordinate systems, the internal symmetries also mean independence of the internal coordinate system. A local symmetry means that you can change the internal coordinate system from one space-time point to another. Since differential equations involve derivatives which are rates of changes of fields with space and time, these local symmetries can hold only if there is an associated set of gauge fields coupled to it in a specific manner. Thus a gauge principle can introduce natural interactions. The unification of electric and magnetic interactions by Maxwell into the theory of electromagnetism may be viewed as the natural gauge symmetry-induced interaction where the symmetry is associated with the electric charge. The electromagnetic potential are coupled to the density and flow of electric charge in this theory, somewhat like the gravitational potentials were coupled to the density and flow of energy-momentum.

Unified Theories

In both gravitation and electromagnetism we have long-range forces; the familiar inverse square law forces are an expression of this feature. It does appear that these are the only long-range forces in nature. The interactions responsible for holding the nucleus together (the «strong interactions») and the interactions responsible for radioactive transmutations (the «weak interactions») seem to be definitely short-range interactions. While the strong interactions appeared to be rather complicated the weak interactions are really quite simple. Twenty-five years ago Sudarshan and Marshak showed that they are very similar to electromagnetism: the coupling involved a flow, this time a flow of only left-handed objects. The weak interaction is therefore called «chiral». About fifteen years ago Weinberg, Salam and Glashow discovered that the Sudarshan-Marshak interactions too could be subsumed in a theory of gauge interactions of which Maxwell electromagnetism is a component part, the «unified electro-weak theory».

The strong interactions could be seen as a gauge theory, «chromodynamics», associated with «color». In this theory the nuclear particles are composed of yet more elementary objects, «quarks», which have and can change color. It is the coupling to the flow of color that is generated by local color invariance which in turn lead to all the strong interactions as well as the composite structure of the nuclear particles.

The possibility of accounting for short-range forces using gauge theories suggests the possibility of unifying electro-weak and strong interactions into what is called a grand unified theory. I cannot but quote a Sanskrit stanza:

bahūnām janmanāmante
ǰnāvavān mām prapadyate
vāsu deva sarvamiti
sa mahạtma sudurlabhaḥ [2].

Broken Symmetries

The mechanism of generating short-range interactions using gauge theory makes use of spontaneous symmetry breaking. This is fascinating in that one is working at the edge of causality!

We are familiar with many cases in which the state of lowest energy (the «ground state») is fully symmetric under the symmetries of the problem. If we drop a ballbearing into a circular bowl, the ballbearing will seek the lowest energy when it is at rest in the centre of the bowl. This is a unique configuration, and it looks the same from all horizontal directions. It is rotationally invariant. On the other hand if the bowl had a circular blister in the centre the lowest energy would not be reached at the centre: the centre would not be a location of stable equilibrium. In its place we have a multitude of equilibrium locations, none of which exhibits the circular symmetry of the bowl. A simpler situation is obtained by a ballbearing at rest on a horizontal table. All positions are equally good but none of them exhibits the simple translation symmetry of the table top.

In such a case no choice has the symmetry of the problem and *any* choice breaks the symmetry. The symmetry is broken without cause, broken spontaneously. Motion about the equilibrium position, at least along certain directions becomes very easy: as easy as rolling the ballbearing on the table. As soon as this situation develops in a field theory the interaction becomes a short-range interaction, the inverse of the range being proportional to the departure of the equilibrium state from symmetry. A large symmetry breaking for the ground state implies a short range for the interactions.

It follows that if we started with a large set of symmetries some of which are spontaneously broken, then some of the interactions would be

[2] From the Song of the Lord. It says, in essence: At the end of many cycles of happenings the rare wise one recognizes that the same universal entity permeates all the many splendoured universe.

short range and some long range even though they are all gauge interactions. The possibility of spontaneous symmetry breaking thus generates richer patterns. We recognize that the structure and functioning of the world depends crucially on the short-range nature of the nuclear and weak interactions contrasted with the longe-range electromagnetic effects: the latter (together with the underlying quantum framework) provide a flexible electronic envelope of the atom which accounts for most of the chemical, optical and structural properties of matter. This could not be so if all the interactions were long range or all of them short range. Thus in a curious way the symmetry of an ice crystal or a diamond crystal depends on the spontaneous breaking of a deeper symmetry. The beauty of the world, the very act of creation, involves the breaking of symmetry. Creation is the reduction of symmetry.

Beyond Grand Unification

Yang and Ward have developed the geometric aspects of the theory of general gauge fields in terms of connections over a Riemannian manifold. Sudarshan and Ward start from such a starting point reminiscent of Einstein's formulation of general relativistic theory of gravitation. Start with a four-dimensional manifold with points labelled by x and a primary field $\psi(x)$ defined over it. The differential equations should follow from an action

$$A = \int$$

where the Lagrangian density $L(x)$ is a function of $\psi(x)$ and its covariant derivatives. The covariant derivatives involve connections both Riemannian and «internal» (flavor + color + spin + ...). The lowest invariant is

$$g^{\mu\nu}(x) D_\mu \bar{\psi}(x) . D_\nu \psi(x)$$

but this reduces in the «flat limit» to a quadratic system not suitable for describing fermions. So we seek other alternatives. It turns out that the fourth order invariants

$$g^{\mu\lambda} g^{\nu\rho} D_\mu D_\lambda D_\nu D_\rho$$
$$g^{\mu\lambda} g^{\nu\rho} D_\mu D_\nu D_\rho D_\lambda$$
$$g^{\mu\lambda} g^{\nu\rho} D_\mu D_\nu D_\lambda D_\rho$$

are all invariants and the difference of any two is no more than linear in the derivatives. So we choose $L(x)$ as a linear combination involving the derivative operators

$$g^{\mu\lambda} g^{\nu\rho} [D_\mu, [D_\lambda, D_\nu]]\, D_\rho = \Delta_1$$

and

$$g^{\mu\lambda} g^{\nu\rho} [D_\mu, D_\nu]\, [D_\lambda, D_\rho] = \Delta_2$$

in the form

$$L(x) = \bar{\psi}(x)(a_1\Delta_1 + a_2\Delta_2)\psi(x).$$

It is interesting to note that if all the gauge couplings and hence all connections vanish the action itself would vanish: no Kinematics without Dynamics!

We can now see qualitatively how the various interactions are contained in this action. If we take a limiting case with the vacuum expectation value of the scalar $\bar{\psi}\psi$ vanishing but nonvanishing for the tensor $D_\mu\bar{\psi} \cdot D_\nu\psi$ according to

$$< D_\mu\bar{\psi} \cdot D_\mu\psi > = \Lambda g_{\mu\nu}$$

due to relativistic invariance, we have the effective Einstein Lagrangian

$$\int d^4x \sqrt{-\det g(x)}\, (a_1 + a_2)\Lambda g^{\nu\rho} R^\lambda_{\lambda\nu\rho}$$

where $R^\nu_{\lambda\nu\rho}$ is the Riemann tensor.

If we take the limit of the spinor connection

$$C_\mu = < i\vartheta(x) g^\alpha_\mu \gamma_\alpha > = < \vartheta > g^\alpha_\mu \gamma_\alpha$$

we get the Dirac equation for $\psi(x)$ corresponding to freely moving particles by a suitable rescaling of the fields. But if the spinor connection is treated as spatially variable we get the Higgs limit with a scalar field $\vartheta(x)$. The mass of the fermion is given by the vacuum expectation value of the Higgs field. It is interesting to note that the Higgs field is neither more nor less elementary than the other fields in the problem but is not arbitrarily introduced. It is a part of the general connection.

When the internal symmetries are introduced the color, electroweak and proton decay fields all appear automatically. If the spinor connection and the internal symmetries are to be integrated into a single unified connection the ultra-grand unified symmetry should be a spinor realization of an orthogonal group (like the 0(10) Georgi and Glashow) or some group which has that as a subgroup. Family unification using orthogonal groups is also a possibility.

Towards Natural Law: Symmetry Breaking as Creation

It is important to observe that natural philosophy seems to have reached a new depth in probing nature. Nature's laws themselves are the data for providing a deeper theory. In the search for the cause for the diversity in the universe we are led to the causeless breaking of symmetry. The potentialities are fully describable but the realization of a broken symmetry world is due to the instability of the fully symmetric configuration.

We may make a general comment about natural law. We expect that at a deep enough level, in place of a number of different *laws*, we expect to find a unified *law*. However, any scheme for unification must contain within it the mechanism for the universality to be broken. In other words, both the symmetry and its breaking must be aspects of the formulation of natural law. When one makes a theory about the universe there are no external agents affecting its behavior.

One consequence of such an approach to the basic interactions in nature is that the expression of physical laws depends upon the physical conditions. For example in the conditions appropriate to the early stages of the universe, the symmetry breaking leading to short-range weak interactions is lifted. Weak interactions are then neither weak nor short range. Behind the realized physical interactions is a level at which the laws remain the same but it manifests in accordance with the physical conditions.

Spontaneous Compactification of Higher Dimensional Spaces

It is unaesthetic to have space-time and the concept of gravitation that inheres in the curvature of space on the one hand: and to have internal symmetries like color and charge and the corresponding «unified» theories. It would be desiderable to have one point of view from which both these complexes of ideas could follow. One such possibility is to deal with a higher dimensional space which includes the four dimensions of spacetime but have new dimensions which somehow become compact and reflect themselves at lower energies as internal symmetries. What we need is a unified description of curved higher dimensional space which spontaneously obtains these compactified dimensions. In this kind of theory we can have intrinsic constraints on the «internal» gauge groups and can even ask question like what determines spacetime to be four dimensional and why are there bosons and fermions in the world with an intrinsic connection between spin and statistics. It is no surprise that such «Kaluza-Klein» theories are now being actively studied.

Finally, human law and justice cannot be any different from natural law in physics. We do have many laws, but at a deeper level synthesis must take place in terms of very few principles. May I quote again:

> sarvāgamanāmācāram prathamam parikalpayet
> ācāra prabhavo dharmo dharmasya prabhuracyutaḥ [3].

Unity in Diversity

We now return to the theme of unity in diversity. The gauge theory idea is on the one hand to have the freedom to have independent local symmetry transformations; and on the other to tying the fields at adjacent points in terms of the action function and hence the differential equation. Even the particle itself is a consequence of symmetry breakdown: creation by destruction. I remind you that this unity in diversity is the secret of wave motion itself: disturbance at one locale influences the next and hence nothing can be isolated. It is amazing that equipped with the gauge principle this same simple mechanism has been instrumental in the various

[3] From the Hymn of a Thousand Names of Viṣṇu. One should recognize, first of all, natural law and the principles underlying them. This leads to right functioning which the lord derives from changelessness.

stages of the unified field theory. These developments evoke the sentiments:

akhanda mandalākārām
vyāptam yena carācaram
tat padam dar śitam yena
tasmai śrī gurave namaḥ [4].

The Asian tradition incorporates the search for unity in diversity. It has been my privilege to outline the history of the search in our times.

[4] From the Adoration of the Preceptor. A free rendering is: Salutations to the Principle of Knowledge who shows things as they are through the unity in the diversity of all that is static and dynamic.