Proceedings of the National Symposium on

RECENT DEVELOPMENTS IN THEORETICAL PHYSICS

Kottayam, India, September 15-19, 1986

Editors

E.C.G. Sudarshan

K. Srinivasa Rao

R. Sridhar

World Scientific

Singapore • New Jersey • Hong Kong

Published by

World Scientific Publishing Co. Pte. Ltd.
P.O. Box 128, Farrer Road, Singapore 9128

U. S. A. office: World Scientific Publishing Co., Inc.
687 Hartwell Street, Teaneck NJ 07666, USA

Library of Congress Cataloging-in-Publication data is available.

RECENT DEVELOPMENTS IN THEORETICAL PHYSICS

ISBN 9971-50-475-X

Printed in Singapore by General Printing and Publishing Services Pte. Ltd.

QUANTUM PHASES IN CLASSICAL OPTICS

E.C.G.Sudarshan
Institute of Mathematical Sciences, Madras 600113, India
Center for Particle Theory, University of Texas, Austin Texas 78712, USA

Front Form Wave Optics

Optics is an old branch of physics. Both the corpuscular and wave theories have been with us for a long time. In most problems of optics one is interested in paraxial or quasiparaxial propagation in which the light is confined to a narrow pencil with a more or less well defined path. Despite the study of light having been the pathway to the discovery of quantum theory[1] for most physical phenomena we take light as if it is a classical wave process. That this is not an approximation for high intensities but an exact transcription was proved in a paper[2] that I wrote a quarter century ago.

Despite this equivalence, there are unexpected quantum phase information which would not have been expected from usual classical considerations associated with the system being taken along a closed path so as to return to its initial conditions. This paper is an exposition of this aspect.

We recognize that despite light being a purely relativistic process for paraxial propagation we may employ the nonrelativistic Schrodinger equation to describe its motion. This is based on the "front form" description formulated by Dirac[3]. If P_1, P_2, P_3 are the generators of space translations(momenta), P_0 the generator of time translations(energy), J_{23}, J_{31}, J_{12} the generators of rotations(angular momenta) and J_{01}, J_{02}, J_{03} the generators of pure Lorentz transformations(moments of energy), then for a paraxial beam with axis along the 3rd direction the generators

$$J_3 = J_{12} \; ; \quad G_1 = \frac{1}{2}(J_{01} - J_{31}) \; ; \quad G_2 = \frac{1}{2}(J_{02} + J_{23})$$

leave the system wave number and frequency invariant. The quantity

$$M = \frac{1}{2}(P_0 + P_3)$$

is practically a (large) constant. The variations and propogation as seen from a frame moving with the wavefront is described by the transverse momenta P_1, P_2 and the effective Hamiltonian of the system is

$$\mathcal{H} = P_0 - P_3 \simeq \frac{P_1^2 + P_2^2}{2M} .$$

If we also write

$$G_1 = M\, Q_1 ; \quad G_2 = M\, Q_2$$

then Q_1, Q_2, P_1, P_2 from a canonical set; and the system behaves like a Galilean system in two dimensions with the henochromatic frequency M playing the role of a mass.[4]

Berry's Phase

If a quantum system is taken with a Hamiltonian which depends on some external variables which vary with respect to time "slowly". Then the eigenstates of the Hamiltonian also will vary "slowly"; in time; and if this rate of variation is slow compared to the natural periods we say that we have an adiabatic variation. In this case the time evolution of the system when the Hamiltonian starts with some external variables and, after describing a suitable excursion in the parameters, returns to the original value and form, the eigenfunction(assumed nondegenerate) should also return to the original eigenfunction except for a phase[5]

$$|\Psi(T)\rangle = e^{i\epsilon_n} e^{i\gamma_n} |\Psi(0)\rangle .$$

Here ϵ_n is a dynamical phase which becomes less and less important the more slowly the variation is carried out so that we may as well ignore it. The non-dynamical phase γ_n is called Berry's phase.

The quantity γ_n for a loop can be seen to be purely kinematic by studying the expression for computing:

$$\gamma_n = i \int_0^T \langle n(t) | \frac{d}{dt} | n(t) \rangle \, dt$$

$$= i \int_0^T \langle n(t) | \frac{\partial}{\partial R_j} | n(t) \rangle \frac{dR_j}{dt} \, dt$$

$$= i \int_{R(o)}^{R(T)} \langle n(R(t)) | \frac{\partial}{\partial R_j} | n(R(t)) \rangle \, dR_j(t)$$

$$= i \oint_C \langle n(R) | \frac{\partial}{\partial R_j} | n(R) \rangle \, dR_j = \gamma_n(C)$$

where $|n(t)\rangle \equiv |n(R(t))\rangle$ is the time dependent nth eigenvector and C is the contour in the parameter space over which R_j varies. We may formally write

$$\gamma_n(C) = - \oint_C A_j(R) \cdot d\, R_j$$

if R_j is a 3-vector variable and $A_j(R)$ is defined by the relation

$$\langle n(R) | \frac{\partial}{\partial R_j} | n(R) \rangle = i A_j(R)$$

which exhibits the close connection Berry's phase has with ganges. In fact of $H(\underline{R})$ is a real Hamiltonian with a 3-vector parameter space identified with Euclidean space, then $\underline{A(R)}$ has the precise form of the potential for a Dirac monopole located at such a point $\underline{R_o}$ where the Hamiltonian $H(\underline{R})$ becomes doubly degenerate.

Spinning Particles and Polarized Light

We could look at the geometric aspects of this as follows: for each set of external parameters R_j we have a different Hamiltonian and hence different eigenvectors with generally different eigenvalues. But for a discrete eigenstate and a "tame" perturbation this variation is continuous with respect to R_j. We may therefore follow the nth eigenvector through the excursion in the parameter space. For each point in the parameter space along the circuit there is a complex vector: in mathematical terms we have a "line bundle",[6] when we return to the original point there can be only a phase difference. This phase difference is called Berry's phase, which thus measures the twist in the line bundle. The twist may depend on the eigenvector with which we start the circuit.

A particularly simple example is given by a non-degenerate discrete Hamiltonian for a spin in a magnetic field:

$$H = - \mu B J_3 .$$

Consider a circuit in the space of angles in which the system is "slowly" rotated around a unit vector n. The time dependent Hamiltonian is given by

$$H(\theta(t)) = \exp(i\,\theta(t)\,\hat{\underline{n}}.\underline{J})\,H(0) \times \exp(-i\,\theta(t)\,\hat{\underline{n}}.\underline{J}) .$$

After some elementary calculation we can show that the state with $J_3 = m$ has the phase factor

$$\exp\{i\,\gamma_m(C)\} = \exp\{-im\,\Omega(C)\}$$

where $\Omega(C)$ is the solid angle subtended by C at the origin. In the special case when C is a circle which subtends a cone with apex at the origin and apex semiangle θ the Berry phase

$$\gamma_m(C) = -2\pi\,m(1 - \cos\theta).$$

Kinematrical Phases in Optics

This last example is particularly interesting since it shows that if the circuit is properly chosen the dynamical phase is supplemented by the kinematrical Berry's phase; and under suitable conditions this can induce an additional sign change between two interfering amplitudes.

Unfortunately while optically active media behave towards polarized light in much the same way a magnetic field acts on a particle with magnetic moment, it is not entirely clear that an optical fiber could be thought of as realizing, by twisting it into a helix, the slow rotation of the polarized light. But if we do take this to be so then a simple experiment can demonstrate this additional Kinematrical spin. There have been assertions that such phases have been observed[7].

We have come across nondynamical phases in optics before. When light incident from a rarer medium gets reflected at the interface with an optically denser medium there is an additional phase change of π . This is seen in the centre of a set of Newton's rings being dark rather than white. We have also seen that in total internal reflection as the angle of incidence increases beyond the critical angle an increasing Kinematic phase appears. Berry's phase is a more general context in which such phases appear.

In all these cases if we consider only this single beam the phase is not observed nor is it observable. It is always necessary to do an interference with another reference beam. So for purely classical events the phase makes no difference. This is made very clear in the Aharanov-Bohm effect in which we compare the phases of two partial beams which skirt a magnetic flux in opposite ways.

Recent researches have shown that Berry's phase is very general and encompasses many interesting special cases. We shall however not be able to survey them in this presentation.

References

1. M.Planck, Verh.d.deutsch Phys. Ges. 2, 202,237(1900); Ann. d. Physik(4) 4, 553(1901). A .Einstein, Ann. d. Physik(4) 17, 132(1905); 20,199(1906).
2. E.C.G.Sudarshan, Phys. Rev. Letters 10, 277(1963).
3. P.A.M.Dirac, Rev. Mod. Phys. 21, 392(1949).
4. E.C.G.Sudarshan, R.Simon, and N.Mukunda, Phys. Rev. A28, 2921(1983); N.Mukunda, R.Simon, and E.C.G.Sudarshan, Phys. Rev. A28, 2933(1983).
5. M.V.Berry, Proc. Roy. Soc. Lond. A392, 45(1984).
6. B.Simon, Phys. Rev. Lett. 51, 2167(1983).
7. A.Tomita and R.Y.Chiao, Phys. Rev. Lett. 57, 937(1986).