DEDUCTION OF PLANCK'S FORMULA FROM MULTIPHOTON STATES

Luis J. Boya[1], **Ian M. Duck**[2] and **E. C. G. Sudarshan** [3]
Center for Particle Physics
Department of Physics
The University of Texas at Austin
Austin, Texas 78712-1081, USA

Key Words: photons, radiation law, old quantum theory.

Abstract

We obtain the black body radiation formula of Planck by considering independent contributions of multiphoton entities

[1]Permanent Address: Departmento de Física Teórica, Facultad de Ciencias. Universidad de Zaragoza. E-50009 Zaragoza, Spain. E-mail: luisjo@posta.unizar.es
[2]Physics Department, Rice University, Houston, TX. 77251-1892
[3]E-mail: sudarshan@utaphy.ph.utexas.edu

are created equal (identical), and that in an occamian, economical description, we should abstain from considering situations in which the photons are individually distinct (indistinguishability); that is, there is a "state" with two photons *different*, for the counting, from the naive juxtaposition of two distinguishable photons; same thing for the three photon states, etc. So the multiphoton molecules should be thought of as entities only to the effect of counting, and not as claiming for the existence of some force which really binds the photons together.

In this way one gets the correct radiation formula at once. We are unaware of any complete proof of this statement, so we now show the deduction, leaving some historical remarks for later.

For the n-photon " molecules" of energy $nh\nu, (n = 0, 1, 2, ...)$ the probability is of course given by the Boltzmann factor

$$p(n) = c\ \exp(-nh\nu/kT) \tag{3}$$

$$\sum_{n=0}^{\infty} p(n) = 1 \text{ gives } c = 1 - x, \quad x \equiv \exp(-h\nu/kT). \tag{4}$$

The multiphoton states still have frequency ν, so the common phase space factor is 2(polarization)$\times 4\pi$(solid angle)$\times \nu^2$ (radial 3D factor); hence the density of energy per inverse wavelength is

$$\begin{aligned} I(\nu, T) &= 8\pi(\nu^2/c^3)\Sigma_0^\infty nh\nu(1-x)\exp(-nh\nu/kT) \\ &= \frac{8\pi\nu^3}{c^3}\frac{hx}{1-x} \\ &= \frac{8\pi\nu^3 h}{c^3}\frac{1}{\exp(h\nu/kT)-1} \end{aligned} \tag{5}$$

i.e., the correct complete radiation formula.

The two traditional limits of the radiation formula do have a particle interpretation, as already remarked by Wolfke [2]. E.g. at low temperature only the lightest "molecule" is excited, that is, the one-photon mode, of energy $h\nu$, that leads to the Wien's limit law (1), which was Einstein's starting point [1].

On the other hand, for high densities or temperatures all the n-photon states contribute together and sort of coalesce, giving a scale-invariant power law distribution $I \propto \nu^2$, i.e. the classical electromagnetic (Rayleigh-Jeans) limit formula.

References

[1] A. Einstein.*Ann.d.Phys*(4),**17**, 132-148 (1905)

[2] M. Wolfke. *Phys.Zs.* **15**, 308-311 (1914). *Ibid.* **22**, 375-379 (1921).

[3] S.N.Bose. *Z.f.Phys***26**, 178-181 (1924)

[4] A. Einstein *Sitz.ber.Preuss.Akad.Wiss.(Berlin)*, pp.261-267 (session of 10-VII-1924). *Ibid* pp.3-14 (session of 8-I-1925)

[5] P.Debye,*Ann.d.Phys.*(4), **33**, 1427-1434 (1910)

[6] Some historical works contain more detailed references; we mention three: (1)J. Mehra and H. Rechenberg, *The historical Development of Quantum Theory* (Springer, New York 1982-2000,several volumes), esp. Vol I, p.59 ff and p.554 ff. (2) M. Jammer, *The Conceptual development of Quantum Mechanics*, (Mc Graw-Hill, New York 1966), esp. pp.51-53 , and (3)E. Whittaker, *A History of the Theories of Aether and Electricity* (Nelson, London; rep. Humanities Press, New York 1953), Vol. II, pp.87-89 and 102.

[7] P.Ehrenfest. *Ann.d.Phys*(4)**36**, 91-118 (1911)

[8] L. de Broglie. *J.phys. et rad.*(6) **3**, 422-428 (1922)

[9] L. de Broglie. *These*, Paris (25-XI-1924).(Masson et Cie., Paris 1925).

[10] J.W. Gibbs, *Elementary Principles in Statistical Mechanics* (Yale University Press, New Havem (1902)); Gibbs proposal was close to the modern indiscernibility.