QUANTUM REALITY AND NON-DUALITY

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ABSTRACT

Quantum mechanics is discussed in the proper language. Much of the uncertainty in quantum physics is due to the choice of inappropriate language. The notion of quantum entanglement first defined by Erwin Schrodinger but revived in modern atomic physics is illustrated and its application to quantum computing outlined, along with quantum teleportation. The EPR involves entangled states. Bell inequalities are traced to the absence of a master probability distribution to quantum mechanics, not to "locality." The generalised schemes of quasiprobability are discussed. The arrow of time and its relation to boundary conditions are treated; along with probability for consistent histories. The status of quantum reality is the subject of the concluding section.

QUANTUM MECHANICS : A SEPARATE REALITY

We physicists are supposed to be experts on the nature of the physical world. But when we proceed to study subatomic physics, the modes which were useful for solids, liquids and gases no longer suffice. This new regimen is called quantum mechanics which is now approximately a century old; and its precise formulation was seven decades ago. Yet too much familiarity with the life style (macroscopic) mechanics made, and still makes, the subatomic (microscopic) physics somewhat enigmatic.

Quantum mechanics associates a vector of unit length with a physical state. The linear operators which take you from one vector to another are the dynamical variables. The most characteristic property of "superposition" is, the existence of states which can be thought of as linear combinations of two or more distinct states. The component states remain as they are but superpose. To see the difference between this quantum property and the corresponding situation of (classical) non-quantum mechanics: If 'A' and 'B' are two points an object moving from 'A' to 'B' must pass intermediate points at intermediate times; the object remains the same throughout. But in the superposition of the state of the object at A and that at B, the change from A to B can be through a sequence of superpositions of the two states. [It would be like blowing up a long cucumber balloon; if you blow it out it tends to form a bubble near the end where air is blown. The experienced person would coax this air to the other end. When this is done one bubble grows and the other shrinks, until finally all the air is at the far end. But at no time is the bubble at intermediate locations. Dirac has used the analogy with the states of polarization of light familiar to most people.]

If we find some dynamical variable which has the value 'a' for the state of location A; and 'b' for the state of location B, what should be its value in a superposed state? The answer is that, in general, we cannot specify a value for the superposed states. In other words, the operation of superposition and the operation of measurement of characteristic dynamical variables do not commute. Thus the principle of superposition implies noncommutability of its dynamical variables. Two familiar examples of such non-commuting variables are multiplication by X and differentiation with respect to X. Another such set are matrices Matrix multiplication is noncommutative.

MODELS IN QUANTUM MECHANICS

Both these models have been used in the earlier constructions of Schrodinger and of Heisenberg. One deals in simple cases with wave equations, so it was called wave mechanics; and the other as matrix mechanics. In the several cases where quantum mechanics was applied both versions gave identical results, so they must be alternate forms of an abstract theory. Schrodinger himself showed their equivalence. But the formulation of Dirac makes use of any familiar models.

To make the unknown in terms of known, the use of models is natural. But it has its limitations in trying to visualize quantum mechanics in terms of the models of non-quantum physics. If we do we will come across paradoxes and puzzles, absent from quantum mechanics itself. These distortions are similar to the distortions of the map of the earth (approximately spherical) earth on the flat map. The two surfaces are inequivalent and no true picture can be constructed. We make maps to suit our needs for most people living near the equator the maps are either one or two ovals are to be identified. So maps in the US contain North America in the center, including Alaska, but the Japanese islands and Korea are at the other end; while in maps in India, the subcontinent is at the center but Alaska and Kamchatka are far apart. We may use a Mercator projection which exaggerates the areas but preserves the correct directions; Greenland becomes as large as Africa. Since we can visualize the globe with maps on it, we recognize the various map projection as limited validity distortions of the globe map. If we ask when did the separation between Alaska and Kamchatka happen; or who is responsible for such a dramatic distortion., the only answer is that it is the incompatibility between the sphere and the plane surfaces. Very similar is the situation in quantum mechanics.

UNCERTAINTY : IN LANGUAGE OR IN PHYSICS ?

While quantum mechanics is a precise theory that has been tested for numerous physical systems, people talk about the "uncertainty principle" and that quantum theory does not give definite outcomes in experiments but only probabilistic statements. A radioactive atomic nucleus can undergo a spontaneous decay, but we cannot say precisely when. We can only give a probability distribution for the remaining life of the radioactive nucleus. Similarly if we try to measure the momentum 'p' and the position 'q', these measurements can only give probability distributions that make the meansquare fluctuations obey the relation

$(\Delta p)^2. (\Delta q)^2 > h^2/4$

where 'h' is a constant introduced by Planck to explain the spectral composition of light from a heated (black) cavity. This is the Heisenberg uncertainty relation. It was almost immediately improved by Robertson and Schrodinger.

These remind us of the twin processes of *Åvaracea*(masking) and *adhyÅsa*(false assignment) that are familiar from the Indian traditional literature. The *adhyÅsa* of classical like properties to quantum system produces linguistic paradoxes and puzzles.

QUANTUM ENTANGLEMENT

We can consider the quantum mechanics of a composite system Q with components R and S. The state vectors of Q are obtained by taking products of the vectors in R and in S. A state in Q may be simply the product of a state in R and one in S. For such states if we study only S, it will continue to have a state, to vector. But we could also have states which are superpositions of products. In this case we the state of the system S cannot be assign a vector but it is a probabilistic combination of states. Such states we call "entangled states". Examples are the spins of two particles that emerge from the decay of a spinless particle.

If we measure the spin of one particle R, then its spin is randomly oriented: it is unpolarized. But this is true for the particle S also. Yet if we measure the spin of particle S the spin of the particle R is precisely opposite. Therefore, the combined system Q has properties that cannot be discovered by measurements on R and S separately. Entanglement is physically relevant.

In modern atomic physics people are able to do experiments on simple atoms and the entanglements between the atom and the light that it emits can be brought about. It then turns out that this entanglement can be transferred to a distant system. This remarkable feat is called "Quantum Teleportation". Both entanglement and teleportation are essential ingtedients for the fabrication of "Quantum Computers".

As another version of †*Åvaracea* and *adhyÅsa* there are people who talk about "information". Information is not a material object and their belief is that in terms of information quantum mechanics is properly stated. But quantum information is not classical and has its own law, derived from vector space quantum mechanics. The property entanglement is entirely a quantum property but correlations " in non-quantum physics shares, with entangled systems a manifestation of the principle: Whole is greater than all its parts (*akhaceda macedalÅkÅram*).

QUANTUM TELEPORTATION

In "quantum teleportation" we send a specific 'signal' from Alice to Bob; which signal is sent is unknown to Alice and to Bob. Yet making use of an entangled state some Victor(the verifier) verifies that the signal sent by Alice reaches Bob. The importance is that no material is transported yet the signal gets through. The implications of this for quantum computers is clear, since we want to process information in sequence. Unlike classical information measured in "bits", quantum computers can deal with large amount of information per transmission. Note that Western Union Telegraph company send telegraphic money orders and Interflora delivers flowers anywhere without actually sending the material. In both these cases of classical teleportation the signal carries only a few bits of information. But even in such a simple application of quantum computing like Shor's Algorithm for factorzing large numbers, the "signal" has an enormous number of bits. While the question of the stability and physical realizability of a multistage quantum computer are yet to be demonstrated, the possibility of such is a clear manifestation of the "separate reality" of quantum mechanics from familiar "ordinary reality" of non-quantum world view.

LOCALITY IN QUANTUM MECHANICS: EPR AND BELL

The nature of quantum entanglement was given by Einstein-Podolsky-Rosen (EPR) in a paper in which they argued that either the composite system Q is indivisible, or that quantum mechanics is incomplete. The Systems R and S are two particles with total momentum zero. EPR suggests that the correlation between measurements on R and S show that they are entangled, and correspondingly however far the particles have travelled they are entangled. This paper generated a tremendous amount of discussion. Most people chose to ignore the first alternative of indecomposability and concentrated on the possible incompleteness of quantum mechanics. Since the total momentum is zero in EPR, the measurement of momentum on R gives us an unambiguous value of the momentum on S without disturbing particle S. Not only that, we could measure the position of particle S. Thus we have simultaneous measurements of both position and momentum. But this is not possible since it would violate Heisenberg's uncertainty relation. Physicists are not sure that the EPR objections are without merit. But it is also not clear how to meet the EPR objections. It is fashionable to ask whether quantum mechanics is "local". That is, if a system of two particles has a definite state can we make measurements on one we can get information about the other only because a mysterious "signal" goes from one particle to the other. But people who talk about such locality have to ignore the evidence of a handheld radio detecting a radiowave more than a million times its size.

In place of particle R and S with a continuism of position and of momentum values it is simpler to deal with two spins (actually spinning particles) in a well defined state, say in the entangled state of spin zero. David Bihm suggested this as a simplification of the EPR paradox. By measuring spin R we know what spin S is along the same axis; but then we seem to be able to measure the spin of S in another direction.

But quantum mechanics of spin asserts that this is impossible. It can happen only if the spin measurements on R and on S affect the combined system Q.

These aspects were made more explicit by John Bell, who suggested using a two spin system with R having a spin component made precise. Then, if the total spin was zero, we could determine the spin of S to be the opposite of the spin of R. But we may also make a measurement of another component of the spin of S. If we compute the correlation between the spin direction of R and that of S as measured we get an "angular correlation". If the probabilistic aspect is due to some "hidden variables", otherwise the system behaved in a non-quantum manner, then Bell was able to derive several inequalities. Quantum mechanics also predicts some inequalities but different from the Bell inequalities. Careful experiments were carried out with the result that quantum mechanics predictions were borne out, but the Bell inequalities are violated.

Many physicists consider the Bell inequalities and their incompatibility with experiments as proof of "local hidden variables". They wax lyrical and assert that Bell's work is the most important work of the twentieth century. But it should be pointed out that Bell's in-equalities are derived from the assumption of a master multivariate probability distribution from which all correlations can be calculated. If the master probabilities are non-negative Bell's inequalities obtain: so they may be a quasi distribution with positive and negative values. In fact by a simple calculation in quantum mechanics we can verify that this is indeed the case.

MECHANICS IN PHASE SPACE: NEW WINE IN OLD BOTTLES

The efforts to treat quantum mechanics as a (explain) 'phase space' distribution in non-classical dynamics have a long history. Eugene Wigner gave a precise method of constructing such a phase space density. His method was to compute the characteristic function from quantum theory and then invert it to obtain an equivalent classical phase space density. This is not a true probability since the density becomes negative for small regions. But the density fully characterises the state. Joseph Moyal showed how to construct the dynamics of such a system in terms of "Moyal Brackets"?

The interesting thing about the Wigner-Moyal phase space picture is that only familiar non-quantum variables are involved; but the quantum features are incorporated into the Moyal formalism of dynamics.

One can also construct other versions of phase space dynamics better suited to certain physical problems for example the coherent state representation of states in quantum optics. But in all these cases, like in the map projections, there are distortions of the non-quantum mechanics. While in the Wigner distribution, the only pathology was non positivity, in the coherent state formalism the distribution can be both negative and highly singular distributions. In the quantum optic case we can use this representation to show equivalence between quantum states and classical distributions and thus recover many relations like the interference or diffraction patterns for a beam of light in quantum optics.

QUANTUM TOMOGRAPHY

Yet another method of making a physical picture of the quantum state in terms of a phase density is to subject it to "quantum tomography". A tomograph evaluates the integral of a two-dimensional density along an infinity of lines with various impact parameters and varying slopes. This is what is done in brain scan. In industry it is used for non-destructive testing.(for example to see if enough steel reinforcement bars are in a bridge without destroying the bridge). It turns out that while the Wigner-Moyal density is not positive definite, all the line integrals along any straight-line in phase space is positive, just as in the usual tomography. The non-negative tomograms characterize the state; one uses in this manner both phase space representations and positive tomograms. Quantum tomography is a rapidly developing field of modern atomic physics.

ON ONE BECOMING TWO: DECOHERENCE

Let us return to the other alternative that EPR suggested: namely, that even if the component particles separate widely, they continue to be one system; and to measurements on *R* or S is a measurement of the whole system. If this is recognized as being true, the EPR puzzle is resolved. We have one system which changes by expansion in time. But a long time later we will think of it as two separate states with no definite phase relation or entanglement. The mechanism by which the quantum corelations die down is called "decoherence". A system of two particles does not abruptly become two separate one-particle states; rather it proceeds by successive stages of decoherence. But how does decoherence obtain? Interaction of a particle with the surroundings can change the phase relations especially as successive interaction with the surroundings but the surrounding does not react. In the language of correlations we can say that the primary corelation between the two particles is now extended to a whole lot of degrees of freedom of the surroundings, which imply many particle correlations. Thus passage from two particle correlations to many-particles is not a dynamical version but an irreversible approximation since the reaction of the sorroundings is neglected. Thus a two-particle system eventually becomes two one-particle system. One becomes two!

ARROW OF TIME AND BOUNDARY CONDITIONS

The irreversibility of decoherence is an aspect of irreversibility that we observe in nature. Thermodynamic processes like ice melting or steam condensing take place on their own; but the reverse transitions of water into ice or into steam requires expenditure of external energy. In any thermodynamic process in addition to conservation of matter, momentum and energy there is a new variable called the "entropy" which tends to increase in the course of any natural process. This provides us with an "entropy arrow of time". The remarkable thing is that as far as we have observed the arrow is universal applying to any thermodynamic system large or small, nearby or far away. The entropy of the world tends to increase.

How could we obtain a thermodynamic system from a purely mechanical system with reversible evolution in time? If an excited atom can spontaneously deexcite with the emission of a photon why not the reverse process occur spontaneously? Usually we arrange things so that the light emitted by the atom goes off to a great distance and does not comeback. So there is no chance for the photon to come back and reexcite the atom. But if we enclose the excited atom in a small cubic box with perfectly reflecting walls, so that the photon cannot escape? In this case we find that there are two time independent states of the system: one mostly ground state plus light of a specified intensity, and the other mostly the excited state with a small but definite intensity light. Any other combination would have a see-saw effect cyclically changing in time.

What about other processes like spontaneous decay of atomic nuclei and of elementary particles? The same considerations apply. In empty space devoid of any appreciable amount of the decay products, the object decays. But if we increase the density of the decay products sufficiently the reverse transition also can take place. So the question of "spontaneous decay" is dependent on the boundary conditions. Since usually the density of decay products is negligible all unstable objects decay. If we have sufficient density of light, atomic transitions can be reversed. This applies, for example to a heated (black) cavity which absorbs all the radiation falling on it. The atoms inside a cavity in equilibrium do not, on the whole undergo spontaneous excitation or spontaneous deexcitation. Both these processes take place but they balance each other. Meghnad Saha obtained such an equilibrium between the atom and its component ion and electrons in a hot stellar atmosphere.

The dependence of the direction of "irreversible processes" is dramatically illustrated by the night sky, say on a new moon day. The stars are almost point like with the rest of the sky dark. Olbers marvelled at this, since in equilibrium all the sky should have the same apparent brightness, since further and further away stars pour out their light. An approximate calculation gives a value close to that of the stars. After all we may consider the stars and the sky as a black cavity. Since the night sky is dark Olbers concluded that equilibrium has not set in; so either the density of distant stars dies down, or many stars were recently created. The parts of the universe seem to be young and out of equilibrium. Therefore in this situation spontaneous transoformations increasing entropy take place. A candle flame that usually illuminates the dark casts a shadow in noonday sun.

Thus history and unidirectional arrow of time are concomitants of the disequilibrium boundary conditions. When there is equilibrium there is no history, no arrow of time.

THE NATURE OF QUANTUM PROBABILITY: CONSISTENT HISTORIES

After these rambles in quantum mechanics perhaps we should look at our own awareness under varying modalities. Normally we are aware of history and the unidirectional flow of time. Events happen to us that do not unhappen. Yet there are times of quietness (*±Ånti*) when time tends to lose its directionality and processes leave no mark in our awareness. So the outside world is not so different from the inside world.

It is generally accepted that quantum mechanics leads to probabilistic predictions and therefore a quantum state is associated with one or more probability distributions. But we have already seen that quantum probability distributions may be indefinite. In the usual framework a random variable is given a statistical state in terms of a probability distribution. In quantum framework, each set of commuting variables has a probability distribution, but since such commuting sets are infinitely variable, that many probability distributions are there, even in those cases like spin, where the allowed values are discrete and finite.

We need to reexamine the question of probabilistic description in quantum mechanics. From the phenomenon of interference in a two slit-illuminated by coherent light we know that light added to light can produce darkness, and the only probability distribution is the Wolf function which contains bright and dark rays. If we are to estimate the probability of light passing through slit A it is the same as light passing through slit B. But this equal probability of _ for each slit does not suffice, since it would not give interference. Does this mean we cannot describe it in terms of physical probabilities? This can indeed be done provided we consider the probability of the symmetric passage through both the slits and of the antisymmetric passage: these are equal to _ each. The interference effect is already contained in the varying strength of the symmetric and antisymmetric components. For the passage through either slit we cannot assign any probability.

The Two-Slit interference in the simplest example of a two "event" quantum "history". With suitable choice of the various events we can generalize to any number of events. A consistent set of histories is one in which probabilities can be assigned the any set of these event: the development is a "tree" branching out as often as necessary. We can show by a simple analysis that at most one recombination per pair of histories is allowed. In other cases we cannot assign probabilities in a consistent fashion. Coherence is in opposition to histories; the more decoherent a system becomes more does it approximate consistent histories. The interaction of a quantum system with its environment results not only in the transfer of energy and momentum but also of coherence. When the coherence has been destroyed, then we have consistent histories. Thus from a purely quantum system, open to interaction with its environment, classical probabilistic (stochastic) description would emerge. This is taken as the mechanism for a classical world view to emerge from a quantum world.

IS QUANTUM MECHANICS A REALISTIC THEORY?

Much has been written and discussed about "realism" in quantum mechanics" initially all these discussions commit to" fallacy of four terms". When someone talks about a quantum particle the mental image is what is called a particle in classical physics. Then the uncertainly relations have to be ascribed to interference with the system by measurements. In turn this creates problems of action at a distance in EPR kind of situation. But if one avoids the temptation to use ambiguous models like "particle" without qualification all these problems disappear. If we want to locate a cloud and measure its motion we must recognize that the cloud is an extended deformable body. Much of the discussion of "realism" in quantum mechanics is frustrating since the language used is totally inappropriate. This is like the question: When a person mistakes a rope in the dark for a snake, and on looking at it in good light finds that there is no snake but only a rope, where did the snake come from and why; and where did the snake go? We recognize the twin mischief of hiding (Åvaraœa) and superimpositon (adhyÅsa). Philosophy and Physics are not that different!

I must call attention again to the problem of map projections. Every projection leads to distortion in its map but on the surface of the earth there is no indeterminancy. We can comprehend a distortion(and even predict it) provided we know the projection. The incompatiblity of the topology of the spherical surface of the earth with the flat surface and confines of a map is , in the final assessment the "cause" of the distortion. Loose language causes paradoxes and misunderstandings.

Note1: OUTLINE OF QUANTUM MECHANICS

Classical physics has too type of motions: those of particles which are discrete and separate; and those of waves which are discrete and separate; and those of waves which are extended and correlated. In quantum mechanics, the "particles" exhibit both these properties under suitable circumstances. Light waves are familiar, under suitable circumstances. Light waves are familiar, but in photo electric effect (light falling on a metal plate liberating electrons) or compton effect (elastic collision of an X-ray quantum with an electron) light exhibits its particle properties; in this context they are called light quanta or "Photons". Similarly electrons behave like particles in campton effect or in a cathode ray tube, but exhibit wave properties when it impinges on a crystal. Now the important distinction between a particle and a wave are the following a particle can have a definite position and a definite momentum, but a wave is extended in space, and its various parts may not with differing velocities. Another distinction is that a particle aded to another particle double their population. But two waves superposed may give four times the effect of a single wave, or no wave at all. This latter properly is a consequence of "superposition", which we can see watching a pond with two different prbbled thrown into it.

Quantum mechanics identipis the "state" of a partile with the amplitede of a wave. A quantily with magnotrde and direction is called a "vector". We are familiar with vectors in there dimensions forces that add by the parallelograry rule, or velocities that add the same way. But there are infimlelly many independent vectory, so the vector space has an intinite number of dimensions, since all waves have an "amptitude and a "phase"(state of the wave motion) we need camplex vectors. The enifinite dimensional complex space ism the state space. Since the anly thing that any action on the sistem can do is to replace are state by another we associate operators with dynamics. If the "map" of a vector by an operator is such that any multiple gets to be the same multiple of the new state; and the sum of two state (superposition!) gets to be the sum of their corresponding new states. We call them "linear operators". Quantum dynamical vasiables like position, momentium or energy corrrepond to linear operators in its vector space.

The classical quentities position and maneintars are comminting quantite; theis product is either order is the same. But as soon as we have superposition we have non-comminting livear operators. If A is a dynamical variable which has adepinite value a_1 for a state and a_2 for another state, then the superposition of the two states has also definite value hence the operations of meascesing A and forming the superposition do not comminte. Superposition impher non-comminting livear operators and conversely.

Notes 2: Map Projections:

In representing largescale maps of the world many kinds of projection are used . The most familiar one has the longitudes equally spaced, but the latitudes are horizontal parallel lines. This tends to enlarge the high latitudes in realtion to the equatorial area; we would also have to identify the left most longitude with the rightmost longitude; where this "Cut" comes is our choice. This projection preserves neither area nor direction. To get the directions right but with even higher areal distortion we can use the Mercator projection the longitudes are parallel vertical lines with the rightmost identified with the left most. But the latitudes are not equidistant but they are parallel horizontal lines. The separation between two latitude is increased in proportion to the latitude is made to appear longer. This Mercator projection was used by sailors, since it gives the directions correctly. Both these projections distort the polar region very seriously. So if we are interested in the north pole and surroundings we can use a stereographic projection with the north pole at the centre, the latitudes being concentric circles around the northpole; and the longitudes being radial lines. If is a good approximation for the high northern latitudes, but terribly distort the southern hemisphere. There is the complementary sterographic projection with the south pole at the centre. To get the distortions more or less the same over the globe, there is the Bertholomen projection which consists of a collection of rounded triangles with the poles at the top vertex and the button vertices; it looks like an orange peel flattened out on a flat surface. The projection that distributes the distortions in angles, distances and areas is the dodecatedral projection due to Buckminster Fuller; it approximates the sphere by a dodecahedron whose surface can be flattened out on paper. When we flatten it we get a number of regular hexagons with the corresponding sides of two hexagons being identified.

Any map provides geographical paradoxes. These may be understood by recognizing it to be the map of a sphere. There is no agency which causes the distortion it is the mismatch between the sphere and the plane. In one language we have paradoxes in the natural language there is none.

Note 3: Entanglement : One not becoming two

If we have a system Q consisting of two subsystems R and S (may be particles) then the product of a state of R and a state of S is a possible state of Q. Since superposition are possible for Q we can have a state which cannot be factorized into a state of R and a state of S. Such states are called "entangled states". This is purely a wave like property, not possessed by classical particle Q spontaneously decays into two photons, R and S, then right and left circular polarizations of R and S must be the same, but if we measure only S or R, they will exhibit no polrization. R, they will exhibit no polarization. David Bohm suggested the use of such entangled states (first identified Edwin Scrodinger) for the thought experiment suggested by EPR (Albert Einstien, Boris

Podolsky and Nathan Rosen). As long as there is no interaction of Q with anything outside the state continues to be entangled, even of R and S separate out as far as we choose. If we allow for interactions of Q with the surrounding we disturb the entanglement and we may find that the state of Q is the product of state of R and of a state of S; all these states are probabilistic combinations. This mechanism of destruction of entanglement is called "decoherence".

When we have an entangled state we have non-local correlations: the locations of R and of S are correlated even when these locations are far apart.

If victor sends such an entangled state with R of the location of A (say Alice) and S at the location B (say Bob). The scientist Alice creates an entangled state of a particle M with R; and this propogates to the location of S. Where Bob deals with etc., The message is the correlation of M and S. The correctness of the transmission can be verified by Victor.

Entangled states can be created with Varying degrees of entanglement using modern atomic physics techniques.

Note 4: Quantum Information:

In engineering practive and communication net works we can breakup any message into a number of bits, a "bit" corresponding to yes-no question. So it is customarry to count information in terms of the number of bits (or megabits or geabits). This suffices for classical information transer and can be modeled in terms of electrical currents with yes-no connections, as first observed by claude shannon. But quantum information is diffecent: it contains yes-no answers together with entanglement. The calculus of propositions in quantum mechanics in different from that of the propositional calculus for classical mechanics. This was first pointed out by Birthoff and John Von Neumann. Unfortunately their characterization of propositions in classical physics and in quantum physics is very inappropriate.

We have the choice of treating classical and quantum systems in terms of their own behaviour; then the usual Aristotalean information theory is sufficient. But if we model the quantum system by a classical system, then we have paradoxical behaviour; a method of coping with these paradoxes is to change the propositional calculus and the method of inference.

Note 5: Entanglement and Teleportation

Suppose we have an entangled system with R and S as the subsystems; Alice observes R and Bob obeserves S. For a system of two electrons with spin_ (intrinsic angular momentum h/2) a total spin O state is an entangled state. Let Alice observe the spin of R; with respect to a fixed direction and B observes the spin of S along an orbitary direction making a non zero angle with Alice's chosen direction. Let Bob make may measurements, one set for a set of angles $\theta,2\theta$. Then John Bell showed that classical hidden variable formalism there is an inequality obeyed by the correlation measured by Alice and Bob; these inequalities. Experiments have shown that the Bell inequalities are voilated but the quantum mechanical ones are obeyed. Bell gave in equalities for more complicated measurements also. According to many physicsts Bell's work proved that quantum theory cannot be a classical theory with local "hidden variables". But it can be shown that for entangled states, there is no nonnegative multivariable probability distribution. If we had these multivariable statistical distributions are not positive defnite and hence not probability distributions.

A simple way of seeing the impossibility of a hidden variable theory is an direct implication of the principle of superposition. By superposing states with spin ±_ along the z-direction we can produce a state with spin _ along the x-direction. But both ingradients, namely the spin along z-axis +_ and -_ have components along the x-spin +_ and -_. How come, if it is a classical spin with hidden variables. The failure is due to the existenceof non commuting operators which is an immediate consequence of the principle of superposition.

Note 6: Tomography

The tomograph is an instrument used for noninvasive testing. In the medical version we have a source of X-rays (or radioactive gamma rays) placed outside the head (which is being scanned) and the recieved intensity on a bank of detectors. By changing the location of the source(which is easier than manipulating a person's head) we can do this for as many location and directions of the line joining the source to a detector. Now more the material traversed the less the transmitted intensity. So the ratio of the intensity (or rather its logarithm) tells us about the "line integrals" (the sum along the lines). This result can be inverted, usually by a computer (implementing a "reconstruction algorithm") to obtain the profile of the material distribution in the "slice" of the head. In quantum tomography we extend this method to quantum distribution. As stated in the text, the uqantum tomogram, are all positive but the quantum distributions are not so.