NOTE OF THE SPIN 2+ HYPOTHESIS FOR THE K MESON

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Note on the Spin 2+ Hypothesis for the K Meson

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One of the dilemmas of elementary particle physics at the present time is the apparent identity of masses and lifetimes of all charged K mesons, within experimental error, which in some cases is now 2 electron masses\(^1\), the positive \(\tau, \tau', K_{\pi2}, K_{\mu3}, K_{e3}\) all seem to possess a mass close to 965 m\(_e\) and a lifetime\(^2\) of 1.3 x 10\(^{-8}\) sec.; the measurements of the masses and lifetimes of the negative K mesons are much less accurate\(^3\) but are consistent with the values for the \(K^+\) mesons. The masses of the neutral K mesons are approximately equal to the \(K^+\) mass although the lifetimes are 100 times shorter.\(^4\) The simplest explanation of all these results would be, of course, a single type of K meson with a single mass and lifetime for the positive and negative varieties; the neutral meson could possess a slightly different mass (due to electromagnetic effects) and a different lifetime by as much as a factor of 100 (due to special selection rules — see below).

However, the \(\tau\) and \(K_{\pi2}\) modes of decay of the charged K meson preclude spin 0 for the K meson and the existence of the \(2n^0(4)\) mode of decay of the neutral K meson excludes spin 1. The lowest spin of the K meson which can therefore be reconciled with the afore-mentioned facts is 2 and the parity must be +. We propose to examine briefly the consequences of the spin 2\(^+\) hypothesis for the K meson.

The assumption of spin 2\(^+\) for the \(K_{\pi2}\) meson can explain the longer lifetime of

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the charged variety (compared to the neutral variety) as well as the spin 0 assumption. In both cases, the argument of the $\Delta I = 1/2$ (I is the isotopic spin) selection rule depends on the evenness of the spin of the $K\pi2$ and the complete symmetry of the final pion wave function with respect to exchange of the space and isotopic spin coordinates. The spin 2+ hypothesis can also be reconciled with the apparent lack of angular correlation between the production and decay planes of the $K$ meson; Regge has shown that there is sufficient flexibility in the unknown matrix elements to simulate an isotropic distribution. However, the greatest obstacle which must be overcome by the spin 2+ hypothesis is the Dalitz-Fabri analysis of the decay of the $\zeta$ meson.

The Dalitz-Fabri analysis is based on a comparison of the observed energy and angular distributions of the outgoing pions with the theoretical predictions which follow from assumed spins and parities of the $\zeta$ meson. In particular, suppose we consider the $\zeta^+$ meson and denote the angular distribution by $W(\Theta)$ (where $\Theta$ is the angle of decay of the two $\pi^+$'s in their own center of mass system relative to the direction of the $\pi^-$) and the energy distribution by $f(\epsilon)$ (where $\epsilon$ is the kinetic energy of the $\pi^-$ in the laboratory system expressed in units of the maximum energy). Then, if $L$ is the angular momentum of the $\pi^+$ pair and $\ell$ is the angular momentum of the $\pi^-$, the usual procedure is to choose the lowest pair of values of $(L, \ell)$ consistent with the assumed spin and parity of the $\zeta$ and to calculate $W(\Theta)$ and $f(\epsilon)$ assuming that there is no interaction among the outgoing pions. Thus, for spin 0+, the lowest pair of values of $(L, \ell)$ is $(0,0)$ and the predicted $W(\Theta)$
and \( f(\varepsilon) \) are, respectively:

\[
W(\theta) \, d(\cos \theta) = 1. \, d(\cos \theta)
\]

\[
f(\varepsilon) \, d\varepsilon = \frac{2\pi}{\pi} \sqrt{\varepsilon (1-\varepsilon)} \, d\varepsilon
\]

For spin \( 2^+ \), the lowest pair of values of \((L, \ell)\) is \((2,1)\) and one gets:

\[
W(\theta) \, d(\cos \theta) = \frac{3}{2} \cdot \delta_{\ell \ell'} \, d(\cos \theta)
\]

\[
f(\varepsilon) \, d\varepsilon = \frac{25}{2\pi} \, \varepsilon^{3/2} (1-\varepsilon)^{5/2} \, d\varepsilon
\]

The observed \( W(\theta) \) and \( f(\varepsilon) \) are, within experimental error, in agreement with the distributions corresponding to spin \( 0^- \) for the \( \tau \) meson and are certainly in disagreement with (2). However, the structure of the \( \tau \) meson is sufficiently unknown that there is no a priori reason for restricting oneself to the \((2,1)\) choice of \((L, \ell)\) in the case of spin \( 2^+ \); it is permissible to postulate that the next possible pair of values of \((L, \ell)\), namely \((2,3)\), yields a transition matrix element for the decay of the \( \tau \) which is comparable to \((2,1)\). If we denote the ratio of the \((2,3)\) to the \((2,1)\) matrix elements by \( \mathcal{P} \) (which must be real in accordance with the usual time reversal invariance arguments), the expressions for \( W(\theta) \) and \( f(\varepsilon) \) are:

\[
W(\theta) \, d(\cos \theta) = \frac{3}{2(1+4\xi^2)} \left\{ (1-8\xi^2+16\xi^4) \, \sin^2 \theta + (10\xi^2-45\xi^4) \sin^4 \theta \right\} \, d(\cos \theta)
\]

\[
f(\varepsilon) \, d\varepsilon = \frac{25}{15 \pi (1+\xi^2)} \left\{ 5 + 24 \xi^2 \varepsilon^2 \right\} \, \varepsilon^{3/2} (1-\varepsilon)^{5/2} \, d\varepsilon
\]

The energy distribution is simply the sum of the contributions from the \((2,1)\) and \((2,3)\) terms whereas the angular distribution also contains an interference term.

Eqs. (3) and (4) contain the additional flexibility needed to obtain a better fit to the experimental data. We have plotted in Fig. 1 the latest observed angular distribution together with the predictions of Eq. (3) for \( \mathcal{P} = 0 \), \( \mathcal{P} = 1.5 \) and \( \mathcal{P} = \infty \); in Fig. 2 are plotted the curves for \( f(\varepsilon) \) corresponding to Eq. (4).

It is evident that the presence of a \((2,3)\) matrix element improves the theoretical predictions for a spin \( 2^+ \) meson. Indeed, despite the fact that every term in Eq. (3) contains a \( \sin^2 \theta \) factor, \( W(\theta) \) can give a reasonably isotropic distribution up to \( \cos \theta = 0.3 \), after which it drops rapidly to zero. The energy
distribution, $f(\varepsilon)$, is still not too satisfactory at the lower and upper ends (i.e. for $\varepsilon < 0.1$ and $\varepsilon > 0.9$) but considerable improvement\textsuperscript{11} (by a factor of two) can be achieved by taking into explicit account the possibility of a strong pion-pion interaction.

The predictions for a spin 0- meson are also plotted in Figs. 1 and 2 and still provide the most satisfactory explanation of the experimental data. However, as we have already remarked, the $K_{\pi 2}$ meson cannot have spin 0- and it has been necessary to invoke several ingenious hypotheses\textsuperscript{12} in order to explain the apparent identity of the masses and lifetimes of the various types of charged $K$ mesons. Since there is now also evidence that the relative abundance of the different types of $K$ mesons does not change with the incident energy of the proton\textsuperscript{13} nor after scattering\textsuperscript{14}, the hypothesis of the actual identity of all the $K$ mesons (with the different types merely representing alternate decay modes of the same particle) is worthy of the closest scrutiny. We believe that we have shown that the earlier Dalitz-Fabri arguments against spin 2+ for the $\Upsilon$ are not quite so definitive as previously believed\textsuperscript{15} and that the inclusion of the $(2,3)$ matrix element and of a strong pion-pion interaction can bring the theoretical predictions for the angular and energy distributions into closer agreement with the experimental data. Our conclusion is, therefore, that the spin 2+ hypothesis is not now excluded by the experimental data on the $\Upsilon$ meson and that considerable improvement in statistics will be necessary before a final decision can be reached. A direct measurement of the spin of the $K$ meson (just as in the case of the pion\textsuperscript{16}) would, of course, settle the matter.
References


5. Some of these considerations were briefly presented at the Sixth Rochester Conference; however, in view of the importance of the problem, it seemed desirable to publish a somewhat expanded version of those remarks.


11. This would require to a strong D-wave interaction among all pairs of pions.


15. We have reexamined the arguments against spin 2+ for the τ meson; higher spins might also be reconsidered.

Figure Captions

Fig. 1. Angular distribution $W(\theta)$ is the angle of decay of the two $\pi^+$'s in their own center of mass system relative to the direction of the $\pi^-$. Predicted distributions for $2^+$ for values of the mixture parameter $\rho = 0, 1.5, \infty$ and for spin $0^-$. The histogram corresponds to the experimental data (reference 10).

Fig. 2. Energy distribution $f(\epsilon)$ were $|\epsilon|$ is the $\pi^-\pi^+$ energy in the laboratory in units of the maximum energy. Predicted distributions for $2^+$ for values of the mixture parameter $\rho = 0, 1.5, \infty$ and for spin $0^-$. The histogram corresponds to the experimental data (reference 10).
ANGULAR DISTRIBUTION of the $\pi^-$

----- EXPERIMENTAL DATA

FIG. 1
ENERGY DISTRIBUTION of the $\pi^-$

--- EXPERIMENTAL DATA

FIG. 2