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ELECTROMAGNETIC FORM FACTORS OF THE NUCLEON AND PION

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Majorana,[1] over 35 years ago, set out to obtain a relativistic wave equation which had, in contrast to the Dirac Equation, no negative energy solutions. The infinite component wave equation he obtained, described particles whose masses were inversely proportional to their spin. Further, his equation had a continuum of space-like and light-like solutions.

Bhabha,[2] in 1945, attempted to undertake a methodical search for the underlying group structure of relativistic equations. He restricted himself to finite component wave equations (by using the nonunitary representations)[3] and was, consequently plagued with the appearance of negative probability and energy densities.

Abers, Grodsky and Norton[4] have shown a way out of this impasse, by explicitly displaying an infinite component field equation which does not seem to violate any of the accepted general rules - and yet gives a reasonable mass spectrum. The crucial point is that the spin consists of two parts,

$$J_{\mu\lambda}{}^{s} = \sigma_{\mu\lambda} + \Sigma_{\mu\lambda} .$$

Here the $\sigma_{\mu\lambda}$ generates the Dirac Spin $\frac{1}{2}$ representation (henceforth called D) while $\Sigma_{\mu\lambda}$ generates a UIR of 0(3,1). Thus particle states carry two indices, to label these two representations.[5]

We shall follow this assignment of spin for particles and this connects our work to that involving a covariant infinite component field equation - however tenuous this connection may be. In actuality, we need consider this "peculiar" assignment as a postulate of the theory, and need not concern ourselves with the infinite component field equations.

Baryons (and Mesons) at rest are assigned to the representations of the direct product group $G \sim O(3,1) \otimes D$ and the spin of the particle is $\vec{J} = \vec{\Sigma} + \vec{\sigma}$. Since all particles carry a spin $\sigma = \frac{1}{2}$ from D, we must consider the UIR of O(3, 1) with integral (half-integral) Σ spins for the assignment of Baryons (Mesons).[6]

We shall denote the 'spinor amplitude' for a nucleon at rest by $|\alpha' \vec{p} = 0>$ where α' stands for the set of quantum members ($\sigma = \frac{1}{2}\ \Sigma = 0,\ J = \frac{1}{2}\ J_3 = \pm\frac{1}{2}$). The complete wave function, in coordinate space, consists of a factor $e^{ip \cdot x}$ in addition.

The boost operator J^s_{oj} enables us to define the spinors of arbitrary momenta

$$| \alpha \vec{p} > \ = \ e^{iJ^s_{oj} \xi_j} \ | \alpha \vec{p} \ = \ 0 >$$

which is a matrix of finite transformation in an 0(3, 1) group. For a boost in the Z direction the right hand side becomes $\sum_\beta V_{\alpha\beta}(\xi)\ |\beta \vec{p} = 0>$ with $\tan h\ \xi = \frac{|p|}{p_o}$.

Barut and Kleinert[7] are able to show that this $V_{\alpha\beta}(\xi)$ is now a matrix of finite rotation of an O(2, 1) group.

We digress a moment to point out that the O(3, 1) $\otimes$ D is now the group used to classify spinors (of the particles) before interactions are included. The boost operators, being part of the group G, enable us to define states with momentum dependence built into it. We proceed, following our canonical route,[8] to describe interactions in terms of non-invariant generators, and postulate a larger group structure - this time, a dynamical non-invariance group.

Recently, Sudarshan[9] has advanced a theory unifying strong, weak and EM interactions by considering a class of universal primary interactions. In this formulation, such diverse things as the absence of the neutral lepton currents, the universality of the electric charge and the Fermi interaction, the principle features of the nuclear forces and the EM properties of the nucleons are related. This remarkable ability of the theory hinges crucially on the simple proposition that the vector meson fields and the axial vector meson fields are the primary fields which possess interactions with hadrons.[10]

We shall, in this spirit, assume that it is the current that is coupled to the neutral vector meson which interacts with the baryons (and mesons) and cause transitions between the various levels.[11] The source of the neutral vector meson (ρ_o), $j_\mu^\rho = \Gamma_\mu \otimes 1$ is assumed to have the proper algebraic structure.

$$[\Gamma_\mu, \Gamma_\lambda] = - i \Sigma_{\mu\lambda} .$$

That is ($\Gamma_\mu, \Sigma_{\mu\lambda}$) generate an O(3, 2) group. The peculiar UIR of O(3, 1) that we choose, assures us that it is also a UIR of O(3, 2).[3,12] The matrix element of Γ_μ can be obtained, group theoretically, for arbitrary initial and final momenta, and are identified to $G_{BB\rho}$ and $G_{MM\rho}$ vertex functions.

For example,

$$< \beta p_z \,|\, \Gamma_\mu \,|\, \alpha o > \;=\; \sum_\lambda < \lambda o \,|\, V_{\lambda\beta}(\xi) \Gamma_\mu \,|\, \alpha o >$$

$$= \sum_\lambda V_{\lambda\beta}(\xi) < \lambda o \,|\, \Gamma_\mu \,|\, \alpha o >$$

We can now use the theory of primary interactions, or at this point the Gell-Mann Zachariasen[13] vector dominance model to obtain $G_{BB\gamma}$ and $G_{MM\gamma}$.

We have identified Γ_μ with the conserved current that is coupled to the neutral vector meson ρ_μ^o and so the space integral of the fourth component is proportioned to I_3. In the Majorana representations that we have chosen, Γ_o has the spectrum ($\Sigma + \frac{1}{2}$), the space integral being taken care of by the orbital part of the wave function. We arrive at the pleasant result that the various spins are now associated with isospin values as well.[14]

Barut and Kleinert[7], identify Γ_μ with the EM current and assign spins ($\sigma = o, \Sigma = J$). This raises the immediate problem that Γ_o is now proportional to charge $Q = \lambda(J + \frac{1}{2})$ and mesons have either half integral charge ($\lambda = 1$) or a completely unrealistic charge spin spectrum ($\lambda = 2 \ldots$). On occasions, the vertex functions calculated from these theories have been called isoscalar form factors,[15] and elaborate comparison with experiment made. This case, by a reason similar to the above, not only gives an unrealistic hypercharge-spin spectrum for the tower, but also implies that the entire tower of (infinite) particles has one value of isospin!

Direct computation of the Electric and Magnetic form factors for the proton give us

$$G_E^V(t) = \frac{1}{\left(1 - \frac{t}{M_p^2}\right)} \quad \frac{1}{\sqrt{1 - \frac{t}{4M_p^2}}} \; ; \; G_M^V(t) = 0 \; .$$

And the $(\pi\pi\gamma)$ vertex function

$$F_\pi(t) = \frac{1}{\left(1 - \frac{t}{m_\rho^2}\right)} \cdot \frac{1}{\left(1 - \frac{t}{4m_\pi^2}\right)^{3/2}}$$

We note that the theory is restricted in the sense that isospin invariance has not been incorporated and we consider only a subset of the real world - one value of $I_3 = (\Sigma + \frac{1}{2})$ for each spin state. However, the description itself is exact in the sense that we get an explicit form factor, with all its t dependence displayed. In fact a quick estimate of the charge radius yields a value $\langle r^2 \rangle_p \simeq 0 \cdot 26\, \mu_\pi^{-2}$ for the proton and $\langle r^2 \rangle_\pi \simeq 2 \cdot 49\, \mu_\pi^{-2}$ for the pion.

Universality of the ρ^0 coupling can now be interpreted in terms of the group structure. The explicit structure of the vertex function for $(MM\rho)$, $(BB\rho)$ can perhaps be used to redo the ρ dominance calculations.

The difficulty with the magnetic form factor can easily be remedied by considering $j_\mu^\rho = 1 \otimes \gamma_\mu$. Here, we find that, for the proton,

$$G_E^v(t) = G_M^v(t) = \frac{1}{\sqrt{1 - \frac{t}{4M_p^2}}} \left(\frac{1}{1 - \frac{t}{M_\rho^2}} \right)$$

and, for the pion,

$$F_\pi(t) = \left(\frac{1}{1 - \frac{t}{M_\rho^2}} \right) \left(\frac{1}{1 - \frac{t}{4M_\pi^2}} \right)^{3/2}$$

In this model the towers consist of particles with the same I_3 value and in principle one could identify $j_\mu = 1 \otimes \gamma_\mu$ with the isoscalar current that is coupled to ω (or ϕ). One should then change v to s and replace M_ρ by M_ω (or M_ϕ). Alternatively one could identify $j_\mu = 1 \otimes \gamma_\mu$ with the electromagnetic current[16] in which case the vector dominance denominator $\left(\frac{1}{1 - \frac{t}{M_\rho^2}} \right)$ will not be present.

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FOOTNOTES

1. E. Majorana, Nuovo Cimento 9; 335 (1932).

2. H. J. Bhabha, Reviews of Modern Physics 17 200 (1945) ibid 21, 451 (1949). See also I. M. Gelfand and A. M. Yaglom, Zh. Eusterim, i Teor. Fiz. 18 703 (1948).

   The more recent attempts by Y. Nambu Prog. Theoret. Phys. 37 368 (1966) have been along this line.

3. Majorana had, earlier, overcome this problem by considering two special representations of the Lorentz [O(3, 1)] group.

4. E. Abers, I. T. Grodsky, and R. E. Norton, Physical Rev. 159 (5) 1222 (1967).
   See also C. Fronsdal, Phys. Rev. 156 (5) 1653, 1665 (1967).

5. A Similar spin assignment is used by Dashen and Gell-Mann, Phys. Rev. Lett. 17, 340 (1966)
   Gell-Mann, Erice Lectures.

6. A typical tower will consist of ($J = \frac{1}{2}\Sigma = o : p$), ($J = \frac{3}{2}\Sigma = 1 \cdot N^*$) and so on. $A(\Sigma \cdot \sigma)$ coupling will lift the degeneracy between the states ($J = \frac{1}{2}\Sigma = 0$) and ($J = \frac{1}{2}\Sigma = 1$).

7. A. O. Barut and H. Kleinert Phys. Rev. 156 (5) 1546 (1967)
   H. Kleinert Ph. D. Thesis, Univ. of Colorado (Unpublished).

   The "states" $|\alpha o\rangle$, $|\alpha p\rangle$ are the analogues of the spinors u(o), u(p) in Dirac theory. The orbital part of the wave function is ignored since we have chosen not to include $J_{\mu\nu}^{o} = \frac{1}{i}\left(p_\mu \frac{\partial}{\partial p_\nu} - p_\nu \frac{\partial}{\partial p_\mu}\right)$ in our boosts. This ingenious generalization of the Dirac theory is due to Barut and Kleinert (op. cit.).

8. J. G. Kuriyan and E. C. G. Sudarshan Phys. Rev. (to be published).

9. E. C. G. Sudarshan, Proceedings, Rochester International Conference, August 1967. Proceedings of the 14th Solvay Conference, October 1967.

10. J. J. Sakurai, IV Coral Gables Conference (1967). W.H. Freeman and Company, San Francisco.

11. In this connection see N. M. Kroll, T. D. Lee and B. Zumino Phys. Rev. 157 (5) 1376 (1967). T. D. Lee and B. Zumino (to be published).

12. These are the two Majorana representations with (jo = 0 · C = $\frac{1}{2}$) (jo = $\frac{1}{2}$ · c = o).

13. M. Gell-Mann and F. Zachariasen, Phys. Rev. (24 953 1961)

14. The spectrum we obtain is thus ($I_3 = \frac{1}{2}$ $J = \frac{1}{2}$ = p) ($I_3 = \frac{3}{2}$ $J = \frac{3}{2}$ N*++) for the Baryon tower and

($I_3 = 1$ $J = 0$ π^+), ($I_3 = 1$ $J = 1$ ρ^+) for the Meson tower.

15. A. O. Barut and H. Kleinert, Phys. Rev. 161 (5) 1666 (1967).

16. E. C. G. Sudarshan - Invited Paper at the Joint A. P. S., C. A. P. and S. M. F. Meeting at Toronto, June 1967. This is also the form of the EM current that M. Gell-Mann arrives at in connection with the representations of current algebra at the infinite momentum limit (private communication).