DO SUB-POISSON PHOTOCOUNTS IMPLY NONCLASSICAL DIAGONAL WEIGHTS?

S.K. Srinivasan† and E.C.G. Sudarshan

Center for Particle Theory, University of Texas, Austin, Texas

An ensemble of Poisson processes with positive weights will exhibit super Poisson fluctuations. Therefore the observed sub-Poisson fluctuations are interpreted as evidence for nonclassical ensembles with indefinite weights. We show that double stochastic processes including deadtime distributions can lead to sub Poisson fluctuations even for a deterministic intensity. More detailed analysis is therefore required to conclude nonclassical diagonal coherent weights.

†Permanent address: Department of Mathematics, Indian Institute of Technology, Madras 6000 036, India

For an ideal photocounter (with no deadtime) with counting efficiency α illuminated by light of a fixed intensity the probability of n counts in time T is Poisson distributed:[1]

$$p(n) - e^{-\mu}\mu^n/n!\,; \quad \mu =< n >= \alpha \int I dt.$$

This has a variance equal to the mean:

$$< n^2 > - < n >^2 - < n >= q(n) = 0$$

If we have an ensemble of complex amplitudes that are Gaussian distributed we get for the effective photocount distribution

$$\begin{aligned} \pi(n) &= \int \phi(I(\cdot))[e^{-\alpha_o^T I(t)dt}\alpha \int I(t)dt)^n/n!]dI(\cdot) \\ &= (e^{-\alpha\mathcal{I}T)}(\alpha\mathcal{I}T)^n \end{aligned}$$

with mean and super-Poisson variance:

$$\begin{aligned} < n >_{.} &= \alpha\mathcal{I}T \\ < n^2 > &- < n >^2 - < n >=< n >^2 . \end{aligned}$$

So the fluctuations are greater than for Poisson counts by the wave noise term. More generally any ensemble of Poisson processes

$$\pi(n) = \int p(n; I)dF(I), dF(I) \geq 0$$

would lead to excess fluctuations.

One of the present authors[2] had shown that any quantum optical field can be realized in the diagonal coherent state representation in terms of linear combinations of projections to coherent states $\mid z >< z \mid$ with a distribution of the class Z. These are not pointwise positive but assure that the density matrix ρ is nonnegative:

$$\begin{aligned} \rho = \int \phi(z) \mid z >< z \mid d^2 z \geq 0; \\ \int \phi(z) \mid P(z) \mid^2 d^2 a \geq 0, \end{aligned}$$

where P(z) is any complex polynomial in z. The possibility that $\phi(z)$ could be negative so that the distribution of the intensity could be pointwise negative leads to the possibility of sub-Poisson photo count statistics and photon antibunching.

Such sub-Poisson photocounts have been found in quantum optics[3]. The natural conclusion has been drawn that these findings give direct evidence of quantum ensembles which cannot be classically realized.

In arriving at the counting rate formula one has implicitly accepted the assumptions of constant counting efficiency and no deadtimes. We have examined the consequences of these effects and find that sub-Poisson photocounts can be obtained even with classical ensembles (even with fixed intensity!) if deadtime effects are included.

Let the active counter have a Poisson no-count lifetime distribution:

$$f_1(t) = \rho e^{-\rho t}$$

while the deadtime distribution is $f_2(t)$. Then, starting with a deadtime, the no-count lifetime distribution[4] is given by

$$f(t) = \int_o^t du f_2(u) f_1(t-u).$$

Then the time upto (and including) the n^{th}-count is distributed according to a probability distribution function $f^{(r)}(t)$ whose Laplace transforms $\tilde{f}^{(r)}(s)$ is related to the Laplace transform $\tilde{f}(s)$ of $f(t)$ by

$$\tilde{f}^{(r)}(s) = (\tilde{f}(s))^r.$$

The counting distribution, the mean and variance can be computed. The general computations are too complicated to reproduce here but we take a very special example to illustrate the possibility of sub-Poissonian photocounts.

For this purpose we choose the intensity to be constant so that the mean count per unit time is ρ^{-1}. Then

$$f_1(t) = \rho e^{-\rho t}\ ;\ \ \tilde{f}_1(s) = \frac{\rho}{s+\rho}$$

Choose

$$f_2(t) = \rho e^{-\rho t}\ ;\ \ \tilde{f}_2(s) = \frac{\rho}{s+\rho}$$

Then the interval of no count is determined by

$$\tilde{f}(s) = \rho^2/(\rho+s)^2$$

which is the special Erlangian distribution with two stages. By virtue of stationarity the mean and variance of the counts in an arbitrary interval [t,t+T] are given by (see [4,5])

$$\begin{aligned} < n > &= CT \\ < n^2 > &= < n > + 2C\int_o^T h(u)(T-u)du \end{aligned}$$

where h(.) is the renewal density and

$$C = \lim_{x\to\infty} h(x).$$

In this case, h(.) is given by

$$h(x) = \frac{\rho}{2}(1-e^{-2\rho x})$$

so that $h(\infty) = \frac{\rho}{2}$. Thus we find

$$< n^2 > - < n >^2 = \frac{1}{2}(1-e^{-\nu}) < \frac{1}{2}^{\nu}$$

leading to sub-Poisson behavior. A more general model of a population point process generated by an Erlangian process subject to a constant death (cavity absorption) rate has been proposed recently by one of us[6] who has shown that the resulting population has a steady state distribution with antibunched/sub-Poisson characteristics. Apparently the existence of processes with sub-Poisson variance is known in the context of space charge limited streams of electrons[7] where the stream is a stochastically inhibited Poisson stream. Doubly stochastic Poisson processes were introduced by Cox[8] and Kingman[9] had shown that if the intensity process is an alternating two-valued process taking values 0 and 1, the resulting counting process is a renewal process provided the spans of intervals $(X_i, Y_i) = 1, 2, \ldots$ over which the values respectively 0 and 1 are taken, are completely independant and identically distributed for $i = 1, 2, \ldots$ with the further condition that the common distribution of X_i is exponential in character. Thus we can conclude that there exist classical situations where a doubly stochastic Poisson process can lead to a stationary stream with a sub Poisson counting variance.

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