

Leptonic Resonances*)

by

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ABSTRACT

Leptonic resonances are predicted in the GeV region on the basis of finite quantum field theory with an indefinite metric.

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We have systematically developed the theoretical framework for finite relativistic quantum field theories¹ during the past decade. In our work we have employed the generalization of the standard framework of quantum mechanics by the use of the indefinite metric; the latter was first employed by Dirac² and Heisenberg³ in other contexts. By using the indefinite metric, we have shown how to construct a finite quantum electrodynamics⁴ in excellent agreement with all available experimental results, as well as how to use it for leptonic weak interactions.⁵ In our work the emphasis has been on the possibility of constructing finite quantum field theories in agreement with the general, empirical results, and we have sought to meet squarely the logical problems encountered by the introduction of apparently negative probabilities. The experimental tests outlined in earlier papers do not seem to have been adequately appreciated. The purpose of this brief note is to emphasize the decisive predictions of leptonic resonances on the basis of our theory.

In the usual formulation of quantum field theory; the propagator of a lepton field is of the form

$$S(p) = \frac{1}{\not{p} - m + i\epsilon} \quad , \quad (1)$$

apart from numerical factors. This has the consequence that in the vicinity of any lepton source, the lepton wave function increases as $\frac{1}{r^2}$ as one approaches the source. It is even more singular than the $\frac{1}{r}$ law for electrostatic and meson fields. The theory thus leads to quadratic divergences when coupled to point sources. To eliminate these divergences one uses an effective propagator of the form

$$S(p) = \frac{(M_1 - m)(M_2 - m)}{(p - m + i\epsilon)(p - M_1 + i\epsilon)(p - M_2 + i\epsilon)}, \quad (2)$$

where M_1 and M_2 are two large auxiliary masses. This choice makes the lepton wave function behave as $\log r$ near its sources. If we rewrite eq. (2) in the form

$$S(p) = \frac{1}{p - m + i\epsilon} - \frac{(M_2 - m)}{M_2 - M_1} \times \frac{1}{p - M_1 + i\epsilon} + \frac{(M_1 - m)}{M_2 - M_1} \times \frac{1}{p - M_2 + i\epsilon}. \quad (3)$$

We note that each lepton of mass m has two satellites with masses M_1 and M_2 . Here M_1 and M_2 are either two real (and distinct masses) or two complex conjugate masses. At least one of the two states has negative weight for the propagator for real masses, necessitating an indefinite metric. For the case of $M_1 = M$, $M_2 = M^*$, both the satellites have zero norm one-particle states, again making the indefinite metric necessary. This theory of relativistic fields with indefinite metric is now fairly well known.

We now observe that if we start out with real lepton satellite masses M_1 and M_2 , they would become "unstable" and the corresponding renormalized poles would move off the real axis. If complex masses are employed, renormalization would only slightly alter their values. In either case the propagators in such a theory would have complex poles.

Since the theory has an indefinite metric there is no reason why these poles should not appear on the physical sheet. With complex satellite masses, the poles are definitely on the physical sheet. If we start out with real masses the negative weight poles would migrate to the physical sheet, the others into the second sheet.

Therefore, in all cases we expect complex poles for the lepton propagators. These poles would manifest themselves as leptonic resonances. They should appear in any channel where single lepton exchange is possible with positive time like four momenta. For example they would exhibit themselves in γe , πe , $\pi \mu$ and $\pi \nu$ channels. Since it is a genuine complex pole, the resonance position would be independent of the channel as long as single lepton quantum numbers are reproduced.

Unlike dynamical resonances in strong interaction physics these are purely kinematic. The width of the resonance bears no immediate relation to the coupling of the lepton resonance to the decay channels. In particular, a significant width can be expected in, say a $\pi \nu$ resonance; it is controlled entirely by the choice of the imaginary part $\frac{1}{2} (M-M^*)$ of the lepton satellite mass. For a charged pion-neutrino resonance it would, of course, be the charged lepton satellite that would be relevant. The cross sections could become arbitrarily large, approaching resonance limits.

Some estimates of the masses are in order. For charged leptons coupled to the electromagnetic field our calculations⁴ have shown that the lepton satellites should not be too light. The estimated ratio was in the neighborhood of (M/m) values of no smaller than 300, but it could be considerably heavier. The preference was for M_1 and M_2 to be of opposite signs (opposite parity); hence both of them should be negative weight states. We conclude that the electron satellite resonances should be at several hundred MeV or higher.

In any case, the search for lepton resonances is a very sensitive and very decisive test of the fundamental postulates of finite relativistic quantum field theory. Conclusive evidence of any such resonances would eliminate the only ground for skepticism of this alternative theoretical framework.

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- (6) The possibility of complex poles in indefinite metric theories has also been studied in a series of recent papers by T. D. Lee and G. C. Wick: Nucl. Phys. B9, 209 (1969); B10, 1(1969); Phys. Rev. D2, 1032 (1970). These authors have reproduced many of the results of the present author, but they prefer to use boson satellites. In such a theory all infinities are not eliminated; and no leptonic resonances are predicted.