FACTORIZATION THEOREM FOR DECAYING SPINNING PARTICLES

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We demonstrate that the differential cross section for multiparticle production via the production and subsequent decay of a heavy, spinning particle factorizes into a production piece and a decay piece provided we choose our kinematic variables properly and the parent decays into at least three particles.

The availability of very high energy particle colliders such as the CERN $\bar{p}p$ machine, the SLC, the LEP collider, and possibly, the SSC may lead to the production of, as yet, unseen heavy particles. If produced, these would, in general decay before leaving the collision region. Their detection would be possible only via a detailed analysis of the debris of ordinary particles produced as a result. A general analysis of the structure of the cross section for multiparticle production via the production and subsequent decay of a (real) heavy, unstable particle has thus assumed added significance [‡1].

In a complex reaction involving many particles amongst which are one or more unstable particles the differential production cross sections have intricate dependences on the energies and angles of the final particles. To the extent that the decaying particles have sufficiently narrow width we expect a simplification since the matrix elements must have a product structure at least when the decaying particles have no spin. Undoubtedly there are also nonresonant background amplitudes; and the dominance of the resonant amplitude must be empirically tested. The dominance is more believable (but not proved) if there are no angular correlations between the frame defined by the decay products and the remaining particles; and can be disproved by finding such a correlation. This test would strengthen the numerical agreement for the factorization of the differential cross section. The dependence of the density matrix for the final state on the gamma ray momentum in a (d, pγ) reaction was worked out by Biedenharn, Boyer and Charpie [1]. The implication of a one-pion exchange is the t-channel wave discussed by Treiman and Yang [2]. For a t-channel exchange the widths are less of a problem but the nonresonant backgrounds are larger.

In the limit of the entire amplitude being expressible as the product of two subamplitudes and a propagator, we must then look for two simplifications. One, the dependence of the entire amplitude on the bilinear combination of momenta which represent the squared momentum transfer by the one particle would have a resonance dependence typical of this propagator. But in addition to this the subamplitudes should transform according to some suitable representation of the little group of the exchanged particle. It is this latter dependence on which we wish to focus our attention.

‡1 By "real" we mean the particle is produced on its mass-shell. This notion makes sense for weakly decaying heavy particles whose width is small in comparison to their mass.

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In the case where the unstable particle has spin zero, it is well known [3] that the differential cross section factorizes provided the total decay width of the parent is small in comparison to its mass, i.e. for the process ab $\to$ cd $\to$ c 1 2 ... n,

$$d\sigma(\mathrm{ab} \to \mathrm{c}\,1 \ldots n) = \sigma(\mathrm{ab} \to \mathrm{cd})\,(\mathrm{d}\Gamma/\Gamma)\,(\mathrm{d} \to 1 \ldots n)\,. \tag{1}$$

This factorization is valid even at the differential level, i.e. when the momenta of the final particles are specified to lie in an infinitesimal volume in momentum space, the only proviso being d to be spinless. Eq. (1) has a simple interpretation in terms of probabilities, viz. the probability of producing the system {c, 1 ... n} via d is equal to the probability of producing the system {c, d} times the probability of d to decay into {1, 2 ... n}.

For the case where d is not a spinless particle, we do not expect a simple relation of the type of eq. (1) to be valid. In this case, the amplitude for the production of {c, 1 ... n} via d is a coherent sum of *amplitudes,* the sum extending over all possible polarization states of d. In squaring this total amplitude to obtain the cross section, we obtain terms involving interference between various polarization states of d. It is these terms which preclude the possibility of a simple general formula of the type of eq. (1) [‡2]. The structure of the cross section for ab $\to$ c 1 ... n for the case where d has spin forms the subject of this paper.

If the reaction process is of the form

$$\mathrm{ab} \to \mathrm{cd} \to \mathrm{c}_1\mathrm{c}_2 \ldots 1\,2 \ldots n\,,$$

then the amplitude is of the form

$$A(\mathrm{ab} \to \mathrm{c}_1\mathrm{c}_2 \ldots 1\,2\,3 \ldots n) = A_{\mathrm{I}}(\mathrm{ab} \to \mathrm{c}_1\mathrm{c}_2 \ldots)\,D(\mathrm{d})\,A_{\mathrm{II}}(\mathrm{d} \to 1\,2 \ldots n)\,,$$

where D is some scalar function of the transmitted four-momentum of particle d if d is spinless. More generally D(d) is a matrix characteristic of the little group of the exchanged particle. The differential cross sections, in general, will not factorize. There is however the possibility of obtaining a product form by taking a group integral of $|A_{\mathrm{II}}|^2$ over the little group of d: for d being a massive particle in the s-channel the little group is SU(2) and the group integration is simple.

The S-matrix element for the process ab $\to$ c 1 ... n is given by

$$S = \frac{\mathrm{i}(2\pi)^4}{\Delta(\mathrm{a},\mathrm{b},\mathrm{c},1 \ldots n)}\,\delta^4[P_\mathrm{a} + P_\mathrm{b} - P_\mathrm{c} - P_1 - \ldots - P_n]\,T(\mathrm{ab} \to \mathrm{c}\,1 \ldots n)\,, \tag{2}$$

with

$$\Delta(\mathrm{a},\mathrm{b},\mathrm{c},1 \ldots n) \equiv [(2E_\mathrm{a}V)\,(2E_\mathrm{b}V)\,(2E_\mathrm{c}V)\,(2E_1V) \ldots (2E_nV)]^{1/2}\,.$$

T is the transition amplitude between free particle states, each one-particle state being normalized so that there are $2EV$ particles in the box of volume V, i.e. $\langle p|p'\rangle = (2\pi)^3 2E\delta^3(p-p')$. Proceeding as Pilkuhn did for the case of spinless d [3], we write [‡3]

$$S = \frac{\mathrm{i}(2\pi)^4}{\Delta(\mathrm{a},\mathrm{b},\mathrm{c},1 \ldots n)}\,\delta^4\left(P_\mathrm{a} + P_\mathrm{b} - P_\mathrm{c} - \sum_{i=1}^{n} P_i\right)\int \mathrm{d}^4P_\mathrm{d}\,\sum_m\,\delta^4(P_\mathrm{a} + P_\mathrm{b} - P_\mathrm{c} - P_\mathrm{d})$$

$$\times\, T(\mathrm{ab} \to \mathrm{cd}_m)\,\frac{1}{(P_\mathrm{d}^2 - M_\mathrm{d}^2) - \mathrm{i}M_\mathrm{d}\Gamma}\,T(\mathrm{d}_m \to 1 \ldots n)\,. \tag{3}$$

‡2 The cross section $d\sigma(\mathrm{ab} \to \mathrm{c}\,1 \ldots n) \neq \Sigma_m\,\sigma(\mathrm{ab} \to \mathrm{cd}_m)\,(\mathrm{d}\Gamma/\Gamma)\,(\mathrm{d}_m \to 1 \ldots n)$ (m denotes the polarization state of d) as may be expected from naive probability considerations because of quantum-mechanical interference effects.

‡3 The formalism we use is that of Chapter 3, §1 of ref. [3] but our results go beyond those presented therein.

Here M_d and Γ are the mass and total decay width of d, respectively, and m denotes the spin state of d. Recalling that the momentum space volume d^3p contains $Vd^3p/(2\pi)^3$ states, the transition *probability* can be obtained by squaring (3) and multiplying by the appropriate number of states. This is equal to

$$\frac{(2\pi)^8}{(2E_aV)(2E_bV)}\,\delta^4(0)\sum_{m,m'}\int d^4p_d\,\delta^4(P_a+P_b-P_c-P_d)\,\delta^4(P_d-P_1-\ldots-P_n)$$

$$\times\, T(d_m\to 1\ldots n)\,T^*(d_m\to 1\ldots n)\,\frac{1}{(P_d^2-M_d^2)^2+M_d^2\Gamma^2}\,T(ab\to cd_m)\,T^*(ab\to cd_{m'})\,\mathrm{dLips}(c,1,\ldots,n)\,,$$

where

$$\mathrm{dLips}(c,1,\ldots,n)\equiv[d^3P_c/(2\pi)^3 2E_c]\;[d^3P_1/(2\pi)^3 2E_1]\ldots d^3P_n/(2\pi)^3 2E_n\,.$$

Notice that although the δ-functions have rendered the above expression "diagonal" in momentum of d the spin dependence contains "off-diagonal" pieces. This is what we had referred to earlier. We now convert the expression for the transition probability to that for the cross section. We then find

$$\sigma(ab\to c\,1\ldots n)=\sum_{m,m'}\frac{(2\pi)^4}{(2E_a)(2E_b)}\frac{1}{|v_{ab}|}(2\pi)^3\int_0^\infty\frac{d(P_d^2)}{(P_d^2-M_d^2)^2+M_d^2\Gamma^2}\int \mathrm{dLips}(c,d)\,\delta^4(P_a+P_b-P_c-P_d)$$

$$\times\, T(ab\to cd_m)\,T^*(ab\to cd_{m'})\int \mathrm{dLips}(1,\ldots,n)\,\delta^4(P_d-P_1-\ldots-P_n)\,T(d_m\to 1\ldots n)\,T^*(d_{m'}\to 1\ldots n)\,, \quad (4)$$

with v_{ab} being the velocity difference (v_a-v_b). The phase space $\mathrm{dLips}(c,d)=[d^3P_c/(2\pi)^3 2E_c]\,d^3P_d/(2\pi)^3 2P_d^0$ with $P_d^0=(p_d^2+p_d^2)^{1/2}$, i.e. the mass of d is $(P_d^2)^{1/2}$. It is easy to check that for the case where d has spin zero, eq. (4) reduces to eq. (1) in the narrow-width approximation.

We now proceed with the analysis of eq. (4). For convenience, we work in the rest frame of d. (Such a frame exists since the second δ-function in eq. (4) requires $P_d^2>0$.) In such a frame the little group becomes SU(2). We choose particle a to move along the Z-axis and particle b to move in the ZX plane with a positive X-component of momentum. This completely specifies a right-handed coordinate system. We will now show that if we explicitly perform the group integral over the little group of d, which involves certain final state integrations, the right-hand side (RHS) of eq. (4) neatly factorizes into two parts, one describing the production of d (via the reaction ab → cd) and the other the decay, d → 1 ... n.

To this end, we note that the decay $d_m\to 1\ldots n$ could have been described in a different coordinate system $X'Y'Z'$ with particle 1 moving along the Z' axis and particle 2 in the $Z'X'$ plane with its X' component of momentum being positive. This is always possible if $n\geqslant 3$; for the remainder of this paper we will assume this is the case [‡4]. The amplitude $\tilde{T}$ for the decay $d_m\to 1\ldots n$ in terms of the variables of the primed coordinate system is related to T by (J is the spin of d)

$$T(d_m\to 1\ldots n)=\sum_\mu \mathcal{D}^J_{\mu m}(-\beta,-\theta_1,-\phi_1)\,\tilde{T}(d_\mu\to\tilde{1}\ldots\tilde{n})\,. \quad (5)$$

In eq. (5), ϕ_1,θ_1,β are the three Euler angles that describe the transformation from the XYZ to the $X'Y'Z'$ coordinate systems. ϕ_1 and θ_1 are the azimuthal and polar angles of particle 1 in the XYZ system and β, the third Euler angle, is related to ϕ_2 and θ_2 (the direction of 2 in the XYZ system) and α the angle between particles 1 and 2 by

‡4 If $n=2$, the little group integral referred to in the previous paragraph would be an integral over the complete final state. This would lead to the well known result that the total cross section factorizes. Since the purpose of this paper is to study the factorizability of the differential cross section, we do not consider the case $n=2$ any further.

$$\cot(\phi_2 - \phi_1) = (\cos\alpha \sin\theta_1 + \sin\alpha\cos\beta\cos\theta_1)/\sin\alpha\sin\beta\,, \quad \cos\theta_2 = \cos\alpha\cos\theta_1 - \sin\alpha\cos\beta\sin\theta_1\,. \qquad \text{(6a,b)}$$

$\mathcal{D}^J_{\mu m}$ is the usual matrix representation of the rotation group. The momenta $\widetilde{P}_1, \widetilde{P}_2$ ... in the amplitude $\widetilde{T}$ on the RHS of eq. (5) are P_1, P_2 ... expressed in the $X'Y'Z'$, system, i.e. $\widetilde{P}_1$ is along Z', $\widetilde{P}_2$ in the $Z'X'$ plane and the remaining momenta $\widetilde{P}_3, ..., \widetilde{P}_n$ are expressed in terms of angles measured with respect to $\widetilde{P}_1, \widetilde{P}_2$. *All the θ_1, ϕ_1 and β dependence of $T(d_m \to 1 \,...\, n)$ is contained in the $\mathcal{D}$-function.* In the "decay integral",

$$I \equiv \int \mathrm{dLips}(1,2,...,n)\,\delta^4(P_\mathrm{d} - P_1 - ... - P_n)\,T(\mathrm{d}_m \to 1 \,...\, n)\,T^*(\mathrm{d}_{m'} \to 1 \,...\, n)\,,$$

on the RHS of eq. (4) we can first isolate the ϕ_1, θ_1 and β dependence of the integrand using eq. (5) and then eliminate θ_2 and ϕ_2 in favor of α and β. We find writing d^3P_3 ... as $\mathrm{d}^3\widetilde{P}_3$..., [‡5]

$$I = \int \frac{|P_1|\mathrm{d}E_1 |P_2|\mathrm{d}E_2\,\mathrm{d}\phi_1\,\mathrm{d}(\cos\theta_1)\mathrm{d}\beta\,\mathrm{d}(\cos\alpha)}{(2\pi)^9}\,\mathrm{d\,Lips}(\widetilde{3},\widetilde{4},...,\widetilde{n}-1)\,\delta[(\widetilde{P}_\mathrm{d} - \widetilde{P}_1 - ... - \widetilde{P}_{n-1})^2 - M_n^2]$$

$$\times \sum_{\mu,\mu'} \mathcal{D}^J_{\mu m}(-\beta,-\theta_1,-\phi_1)\mathcal{D}^{J*}_{\mu' m'}(-\beta,-\theta_1,-\phi_1)\widetilde{T}(\mathrm{d}_\mu \to \widetilde{1} \,...\, \widetilde{n})\widetilde{T}^*(\mathrm{d}_{\mu'} \to \widetilde{1} \,...\, \widetilde{n})\,,$$

where we have eliminated $\widetilde{P}_n$ using the δ-function [‡6]. M_n is the mass of the nth particle. Since the amplitude $\widetilde{T}$ is independent of θ_1, ϕ_1 and β, these variables can now be explicitly integrated over. The result is [‡7]

$$I = \frac{1}{2}\,\frac{1}{(2\pi)^7}\,\frac{1}{2J+1}\,\delta_{mm'} \sum_\mu \int |P_1|\,\mathrm{d}E_1\,|P_2|\,\mathrm{d}E_2\,\mathrm{d}\cos\alpha\,\mathrm{dLips}(\widetilde{3},...,\widetilde{n}-1)$$

$$\times\,[(\widetilde{P}_\mathrm{d} - \widetilde{P}_1 - ... - \widetilde{P}_{n-1})^2 - M_n^2]\,|\widetilde{T}(\mathrm{d}_\mu \to \widetilde{1} \,...\, \widetilde{n})|^2.$$

Replacing this in eq. (4), we have

$$\sigma(\mathrm{ab} \to \mathrm{c}\,1 \,...\, n) = \int_0^\infty \frac{\mathrm{d}(P_\mathrm{d}^2)}{(P_\mathrm{d}^2 - M_\mathrm{d}^2)^2 + M_\mathrm{d}^2\Gamma^2}\,\sigma(\mathrm{ab}\to\mathrm{cd})\,\frac{1}{2(2\pi)^4}\,\frac{1}{(2J+1)} \sum_\mu \int |P_1|\,\mathrm{d}E_1\,|P_2|\,\mathrm{d}E_2\,\mathrm{d}\cos\alpha$$

$$\times\,\mathrm{dLips}(\widetilde{3},...,\widetilde{n}-1)\,\delta[(\widetilde{P}_\mathrm{d} - \widetilde{P}_1 - ... - \widetilde{P}_{n-1})^2 - M_n^2]\,|\widetilde{T}(\mathrm{d}_\mu \to \widetilde{1} \,...\, \widetilde{n})|^2\,, \qquad \text{(7a)}$$

with

$$\sigma(\mathrm{ab}\to\mathrm{cd}) \equiv \frac{(2\pi)^4}{(2E_\mathrm{a})(2E_\mathrm{b})}\,\frac{1}{|v_\mathrm{ab}|} \int \mathrm{dLips}(\mathrm{c},\mathrm{d})\,\delta^4(P_\mathrm{a} + P_\mathrm{b} - P_\mathrm{c} - P_\mathrm{d}) \sum_m |T(\mathrm{ab}\to\mathrm{cd}_m)|^2\,. \qquad \text{(7b)}$$

Eq. (7) is the statement of the factorization for the case when the particle d has spin. Physically eq. (7) states that all spin information of d is lost if we integrate out the orientation of the $X'Y'Z'$ system with respect to the XYZ system. This is not surprising, since in doing so, we have uniformly averaged over all possible choices of the spin quantization axis.

‡5 The jacobian of the transformation is unity.

‡6 $\widetilde{P}_n$ in the amplitude $\widetilde{T}$ is identically $\widetilde{P}_d - \widetilde{P}_1 - ... - \widetilde{P}_{n-1}$. Notice also that the one-dimensional δ-function is independent of θ_1, ϕ_1 and β.

‡7 This result appears to be implied by the discussion in Chapter 3 of ref. [3]. No attempt is made to prove it, however, and the emphasis of the discussion appears to be quite different from that in this paper.

We note that in eq. (7a) the factorization holds for "each value" of the mass $(P_d^2)^{1/2}$, of d. This mass is then integrated over with a Breit–Wigner weighting function. This feature would have been present in the spinless case also, had we not made the norrow-width approximation. If we now put d on its mass-shell, i.e. replace $1/[(P_d^2 - M_d^2)^2 + M_d^2\Gamma^2]$ by $(\pi/M\Gamma)\delta(P_d^2 - M_d^2)$, eq. (7a) reduces to

$$\sigma(\mathrm{ab} \to \mathrm{c}\,1 \ldots n) = \sigma(\mathrm{ab} \to \mathrm{cd}) \frac{1}{(2\pi)^3} \frac{1}{4M_d\Gamma} \frac{1}{2J+1} \sum_\mu \int |\boldsymbol{P}_1| \mathrm{d}E_1 |\boldsymbol{P}_2| \mathrm{d}E_2 \, \mathrm{d}\cos\alpha \, \mathrm{Lips}(\tilde{3}, \ldots, \tilde{n}-1)$$

$$\times \delta[(\tilde{P}_d - \tilde{P_1} - \ldots - \tilde{P}_{n-1})^2 - M_n^2] \, |\tilde{T}(\mathrm{d}_\mu \to \tilde{1} \ldots \tilde{n})|^2 , \tag{7c}$$

with (ab → cd) as in eq. (7b) expect that now the phase space of particle d is

$$\mathrm{d}^3P_d/(2\pi)^3 2E_d , \quad E_d = (\boldsymbol{P}_d^2 + M_d^2)^{1/2} .$$

The factorization is then complete.

It is instructive to consider an explicit example, say, for the decay of a spin-half particle. Then, the square of the matrix element is of the form

$$s \cdot p_1 f_1 + s \cdot p_2 f_2 + \ldots + s \cdot p_i f_i + \ldots ,$$

where s is the spin vector for d and the f are functions of all possible dot products of the momenta. If s is along the Z axis in the XYZ system then in the $X'Y'Z'$ system s^μ is given by

$$s^\mu = s(0, -\sin\theta_1 \cos\beta, \sin\theta_1 \sin\beta, \cos\theta_1) .$$

Then $s \cdot p_1 \sim \cos\theta_1$, $s \cdot p_2$ depends linearly on $\cos\theta_1$ or $\cos\beta$, and $s \cdot p_i$ $(i > 3)$ depends linearly on $\cos\theta_1$, $\cos\beta$, or $\sin\beta$. These are all zero when we integrate over β and $\cos\theta_1$. The dot products of the momenta can depend on α but not on β or $\cos\theta_1$.

Therefore, in the rest frame of d, the differential cross section with respect to the particle energies (they are rotationally invariant), or with respect to the angles measured with respect to any two of the decay fragments of d, factorize. For example $\mathrm{d}\sigma/\mathrm{d}E_i \mathrm{d}\cos\alpha_i$ factorizes where E_i is the energy of the ith particle and α_i the angle between particle 1 and particle i. On the other hand, the cross section $\mathrm{d}\sigma/\mathrm{d}(\cos\theta_i)$ where θ_i is the angle of particle i with, for instance, particle a will not, in general, factorize.

At this point it is worth noting that the differential cross section $\mathrm{d}^3\sigma/\mathrm{d}^3P_c$ will factorize in the XYZ system. Therefore, if c were also decaying via $\mathrm{c} \to 1'2' \ldots m'$ with $m' \geqslant 3$, the cross sections $\mathrm{d}\sigma/\mathrm{d}E_i'$, $\mathrm{d}\sigma/\mathrm{d}\cos\alpha_i'$ or $\mathrm{d}\sigma/\mathrm{d}E_i' \cos\alpha_i'$ factorize where E_i' is the energy of i' and α_i' the angle between $1'$ and i'.

We now turn our attention to the possibility of factorization in the laboratory frame, i.e. when d is not at rest. The laboratory frame may be the center of momentum frame of a and b for a colliding beam experiment or the rest frame of b for a fixed target experiment. The θ_1, ϕ_1, and β integrals can be done in the $X'Y'Z'$ frame as before. Then the d rest-frame energies E_1 and E_2 can be written as $(P_d \cdot P_1)/(P_d^2)^{1/2}$ and $(P_d \cdot P_2)/(P_d^2)^{1/2}$, respectively. Further, $|\boldsymbol{P}_1||\boldsymbol{P}_2|\mathrm{d}\cos\alpha$ can be replaced by $\mathrm{d}(-P_1 \cdot P_2)$. The three Lorentz invariants $(P_d \cdot P_1)$, $(P_d \cdot P_2)$ and $(P_1 \cdot P_2)$ are constrained by the δ-function in eq. (7a). Moreover, the amplitude $\tilde{T}$ in eq. (7a) can also be written in terms of these invariants using

$$\cos\alpha = (P_1 \cdot P_2 - E_1 E_2)/(-|\boldsymbol{P}_1||\boldsymbol{P}_2|) ,$$

with E_1, E_2 being replaced by the aforementioned invariants. The energy and angles of the other particles, i, $(i > 3)$ in the rest frame of d can be written in terms of invariants in the same way: $E_i \sim P_d \cdot P_i$, $\cos\theta_i \sim P_1 \cdot P_i$. The azimuthal angle ϕ_i is given by $\cos\alpha_i = \cos\theta_i \cos\alpha + \sin\theta_i \sin\alpha \cos\phi_i$ where α_i is the angle between $\boldsymbol{P}_i$ and $\boldsymbol{P}_2$ in the rest frame. Thus ϕ_i can be expressed in terms of $P_2 \cdot P_i$, $P_1 \cdot P_i$, and, for α, $P_1 \cdot P_2$. All other dependence is already in invariant forms such as $P_i \cdot P_1$, $P_i \cdot P_j$. Then, in an arbitrary frame, (generalized to the n-body decay of d) eq. (7) states that *the differential cross section with respect to all the independent Lorentz invariants formed out of*

$P_d, P_1, ..., P_n$ *factorize*. In general, neither the energy or angular distributions factorize; nevertheless in terms of the kinematics discussed above, the factorization property persists. Analysis of experimental data in terms of these Lorentz invariant variables may prove to be of use since in terms of these variables it is possible to separate cleanly the production from the decay; this, in turn, may make it possible to study the dynamics of the decay process independently of the production [‡8].

At this point, we remark that if c decayed into $1' ... m'$, differential cross sections with respect to Lorentz invariant combinations of $P_c, P_{1'}, ..., P_{m'}$ would also factorize, independently of the factorization of the decay of d. In the event that the decay products of c and d had common particles, this factorization property would be difficult to put to use for it may not be possible to decide whether to associate these common particles with c or d.

To illustrate the utility of the factorization we consider two realistic examples. In the first example, one of the particles is unstable whereas in the other, both the produced particles decay. First, we consider the production and subsequent decay of the supersymmetric partner of the Z-boson (zino) in association with the photino via the reaction

$$e^+e^- \to \tilde{Z} + \tilde{\gamma}_1 \to \mu^+\mu^-\tilde{\gamma}_2 + \tilde{\gamma}_1 .$$

We assume that the photino, like the neutrino escapes undetected. Let k and l denote the four-momenta of the beam electron and p and q denote those of the produced muons. For convenience we have labelled the primary photino by the subscript 1 and the photino produced in the zino by 2. In the rest frame of the zino, the dependence on the polar and azimuthal angles of $\tilde{\gamma}_2$ and the angle β' (see eq. (6)) of the decay amplitude is all contained in the $\mathcal{D}$ -function (see eq. (5)). We recall that in performing this integration we uniformly averaged over all possible orientations of the quantization axis for the spin of the zino. This led to the factorizable form, eq. (7), for the cross section. In the rest frame, the energy distributions of the muons factorized. In order to render the result useful, we rewrote this distribution in terms of independent Lorentz-invariant quantities and concluded that the differential cross section $d\sigma/d(P \cdot p)$ (or equivalently $d\sigma/d(p \cdot q)$ since $\tilde{\gamma}_2$ is unobserved) factorized with P being the zino momentum. The cross sections $d\sigma/d(k \cdot p)$, $d\sigma/d(k \cdot q)$, $d\sigma/d(l \cdot p)$ or $d\sigma/d(l \cdot q)$ do not factorize since the quantities $(k \cdot p)$, $(k \cdot q)$, $(l \cdot p)$, $(l \cdot q)$ would depend on the Euler angles (θ_1, ϕ_1 and β) referred to earlier. Thus, as stated, the differential cross section with respect to the Lorentz invariant quantities formed out of P, p, q alone factorizes. (In this case there is only one such quantity when everything else is integrated out.)

As a more complicated example, we consider the reaction

$$e^+e^- \to \underbrace{\tilde{Z}}_{\hookrightarrow \mu^+\mu^-\tilde{\gamma}} + \underbrace{\tilde{Z}}_{\hookrightarrow e^+e^-\tilde{\gamma}}$$

The muons have momenta p_1 and p_2 and the electrons q_1 and q_2. Since the factorization for the decay of the first zino is quite independent of what happens to the other zino, following exactly the same arguments, we can see that the cross sections $d\sigma/d(p_1 \cdot p_2)$, $d\sigma/d(q_1 \cdot q_2)$ or even $d^2\sigma/d(p_1 \cdot p_2)d(q_1 \cdot q_2)$ factorize whereas $d\sigma/d(p_1 \cdot q_1)$, $d\sigma/d(p_1 \cdot q_2)$ etc. do not.

To summarize, we have shown that the cross section for multiparticle production via the production and subsequent decay of a heavy unstable spinning particle breaks up into a production factor times a decay factor provided we choose the kinematic variables suitably and the parent decays into at least three particles. This generalizes the well-known result [3] for spinless unstable particles. We have pointed out that analysis of data in terms of these suitably chosen kinematic variables will isolate the dynamics of production of the new particles from the dynamics of their decay.

‡8 Reconstructing P_d would require knowledge of all the momenta $P_1, ..., P_n$ or of P_c. The latter may be possible if c is a long-lived charged particle. In the absence of either possibility, our result is still useful even if the momenta of at least two of the decay products can be measured.

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