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CURRENTS, ALGEBRAS AND DYNAMICAL SYSTEMS*

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The Three Faces of Symmetry

Symmetry groups and multiplets in particle physics are almost as old as particle physics itself. Even before the neutral pion was discovered it was made a member of an isotopic spin triplet with a charge independent coupling to the nucleon doublet. With the subsequent discovery of the neutral pion faith in the hypothesis of charge independence grew; and this faith is supported by experimental tests of the hypothesis which are consistent with attributing all deviations from strict charge independence to the charge-dependent electromagnetic interactions.

Charge independence is still the best verified (and most satisfactory) of all particle symmetries (which are approximate). As a symmetry group the isospin group performs three functions: (1) To classify particles into multiplets which furnish irreducible unitary representations of the group. For example, the nucleon, pion, eta and rho are identified with the representations labelled by isospin 1/2, 1, 0 and 1 respectively. (2) To provide selection rules and amplitude relations for "strong" interactions. The hypothesis of charge independence implies conservation of the triplet of dynamical variables of total isospin. In most cases these rules are not directly realized since neither initial nor final states are pure isospin states; but in combination with charge conjugation invariance they lead to the familiar G-parity conservation. More relevant are the linear relations between amplitudes that follow from conservation of isospin. (3) To classify the non-charge-independent interactions according to their tensor character. The electromagnetic and weak interactions are not invariant under isospin but they have definite transformation properties with respect to the isospin group. Thus the electromagnetic interaction transforms like the sum of an isoscalar and a component of an isovector; the strangeness conserving leptonic weak interactions transform as an isovector; and the strangeness violating (leptonic) weak interactions transform as an isospinor. It is also well known that these lead to sum rules[1] for the linear and quadratic electromagnetic properties of hadrons and weak decay amplitudes.

When we go beyond charge independence and consider more approximate symmetries like SU(3), SU(4) or SU(6) we should reconsider the role of such a symmetry. The first function of a symmetry is still satisfied in these approximate symmetries; in fact, the main reason for consideration of the higher groups is the success of the classifications based on them. The second function is not, in general, satisfied and one refers to this by saying that the symmetry is "broken". The conservation laws and amplitude relations are now not expected to be valid as such, since they are also "broken". The third function can still be invoked; in fact not even the strong interaction is invariant under the symmetry group. One may isolate a part which is invariant under the group and consider it as the invariant "very strong" interaction with a "medium strong" part which transforms as a component of a tensor operator. We are thus led to predictions about the deviations from exact symmetry; the most familiar example is the mass formula[2] for SU(3). We can still invoke the third function of a symmetry group, namely, to specify the tensor character of the electromagnetic and weak interaction properties. Once the tensors are so specified we can deduce sum rules for the electromagnetic and weak interaction properties. The tensor character of an operator is completely specified by the commutation relations of the operator with the generators of the group; it depends in no way on the structure of the operator itself. If X_α are the generators of a group and $D_{rs}(X_\alpha; \lambda)$ furnish a matrix reprexentation of these generators, labelled by the set of parameters λ, we say that a particular operator A_r is a component of a tensor operator of type λ if

$$\tilde{X}_\alpha A_r \equiv [X_\alpha, A_r] = \sum_s D_{rs}(X_\alpha; \lambda) A_s$$

Very often, however, we are not given the full set of operators A_r but just one operator, say the Hamiltonian for the medium strong interactions which is postulated to transform like the I = Y = 0 component of an octet. How do we assign the tensor character in this case? To find the answer it is necessary to find the common properties of all the components of a particular irreducible representation.

These properties are fully specified by the eigenvalues of a complete set of Casimir invariants for the group. For the SU(2) group there is only one such invariant which is $I_1^2 + I_2^2 + I_3^2$. For an operator to be a tensor operator A of rank j with respect to SU(2) it is necessary and sufficient to require

$$\sum_{\alpha=1}^{3} (\tilde{I}_\alpha)^2 A \equiv [I_1,[I_1,A]] + [I_2,[I_2,A]] + [I_3,[I_3,A]] = j(j+1)A .$$

The I_3 = m component further satisfies

$$\tilde{I}_3 A \equiv [I_3, A] = m A .$$

Hence, if we have an operator that is invariant under I_3 (like the electromagnetic interaction) which satisfies, in addition, the above relation with $j(j+1) = 2$ we have

$$[I_3, A] = 0 , \quad [I_1,[I_1, A]] + [I_2,[I_2, A]] = 2A$$

and we can identify A with the third component of a vector. We can construct the complete set of components according to

$$A_j^{m\pm 1} = \{ j(j+1) - m(m\pm 1)\}^{-1/2} [(I_1 \pm i I_2), A_j^m]$$

so that we have

$$A_1^{\pm 1} = \frac{1}{\sqrt{2}} [I_1 \pm i I_2 , A] \quad ; \qquad A_1^0 = A$$

For more complicated groups we have a larger set of Casimir operators; and for a particular operator to belong to an irreducible tensor all the "tilded Casimir operators" should have the appropriate eigenvalues. Thus, for example, for SU(3) we have two Casimir operators which are respectively quadratic and cubic in the generators. The corresponding "tilded Casimir operators" are respectively sums of double and of triple commutators with the generators. This is to be contrasted with the tilded Casimir operator

$$(\tilde{I})^2 \equiv \sum_{\alpha=1}^{3} [I_\alpha , [I_\alpha , \quad]]$$

for SU(2) which is the sum of double commutators.

By identifying the electromagnetic current of hadrons with an octet component in SU(3) for example, we may derive several relations between the electromagnetic properties (magnetic moments, electromagnetic mass differences) of the hadrons.[3] By identifying the magnetic moment with an (8,3) component of a 35 dimensional representation of SU(6) we could derive further relations.[4] In deriving these relations we need assume only the classification of the particles and the tensor characterization of the operators. It is not necessary to assume invariance of the Hamiltonian under the group[5]; it is neither necessary nor relevant to know anything more about the operator characteristics of the generators or the (electromagnetic) current.

When the requirement of invariance is relaxed we could let a symmetry group serve other functions. All the familiar symmetry groups are related to multiplets of particles; these are finite - dimensional since the postulated symmetry group is compact. However, the exact groups of microscopic physics (like the Euclidean group and the Poincaré group) are noncompact and their faithful unitary irreducible representations are infinite dimensional. In fact both in nonrelativistic and relativistic physics, a particle could be identified with suitable infinite dimensional irreducible representations, respectively, of the Galilei and the Poincaré groups. We can now extend this notion to more general dynamical systems with an infinite number of states to arrive at the notion of a characteristic noninvariance group.

Noninvariance Groups and Their Uses

Consider an arbitrary dynamical system. If we can define a group such that all the states of the dynamical system constitute a single irreducible representation of the group, we refer to the group as a characteristic noninvariance group of the system.[6] Clearly, such a noninvariance group cannot commute with the Hamiltonian (unless the Hamiltonian is a constant!) so that the generators of the group are not conserved. If the Hamiltonian does possess an invariance group (i.e. has a degeneracy) it is appropriate to choose this invariance group as a subgroup of the noninvariance group of the system.

Since the Hamiltonian is not invariant with respect to the non-invariance group, it is appropriate to specify its tensor character. The mass (or energy) spectrum depends on this specification. Since the entire set of states furnish an irreducible representation of the group, it follows that every dynamical variable can be identified with an element of the (generalized) enveloping algebra of the non-invariance group. It is then a straight-forward matter to identify the tensor character of the Hamiltonian. In the case of systems with an invariance group, the Hamiltonian must be a function of the Casimir operators of the invariance group and this simplifies the identification of the Hamiltonian with an element of the enveloping algebra of the noninvariance group.

Let us consider some simple dynamical systems.(6) Consider first of all the isotropic rigid rotator in dimensions. This has an invariance group which is the orthogonal group $O(n)$ in n dimensions; and the Hamiltonian is proportional to the quadratic Casimir invariant. The noninvariance group which has $O(n)$ as a subgroup is the pseudo-orthogonal group $O(n,1)$. (If we had restricted attention to the even levels only or the odd levels only, the "dynamical system" would be different; correspondingly the noninvariance group also changes to $SL(n,R)$.) For the isotropic harmonic oscillator in n dimensions, the degeneracy groupis $SU(n)$ and the noninvariance group is $SU(n,1)$. The Hamiltonian in this case is an algebraic function of the generators of $SU(n)$. Finally for the negative energy states of the hydrogen atom in n dimensions, the degeneracy group is $O(n+1)$, the Hamiltonian is proprotional to the reciprocal of a quadratic expression in the generators and the noninvariance group is $O(n+1, 1)$.

In all these cases, the noninvariance group serves to define the spectrum of representations of the degeneracy group that occur. Thus, in the case of the rotator in dimensions or the hydrogen atom in $n-1$ dimensions, the only representations of $O(n)$ which occur are the symmetric tensor representations, and each one of these occur once and only once. This information can be transcribed in terms of the identification of the ("pyramidal") representation of $O(n,1)$ that encompasses all the states of the system.

It thus appears that the entire set of properties of a dynamical system can be transcribed into group-theoretic terms to apply to the noninvariance group. The important result in this connection is the equivalence of the standard formulation and the formulation in terms of a noninvariance group; and this may be stated in terms of the "reconstruction theorem."(7) "Given any dynamical system formulated in terms of suitable primitive dynamical variables and associated with a noninvariance group, the primitive dynamical variables may be defined as suitable elements of the generalized enveloping algebra of the generators of the noninvariance group." This theorem is an immediate consequence of the fact that the (generalized) algebra generated by the representative (infinite-dimensional) matrices coincides with the algebra of the dynamical variables. As an illustration, consider the harmonic oscillator in one dimension. The SU(1,1) group is associated with the generators

$$L = a^+ a$$

$$K_+ = \sqrt{a^+ a - 1} \quad a^+$$

$$K_- = \sqrt{a^+ a} \quad a$$

where a, a^+ are the primitive creation and destruction operators. We can now rewrite this to "reconstruct" the primitive dynamical variables according to

$$a = (L)^{-\frac{1}{2}} \; K_-$$

$$a^+ = K_+ \; (L)^{-\frac{1}{2}}$$

The Hamiltonian $H = \omega \, a^+ a$ is now simply

$$H = \omega L.$$

The algebra is more complicated for other systems, but the reconstruction proceeds on the same principles.

Starting with the noninvariance group and reconstructing the primitive dynamical variables can be viewed from a different point of view. In this picture the fundamental entity is the (noncompact) noninvariance group itself; but then the Hamiltonian, and more generally the interaction structure in general should be specified

in terms of the generators of the noninvariance group. For a system, like the system of two hydrogen atoms in collision, the interaction is specified a priori. The transcription of this interaction in terms of the noninvariance groups would lead to rather complicated structures. But for a simpler system like the harmonic coupling between two harmonic oscillators the interaction is equally simple in terms of the noninvariance group (as may be seen from the expression given above for the primitive dynamical variables in terms of the generators). On the other hand, for a system in which the interaction is not a priori known, we may make a dynamical model by postulating a suitable simple expression in terms of the generators for the interaction. Noncompact noninvariance groups thus become dynamical models.[7]

The use of the noninvariance groups is to be contrasted with the Lie algebra of all dynamical variables. The dynamical variables generate an algebra in which both a Lie algebra structure and an associate multiplication are defined. Further, the Lie bracket operation with any element acts as a derivation:

$$[A \cdot B, C] = [A, C] \cdot B + A \cdot [B, C]$$

This associative algebra structure with derivations is the essential structure of the algebra of all dynamical variables. This structure is equally valid in classical mechanics, except that the derivation is a Poisson bracket rather than a commutator bracket. In either case the Lie algebra of all dynamical variables is infinite dimensional even though the system has finite degrees of freedom. The Lie algebra of the noninvariance group is however finite dimensional for a system with finite degrees of freedom.

Local Noninvariance Algebras, Currents and Sources

These considerations should be extended to systems with infinitely many degrees of freedom to apply to systems described by fields. We expect a formal generalization of the noninvariance group to infinite parameter groups. Just as the generalization from a finite number

of canonical variables to an infinite number was effected in terms of a local field labelled by a space point, we may consider the generators of the infinite parameter noninvariance group to be labelled by space points. In this manner we are led to consider the notion of the infinite parameter Lie algebra of "currents."[8] The formal generalization of the noninvariance group would imply a complete set of currents i.e., any operator commuting with the complete set of currents should be a multiple of the identity. Even with infinite number of such current operators, a finite number associated with each space point, we could consider the notion of the enveloping algebra admitting of associative multiplication and derivations. The reconstruction theorem has now only a formal significance; on the other hand, the primitive dynamical variables are themselves normally chosen as field operators. Poincaré invariance of the theory requires the existence of automorphisms of the noninvariance group; again, using the lesson from local field theory we may require the currents to be local operators $j_{(\alpha)}^{\mu}(x)$ which transform according to:

$$U(a,\Lambda)\; j_{(\alpha)}^{\mu}(x)\; U^{-1}(a,\Lambda) = \sum_{\nu} D^{\mu}{}_{\nu}\; j_{(\alpha)}^{\nu}(\Lambda^{-1}x).$$

With the introduction of the algebra of local currents, we could attempt the standard developments of local field theory. We could consider the strong or weak limits of the currents. In analogy to canonical field theory, we may postulate equal-time commutation relations: unlike canonical fields for which the equal-time commutators is a number, the equal-time commutator of two "canonical" currents is to be a current since they constitute a Lie algebra. A particular construction that realizes such a structure is obtained in terms of bilinear expressions in the fields. Such a group-theoretic characterization for fermi and para-fermi fields has been known in the past.[9] However, it is by no means necessary that currents must be bilinear in canonical fields. (If polynomials in canonical fields are to form an algebra of currents with finite multiplicity at each point, then the degree must not exceed two.) But the most natural development is to associate the local currents with sources of the various kinds of fields.

The use of the local operators associated with the sources of particles in field theory is not new. In the conventional LSZ formalism,[10] for example, matrix elements of suitable products of such "currents" (sources) are used. What is new in this connection is the additional structure to be attributed to the currents; that they form a Lie algebra. The combination of the local operator structure with the Lie algebra leads to a large variety of interrelationships between physical quantities. At the same time, they put in rather stringent restrictions on the kind of approximations that can be usefully employed.

Before proceeding to the applications of these ideas we note that the identification of the local "currents" which are generators of the noninvariance group with the "sources" is an implementation of the suggestion made above that we start with the noninvariance group and specify interactions in terms of it.[7]

Uses of Current Algebras

To illustrate the uses of algebras of local currents, we review some of the recent work on the sum rules relating various amplitudes which emerge from the use of the commutation properties of sources. The most interesting and useful such relation is the remarkable Adler-Weisberger relation between (extrapolated) pion-nucleon total cross sections and the renormalization of the axial vector coupling constant in nuclear beta decay. Practically all the results discussed below have been derived from different starting points; the aim in discussing them here has been to exhibit the common framework within which these derivations are to be viewed.

The simplest type of consequence of an assumed current algebra structure is obtained from the assumption that the electromagnetic current $j^{\mu}(\underline{y},t)$ is the sum of an isotopic invariant part $j^{\mu}_{(0)}(\underline{y},t)$ and the third component of an isotopic vector $j^{\mu}_{\alpha(1)}(\underline{y},t)$ which commute with the density of isotopic spin $j^{0}_{\alpha(1)}(\underline{x},t)$ according to:[11]

$$\left[j^{0}_{\alpha(1)}(\underline{x},t) \, , \, j^{\mu}_{\beta(1)}(\underline{y},t) \right] = i \, \epsilon_{\alpha\beta\gamma} \, j^{\mu}_{\gamma(1)}(\underline{y},t) \, \delta(\underline{x}-$$

$$\left[j^{0}_{\alpha(1)}(\underline{x},t) \, , \, j^{\mu}_{(0)}(\underline{y},t) \right] = 0$$

where the isotopic spin operators T_α are the space integrals

$$T_\alpha(t) = \int d^3x \; j^0_{\alpha(1)}(\underline{x},t)$$

which are time independent by assumption. Hence by integration we have:

$$\left[T_\alpha, \; j^\mu_{\beta(1)}(\underline{y},t)\right] = i\,\epsilon_{\alpha\beta\gamma}\; j^\mu_{\gamma(1)}(\underline{y},t)$$

so that $j^\mu_{\beta(1)}(\underline{y},t)$ are the components of a tensor operator of rank one with regard to the isospin group. The consequence of this assumed commutation relation is thus the same as the ones discussed in connection with approximate symmetries. We could thus claim the relations

$$\mu(\Sigma^+) - \mu(\Sigma^0) = \mu(\Sigma^0) - \mu(\Sigma^-)$$

$$\mu(N^{*++}) - \mu(N^{*+}) = \mu(N^{*+}) - \mu(N^{*0}) = \mu(N^{*0}) - \mu(N^{*-})$$

to be consequences of the current algebra commutation relations, though they follow from the tensor characterization of the electromagnetic current alone.(1)

To get a significant result it is necessary to consider the commutation between two currents viewed as sources. In the conventional form of nuclear beta decay interaction the Gamow - Teller part of the interaction is proportional to the axial vector density:

$$G\,\bar{\psi}\,\gamma_5\,\gamma^\mu\,\tau_+\,\psi = G\,j^\mu_A \equiv G\,\psi^\dagger\gamma_0\gamma_5\gamma^\mu\,\tau_+\,\psi$$

with the same coupling constant G as for the vector (Fermi) interaction. If we consider the equal time commutation relation of j^μ_A with its hermitian conjugate we have

$$\left[j^\mu_A(\underline{x},t),\; j^{\dagger\nu}_A(\underline{y},t)\right] = \delta(\underline{x}-\underline{y})\;\psi^\dagger\left[\gamma_0\gamma_5\gamma^\mu\,\tau_+,\; \gamma_0\gamma_5\gamma^\nu\,\tau_-\right]\psi$$

In particular

$$\left[j^0_A(\underline{x},t),\; j^{\dagger 0}_A(\underline{y},t)\right] = \delta(\underline{x}-\underline{y})\;\;\bar{\psi}(\underline{x},t)\;\gamma_0\cdot 2\,\tau_3\;\psi(\underline{x},t)\,.$$

The right-hand side is twice the third component of the vector current of which the charged components have the form of the conventional vector Fermi interaction. The identification of the latter with the component of the isotopic spin density has been known to be successful in explaining the lack of renormalization of the vector coupling constant of beta decay. Let us now postulate, on the basis of the above motivation, the current commutation relation:

$$\int d^3x \int d^3y \left[j_A^{\,0}(\underline{x},t) \;,\; j_A^{\,+\,0}(\underline{y},t) \right] = 2\, I_3$$

If we now return to the current $\bar{\psi}\gamma_5\gamma^\mu\tau_+\psi$, the divergence of this current has the same transformation properties as $\bar{\psi}\gamma_5\tau_+\psi$ which is appropriate to serve as the field of the pion. We can now postulate that

$$\partial^\nu j_\nu^{\,A}(\underline{x},t) = \xi\,\mu^2\,\pi(\underline{x},t)$$

where $\pi(\underline{x},t)$ is the pion field and ξ is a suitable constant. The constant ξ can be evaluated from the pion decay lifetime. By suitable manipulations we can re-express the matrix element of the left-hand side of the last but one equation taken between one-nucleon states in terms of an extrapolation of the total cross sections of positive and negative pions on nucleons and the renormalization of the axial vector beta decay interaction (defined as the ratio of the effective axial vector coupling constant to the universal vector coupling constant). One obtains the Adler-Weisberger relation[12]

$$\frac{1}{g_A^2} = 1 - \frac{\xi^2}{2\pi}\int_{\mu+\frac{\mu^2}{2m}}^{\infty} \frac{d\nu'}{\nu'}\left\{\sigma^{(+)}(\nu',0) - \sigma^{(-)}(\nu',0)\right\}$$

where $\sigma^{(\pm)}(\nu,0)$ is the total charged pion cross section extrapolated to zero pion mass (at the invariant energy variable $\nu = \frac{1}{m}(p\cdot k)$ where p and k are the nucleon and meson momenta, and m is the nucleon mass). Numerical evaluation gives a result in remarkable agreement with the experimental value.

The Adler-Weisberger relation gives an interrelation between the axial vector renormalization and the pion-nucleon scattering amplitudes, based on two hypotheses, one that the axial vector currents have a commutator which is proportional to the isotopic spin density and second, that the divergence of the axial vector current is proportional to the pion field. One could apply similar considerations to the axial vector currents involving strangeness violating decays and obtain a similar result.(13) We can proceed further: if we postulate that the commutator of the space integrals of two distinct pairs of axial vector currents are equal to the same isotopic spin operator, they must be equal to each other. Such an equality now involves a sum rule involving pion and kaon scatterings on the same target, say proton or pion.(14)

The commutation relations between sources can give rise to a variety of other sum rules. Another example is obtained by taking a current having the transformation property of $\bar{\psi}\gamma^{\mu}\tau_{\alpha}\psi$ and commuting two space-like components of this current four-vector to obtain a relation between the axial vector renormalization and the (forward) scattering amplitude for circular polarized photons on polarized nucleons. This sum rule(15) has the merit of determining, in principle, both the sign and magnitude of the axial vector coupling constant; unfortunately there is no available data to compute the quantity!

If instead of evaluating the commutator of the integrated quantity, we allow for a finite momentum transfer, we can, in principle, determine vertex functions.(16) At the present time there is no direct way to use a strong interaction "form factor" but it is a useful quantity to calculate. And for the weak interaction currents it is directly measureable for a range of momenta. If two distinct commutators are being equated, the use of such finite momentum transfer equations yield a one-parameter family of sum rules between different amplitudes. Finally, taking the matrix elements between a one-particle state and a two-particle state one obtains a correlation between production amplitudes and scattering amplitudes.(17) It is also possible to deduce sum rules for weak interaction processes.(18) Work in this general area is still in its infancy.

Within a suitable framework we can use these techniques to gather information about the fundamental constituents of a model of elementary particles.[15] For example, let us make the hypothesis that all currents must be bilinear in terms of the fundamental fields. Then it is clear that the commutator of two charged axial vector currents can be equal to vector current of isotopic spin involving all the fields only if for every field there is a chiral conjugate field of the opposite parity: a fermion field is its own chiral conjugate, but for a meson field the chiral conjugate must have the opposite parity. Thus in a theory with both nucleons and pions as fundamental fields we cannot have the Adler-Weisberger relation. On the other hand, if the pion field is not fundamental but only the nucleon is fundamental, the relation can be recovered. Even if neither the pion nor the nucleon is fundamental, but both have underlying spinor fields, the Adler-Weisberger relation can be obtained only if the axial current of weak interaction involves all the relevant fields. Comparison of such quantities seem to provide a quantitative test of field theory models of elementary particles.

All these consequences of the Lie algebra structure are contingent upon the local nature of the current operators. But the use of such local operators would imply severe restrictions of the type familiar from axiomatic field theory.

Restrictions on Local Current Algebras

The currents satisfy equal time commutation relations which express the commutator as a (singular) linear combination of currents at the same time. By Lorentz invariance we can generalize this to the commutation relations on any space-like surface. However, we cannot extend this to time-like separated currents: if we assume that the commutator of a set of local currents for all space-time points is a linear combination of currents, then the currents must all be linear in a generalized free field. In other words, no nontrivial system of currents can exist with their commutators for all times are linearly expressible in terms of themselves. This result follows from some earlier work on quantum field theory[19] but the proof is nontrivial.

In a conventional symmetry scheme involving a set of fields, we can write down a local current density which satisfies a differential conservation law. The space integral of the time-like component then gives the "charge" which is conserved and hence independent of time. Suppose we now have such a local current but do not require the current to satisfy the differential conservation law. However, the corresponding charge $Q(t) = \int j_0(\underline{x},t)\, d^3x$ is the generator of a family of transformations. Suppose now we have such a set of generators which satisfy a Lie algebra relation and we suppose further that the vacuum is invariant under these charges so that $Q(t)\,|0\rangle = 0$. It is then possible to show[20] that $Q(t)$ must in fact be independent of time and $j_\mu(\underline{x},t)$ must satisfy a differential conservation law. To prove this we consider the matrix element

$$\langle n|\, Q(t)\,|0\rangle = 0 = \int d^3x\ \langle n|\, j_0(\underline{x},t)\,|0\rangle$$
$$= (2\pi)^3\, \delta(\underline{p}_n)\, e^{ip_n^0 t}\, \langle n|\, j_0(\underline{0},0)\,|0\rangle$$

where $|n\rangle$ is any state. Hence,

$$\delta(\underline{p}_n)\ \langle n|\ \partial^\mu j(\underline{0},0)\,|0\rangle = 0 .$$

By Lorentz invariance, we could remove the restriction $\underline{p}_n = 0$ to obtain $\langle n|\, \partial^\mu j_\mu(\underline{0},0)|0\rangle = 0$ or $\partial^\mu j_\mu\ |0\rangle = 0$.

But any <u>local</u> operator annihilating the vacuum must itself vanish. Hence

$$\partial^\mu j_\mu = 0 .$$

A more relevant example is obtained by the action of the generators of the symmetry group on one-particle states. In the limit of exact symmetry such generators must have one-particle to one-particle matrix elements only. With a broken symmetry we may expect not all other matrix elements to vanish but one may anticipate that the other states to which the nonzero matrix elements develop may be those possessing only invariant masses which are in the neighborhood of the mass of the one-particle state. A local current again rules out this possibility. One can prove

One can now ask whether at least the number of [illegible]

can be restricted. More precisely, while the integrated currents do not necessarily leave the vacuum invariant, or connect one-particle states to one-particle states, can we demand that they have matrix elements connecting any state containing ν particles or less with states containing ν particles or less only? The answer to this question also is in the negative. We have to use "behind-the-moon" arguments to prove that if $\langle m | j | n \rangle = 0$ then $\langle m-1 | j | n-1 \rangle$ is also zero (where m, n refer to the number of particles contained in the state). A field-theoretic version of crossing can now be invoked to show that $\langle m | j | n \rangle = 0$ also implies $\langle m+1 | j | n-1 \rangle = 0$.

Another such result concerns currents that contain only a <u>finite</u> number of creation and/or destruction operators when expanded in terms of the asymptotic free fields. In this case, the requirement of locality of the currents requires[22] the currents then to be Wick polynomials in the asymptotic fields and their derivatives. If we now require that the currents now satisfy the commutation relations corresponding to a semisimple Lie algebra, then it follows that the currents must be quadratic polynomials in the asymptotic fields and their derivatives. If the vacuum is to be left invariant (so that the current is conserved) all the particles involved must be <u>degenerate</u> in mas

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