Measurement-damped oscillations

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It is suggested that a recent experiment verifying the quantum Zeno effect be extended to longer times with weakened laser pulses that make the measurements uncertain. This would show more detailed effects of changes of the atomic state made by the laser pulses. It would test the way the pulses change the density matrix when the measurements are uncertain. It is predicted that when the pulses are sufficiently weak, the transition probability will oscillate in time, as it would if there were no pulses, but the pulses will damp the oscillations the same way they inhibit transitions to produce the Zeno effect. The oscillations could also reveal the change of phase between the atomic states produced by the interaction with the laser beam. Transition probabilities have been calculated as functions of time. Representative results are plotted.

von Neumann's "projection postulate" describes the way a measurement changes the density matrix representing a quantum state [1]. It has been the subject of much discussion [2] but few experimental tests. A recent experiment [3] has verified the effect of the projection postulate in producing a "quantum Zeno effect" [4]. Trapped ions are exposed to a rf field that causes transitions between a ground state 1 and a metastable state 2. All the ions are in state 1 at the start, when the rf field is turned on, when t is 0. At time t the probability that an ion is in state 2 is $\sin^2\omega t$. The rf field is turned off at the time T when ωT is $\pi/2$. Then $\sin^2\omega T$ is 1. All the ions are left in state 2. This time T will be our basic unit.

In other runs, the ions are not left undisturbed to make the transition from state 1 to state 2. At n times, $T/n, 2T/n, \ldots, T$, between 0 and T, measurements are made that distinguish between states 1 and 2. A laser beam is turned on briefly to stimulate transitions from state 1 to another state 3. An ion has to be in state 1 to make this transition, so observation of fluorescence from this transition is a measurement that the ion is in state 1, and observation of no fluorescence is a measurement that it is in state 2. The laser pulse is strong enough to produce 72 fluorescence photons per ion, enough to make the measurement certain.

Between laser pulses the change of ion state is the same as before, but at each pulse the density matrix changes according to the von Neumann projection postulate. The effect is to inhibit the increase of probability for state 2. As Zeno predicted [4], the ion is less likely to make a transition when it is watched.

We would like to suggest an extension of the experiment to times beyond T, with weakened laser pulses that make the measurements uncertain. This could verify more detailed effects of the laser pulses. It would test the way they change the density matrix when the measurements are uncertain. The theory of that was described recently by Peres and Ron [5]. We predict that when the laser pulses are weak enough, the probability for state 2 as a function of time will oscillate, as it would if there were no laser pulses, but the pulses will damp the oscillations the same way they inhibit transitions. We show that the oscillations could also reveal the change of phase between states 1 and 2 produced by the interaction with the laser beam.

Peres and Ron [5] show that each laser pulse changes the density matrix for the atom

$$\rho = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} \tag{1}$$

to

$$\begin{bmatrix} \rho_{11} & \lambda e^{i\eta}\rho_{12} \\ \lambda e^{-i\eta}\rho_{21} & \rho_{22} \end{bmatrix} \tag{2}$$

where $\lambda e^{i\eta}$, which they call S, is the amplitude for no change in the state of the photons. For a strong laser pulse, λ is 0. Then the change in the density matrix is what we get from the von Neumann projection postulate. When λ is not 0 but η is 0, we get

$$\begin{bmatrix} \rho_{11} & \lambda\rho_{12} \\ \lambda\rho_{21} & \rho_{22} \end{bmatrix} = \lambda \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} + (1-\lambda) \begin{bmatrix} \rho_{11} & 0 \\ 0 & \rho_{22} \end{bmatrix} . \tag{3}$$

This represents a mixture of the state we would have if there were no laser pulse and the state we would get if the pulse were strong enough to make the measurement certain. We can interpret λ as the probability for no measurement and $1-\lambda$ as the probability of certain measurement. There is no such interpretation when η is not 0.

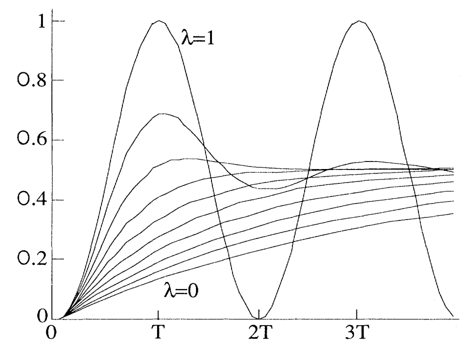

FIG. 1. Probability for state 2 as a function of time from 0 to $4T$ when η is 0 and n, the number of laser pulses in time T, is 16. The different curves are for equally spaced values of λ from 0 to 1.

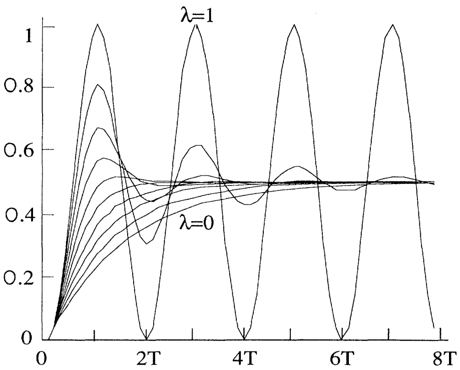

FIG. 3. Probability for state 2 as a function of time from 0 to $8T$ when η is 0 and n is 8. The different curves are for equally spaced values of λ from 0 to 1, the same as for Fig. 1.

There is a change of phase between states 1 and 2. If an atom is in either state 1 or state 2 before the laser pulse, then it is in the same state after, but if it were in a superposition, it would be changed.

We have calculated probabilities for state 2 as functions of time. Our method is similar to the one used for the Zeno effect [3]. For the time between one laser pulse and the next, it is the same; the vector **R** in the density matrix is rotated by π/n, where n is the number of laser pulses in time T. At each laser pulse, the density matrix changes form from (1) to (2). When η is 0, that means R_1 and R_2 are multiplied by λ. The projection is not as severe as for a certain measurement, when λ is 0. The result can be that rotation of **R** is allowed to continue. Each cycle of rotation of **R** is a cycle of oscillation for the probability for state 2. But each laser pulse decreases the length of **R**. That means the oscillations are damped.

When η is not 0, the vector **R** is also rotated by η around the 3 axis at each laser pulse. That changes the oscillations. The effect could be observed if η can be controlled. The calculations of Peres and Ron [5] suggest that η could be controlled by tuning or detuning the laser frequency relative to the atomic transition frequency.

We have plotted results for some representative cases. Figure 1 shows the probability for state 2 as a function of time from 0 to $4T$ in cases where η is 0 and n is 16. The different curves are for equally spaced values of λ from 0 to 1. Figure 2 shows the same for equally spaced values of λ from 0.9 to 1. Note the points where the probability is almost independent of λ. In Fig. 3, for cases where η is 0 and n is 8, the time goes to $8T$. The different curves are for equally spaced values of λ from 0 to 1, the same as in Fig. 1. The effect of a nonzero phase η is typically to decrease the transition probability and shorten the period of oscillation. This is shown in Fig. 4 for cases where n is 16 and λ is 0.9, the same as in Figs. 1 and 2. The different curves are for values of η from 0 to 0.8. They would be the same for the corresponding negative values of η.

Formulas we obtained for the probability when η is 0 show that oscillation occurs when

$$2\sqrt{\lambda}\tan(\pi/n) > 1-\lambda \ . \tag{4}$$

This means λ must be larger than 0.673 if n is 16 and

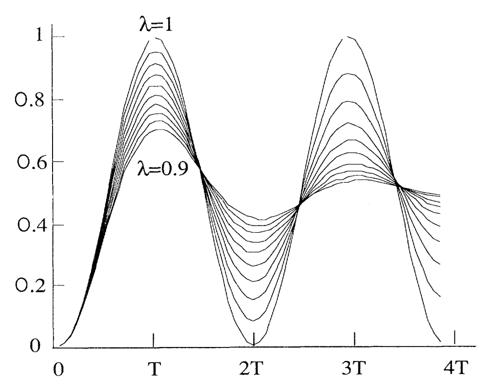

FIG. 2. Same as Fig. 1 for equally spaced values of λ from 0.9 to 1.

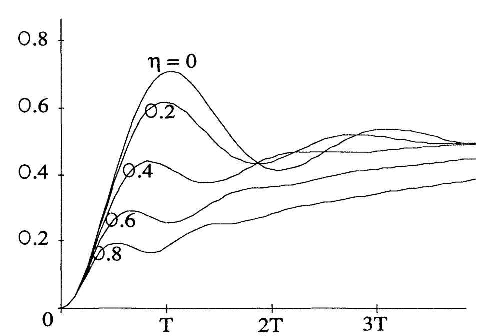

FIG. 4. Probability for state 2 as a function of time from 0 to $4T$ when n is 16 and λ is 0.9, the same as in Figs. 1 and 2. The different curves are for equally spaced values of η from 0 to 0.8.

larger than 0.446 if n is 8. For smaller λ, the probability just increases from 0 monotonically and converges to $\frac{1}{2}$. The formulas also show that the period of oscillation increases from $2T$ as λ decreases from 1. All this behavior can be seen in the figures.

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