On Multiquark Mesons

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It is a well-established fact that observed hadrons fall into multiplets, whose structure can be summarized by the statement that mesons are quark-antiquark composites and baryons three quark composites; quarks being the objects that provide a fundamental representation of the underlying unitary group. Could there exist hadrons that are built up of more quarks (and antiquarks) than these lowest multiplets? With regard to mesons, we had addressed ourselves to this question in 1966 and advocated, since then, the following answer[1 4]: Assign mesons to a completely degenerate representation (CDR) of a non-chiral direct product group SU(n) X SU(n), where one of the two factors in the direct product operate on quarks and the other on antiquarks. Completely degenerate representations are those of the type $(\bar{d},d)$, where d denotes a symmetric tensor representation and $\bar{d}$ is the representation conjugate to d. When the underlying symmetry group is SU(3), the CDRs are of the form $(\bar{3},3)$, $(\bar{6},6)$, $(\overline{10},10)$ and can be viewed as quark-antiquark, diquark-antidiquark, triquark-antitriquark states. If a charmed quark is included in the picutre, the lowest CDRs of SU(4) X SU(4) are $(\bar{4},4)$, $(\overline{10},10)$ etc. In the tensor language the CDRs are represented by tensors of the type $G^{\mu\nu\ldots}_{\alpha\beta\ldots}$ with the properties 1) the tensor is completely symmetric under permutation of the lower (quark) indices, 2) it is completely symmetric under permutations of the upper (antiquark) indices and 3) the number of upper indices is equal to the number of lower indices. The CDRs possess nice mathematical properties. Under decomposition into the 'middle' SU(n), a CDR consists of a sequence of self-conjugate representations, <u>each occurring once</u>. A second property of CDRs is the following. Under the assumption that the symmetry breaking operator has the transformation property (R,1) + (1,R), where R denotes the appropriate component (or components) of the regular representation, the first order mass formula is found to be

$$m = m_0 + a_3(N_3 + N^3) + a_4(N_4 + N^4) + \ldots + a_n(N_n + N^n) \ldots \qquad (1)$$

where N_i (N^i) is the number operator of the lower (upper) index "i" in the meson tensor and a_i are undetermined parameters. In the above n is the dimensionality of the underlying unitary group (number of quark flavors!). The above mass formula leads to a generalized equal spacing rule in an n-demensional hyperspace as well as to the degeneracy of certain meson components. The latter includes the degeneracy of mesons differing in isospin but with other quantum numbers the same. These isospin degeneracies persist under more general symmetry-violations than those considered thus far ---- they remain valid under arbitrary violations of the original non-chiral group as long as the subgroup SU(2) X SU(2) is kept unbroken.

Let us consider coupling schemes for multiquark mesons; specifically those connecting the dequark-antidiquark $(\bar{Q}^2Q^2)$ mesons to conventional mesons and baryons. As regards the trilinear coupling to a pair of $(\bar{Q}Q)$ mesons, the non-chiral groups admits a coupling only when the conventional meson $(\bar{Q}Q)$ pair is in a symmetric state and this is given by

$$g_1 \; G^{\mu\nu}_{\alpha\beta} \quad G^{\mu}_{\alpha}(1)\; G^{\nu}_{\beta}(2) + G^{\nu}_{\beta}(1)\; G^{\mu}_{\alpha}(2) \qquad (2)$$

where G^{μ}_{α} denotes a conventional $(\bar{Q}Q)$ meson multiplet. When the final meson pair is in an antisymmetric state, the coupling is possible only if we violate the original non-chiral group. Retaining now only the 'middle' SU(n) symmetry, the relevant coupling is

$$g_2 \; G^{\mu\nu}_{\alpha\nu} \quad G^{\mu}_{\lambda}(1)\; G^{\lambda}_{\alpha}(2) - G^{\lambda}_{\alpha}(1)\; G^{\mu}_{\lambda}(2) \tag{3}$$

where a possible term proportional to the trace G^{λ}_{λ} has been dropped in accordance with the Okubo-Zweig-Iizuka rule[5]. We should emphasize that eqns. (2) and (3), albeit for the special case when the number of flavors is three, were written down in our 1967 paper[2]. Meson-baryon couplings are similarly constructed. Again, they can proceed only through a violation of the original non-chiral group. The SU(n) preserving coupling that one now obtains is too general to be of any practical use. However, we can look upon the OZI rule to imply that those terms in the coupling that contain the contractions $G^{\mu\lambda}_{\alpha\lambda}$, $G^{\alpha\lambda}_{\alpha\lambda}$ are inhibited as compared to the leading term that does not involve a contraction. Then a unique scheme results:

$$g \; G^{\mu\nu}_{\alpha\beta} \quad \psi^{\mu\nu\lambda} \; \psi_{\alpha\beta\lambda} \tag{4}$$

where $\psi_{\alpha\beta\gamma}$ is the 'three quark composite' baryon tensor which has the properties 1) $\psi_{\mu\nu\lambda} = \psi_{\nu\mu\lambda}$ and 2) $\psi_{\mu\nu\lambda} + \psi_{\nu\lambda\mu} + \psi_{\lambda\mu\nu} = 0$.

References

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