Unified Theory of Weak and Electromagnetic Interactions

With One Single Coupling Constant

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Summary

We discuss a unified theory of weak and electromagnetic interactions. The theory involves the 4-vector fields $W^+_u$, $Z^0$, and the experimentally established leptons. The dynamics of interactions among these physical particles is characterized by one single coupling constant. The theory is covariant, renormalizable, unitary and "gauge independent." All these come about because dynamics and symmetry are interlocked in a simple way. These are to be contrasted with those in the Weinberg unified theory for leptons in which there are, in addition, the Higgs scalars with complicated interactions, and the dynamics of interactions among physical particles is characterized by five different coupling constants.

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1. Introduction

The machinery in this paper is essentially based on the following properties of the Lagrangian: (a) It involves the Lagrange multiplier field $\chi_d$ ($d = 1, 2, \ldots$) for the massive $d$-vector fields; (b) Aside from mass terms, it is (local) isospin-invariant. The property (a) makes the theory manifestly variant and the massive vector boson propagator $\propto k^{-2}$ as $k \to \infty$; the (b) establishes deep connection between dynamics and symmetry, and ensures the couplings of $\chi_d$ to be renormalizable. The usual field-theoretic definition of the $S$ matrix is not unitary $\chi_d$ does not obey the free equation. The interactions of $\chi_d$ will contribute an extra amplitude and upset unitarity. The definition of the Lagrangian of the interacting fields have two independent free parameters $\xi$, and $\xi$: the propagators and the vertices do depend on these parameters. The parameter $\xi$ corresponds to a gauge change in the photon propagator; and $\xi$ is the ratio of the square of the mass of the regd intermediate boson to that of its scalar counterpart. Limit $\xi \to 0$ corresponds to infinity heavy scalar quanta a manifestly unitary $S$-matrix.

In this paper, we consider a unified theory in which there is massive neutral vector field $Z_u(x)$ in addition to $W^u(x)$ and $A^u(x)$ discussed in the previous paper. In this theory, we need not assume heavy leptons and all the leptons are experimentally observed. Aside from the structure of coupling in the Lagrangian, this theory differs from Weinberg theory in two important aspects: (i) This theory has no Higgs mesons and is asymptotically free; (ii) The masses are free parameters, e.g., the $Z_u$ mass is independent of the $W^u$ mass. Consequently, we can construct a unified theory of leptons in which the weak and the electromagnetic forces are characterized by one and the same coupling constant without contradicting experimental data on neutral currents. This is to be contrasted with the Weinberg theory of leptons, in which there are five different coupling constants for the interactions of physical particles. (See Section 5)

2. Lagrangian and Field Equations

Let us consider a physical system involving $W^u(x)$, $A^u(x)$, $Z_u(x)$, $\nu(x)$ and $\bar{\nu}(x)$ in interactions. The Lagrangian of the system is

$$L = L_b + L_t + L_L,$$ (1)
\[ \mathcal{L}_b = -\frac{1}{2} \mathcal{W}_a^+ \mathcal{W}_a^- + \mathcal{M}_a^2 \mathcal{W}_a^- \]
\[ - \frac{1}{4} \mathcal{F}_{\alpha \beta} - i e (\mathcal{W}_a^+ \mathcal{W}_a^- - \mathcal{W}_a^+ \mathcal{W}_a^-) \]
\[ - \frac{1}{4} \mathcal{Z}_a^\alpha - i G \cos^2 \theta (\mathcal{W}_a^+ \mathcal{W}_a^- - \mathcal{W}_a^+ \mathcal{W}_a^-) \]
\[ + \frac{1}{2} \mathcal{L}_a^2 \mathcal{Z}_a^\alpha \]
\[ \mathcal{L}_L = -\frac{1}{2} \zeta (\mathbf{A}^a) \mathbf{A}_a + \frac{1}{2} \frac{1}{2} (\mathcal{Z}_a^2)^2 - \mathcal{L}_b (\mathbf{A}_a + i G \cos^2 \theta \mathcal{Z}_a^2) \mathcal{W}_a^- \]

The presence of \( \mathcal{L}_L \) in (3) implies that the massive 4-vector fields contain both the physical spin 1 particle and the unphysical spin 0 particles having negative metric. By appealing to the action principle, the Lagrangian (1) gives the variational equations

\[ \mathcal{L}_L = 0 \]
\[ \mathcal{L}_b = 0 \]
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where

\[ \mathcal{W}_\alpha^+ = \mathcal{W}_\alpha^+ \]
\[ \mathcal{W}_\alpha^- = \mathcal{W}_\alpha^- \]
\[ \mathcal{F}_{\alpha \beta} = \mathcal{F}_{\alpha \beta} \]
\[ \mathcal{Z}_a^\alpha = \mathcal{Z}_a^\alpha \]
\[ \mathcal{L}_a^2 = \mathcal{L}_a^2 \]

The parameters \( \epsilon, G, \zeta, \ell, \gamma, \) and \( \theta \) are real. Note that \( \mathcal{M}_A \) is independent of \( M \). The Lagrangian \( \mathcal{L}_L \) in (1) could be replaced by

\[ \mathcal{L}_b = -\frac{1}{2} \mathcal{W}_a^+ \mathcal{W}_a^- + \mathcal{M}_a^2 \mathcal{W}_a^- \]
\[ - \frac{1}{4} \mathcal{F}_{\alpha \beta} - i e (\mathcal{W}_a^+ \mathcal{W}_a^- - \mathcal{W}_a^+ \mathcal{W}_a^-) \]
\[ - \frac{1}{4} \mathcal{Z}_a^\alpha - i G \cos^2 \theta (\mathcal{W}_a^+ \mathcal{W}_a^- - \mathcal{W}_a^+ \mathcal{W}_a^-) \]
\[ + \frac{1}{2} \mathcal{L}_a^2 \mathcal{Z}_a^\alpha \]
\[ \mathcal{L}_L = -\frac{1}{2} \zeta (\mathbf{A}^a) \mathbf{A}_a + \frac{1}{2} \frac{1}{2} (\mathcal{Z}_a^2)^2 - \mathcal{L}_b (\mathbf{A}_a + i G \cos^2 \theta \mathcal{Z}_a^2) \mathcal{W}_a^- \]
\[ \dot{a}_a^2 + \frac{1}{2} H_a^2 = 0. \tag{12} \]

The divergence of (9) together with (10), (7) and its adjoint give us

\[ \sigma \chi_0 = 0. \tag{13} \]

Similarly, we have

\[ (\alpha + \eta^{-1} H_1^2) \chi + \frac{1}{2} \sigma M_2^{-1} \gamma \chi = 0 \tag{14} \]

\[ (\alpha + \eta^{-1} H_1^2) \chi + \frac{1}{2} \sigma M_2^{-1} \gamma \chi = 0 \tag{15} \]

by a straightforward but tedious calculation. In equation (15) we have set \( \chi_0 = 0 \), which is consistent with (13). The equations (13) - (15) for the Lagrange multiplier fields are important in our approach. We see that all the source terms correspond to renormalizable coupling. The significance of this fact will be discussed in Section 4.

We may quantize the theory based on (1) according to the usual canonical quantization procedure. We find that \( \chi_0 \) is a zero-norm field while \( \chi_z \) and \( \chi^z \) are negative metric fields. Thus, they are unphysical. Since \( \chi_0 \) is a free field, it has no physical effect. The fields \( \chi_z \) and \( \chi^z \) are not free. So, the problem is how to treat their interactions which upset unitarity. This will be discussed later. For the free fields \( \mathcal{W}_a, \mathcal{Z}_a, \) and \( \mathcal{A}_a, \) we have the following general commutators:

\[ [\mathcal{W}_a(x), \mathcal{W}_b(y)] = -i(\xi_0 + M^{-1} x_2 x_2) \delta(x-y, M^2) \]

\[ + i H_1^{-1} x_2 \delta(x-y, M^2) \]

\[ [\mathcal{Z}_a(x), \mathcal{Z}_b(y)] = -i(\xi_0 + M^{-1} x_2 x_2) \delta(x-y, M^2) \]

\[ + i H_1^{-1} x_2 \delta(x-y, M^2) \]

\[ [\mathcal{A}_a(x), \mathcal{A}_b(y)] = -i \xi_0 \delta(x-y, 0) + i(1 - \gamma^0) x_2 \delta(x-y) \]

\[ \Delta(x, p^2) = -i(2\pi)^{-3} \int d^4 p \epsilon(p_0) \delta(p^2 - p_2^2) e^{-i p \cdot x}. \]

\[ E(x) = -\frac{1}{8\pi^2} \chi(x) \Theta(x^2) = -i(2\pi)^{-3} \int d^4 p \epsilon(p_0) \delta(p^2 - p_2^2) e^{-i p \cdot x}, \]

\[ \partial E(x) = \delta(x, 0). \]
3. The Unitary S Matrix

Let us consider the physical processes in which the initial and final states involve only physical particles. According to the usual field-theoretic definition, the amplitudes for these processes could be expressed in terms of the path integral

$$\mathcal{A} = \int \exp[i \int d^4x (\mathcal{L} + \mathcal{L}_s)] d\tilde{x},$$

$$\mathcal{L}_s = \text{external source terms},$$

$$[\tilde{x}] = [\bar{u}, u, \bar{v}, v, \bar{w}, w],$$

$$[\tilde{\chi}] = [\bar{\chi}, \chi],$$

with external physical particles. Integrating over $\tilde{x}$, we have

$$\mathcal{A} = \int \exp[i \int d^4x (\mathcal{L}_b + \mathcal{L}_\xi + \mathcal{L}_L + \mathcal{L}_s)] d\phi,$$

where $\mathcal{L}_L$ is given by (6). The amplitude (16) or (17) is not unitary because the Lagrangian involves the unphysical spin 0 part of the massive 4-vector fields which could be produced in the intermediate states and contribute unwanted amplitudes.

One way to restore unitarity of the amplitude (17) is to take the limit $\xi \to 0$. In this limit, the masses of the scalar quanta associated with the 4-vector field $\bar{\omega}_\mu$ and $\omega_\mu$ become infinite. These unphysical particles cannot be present in initial and final state. Nor can it contribute in the intermediate states of the physical processes. The unphysical particles associated with $A_\mu$ do not, as usual, affect physics because they are free particles. Therefore, the amplitude (17) is unitary for arbitrary gauge parameter $\xi$ in this limit.

4. Renormalizability and Asymptotic Freedom

The propagators for the free fields $\bar{\omega}_\mu$, $\omega_\mu$ and $A_\mu$ are respectively given by

$$\int d^4k \, e^{ik \cdot x} \langle 0 | T(\bar{\omega}_\mu(x) \omega_\mu(0)) | 0 \rangle = -i (g_{\omega A} - \frac{\xi}{k^2} k_{A B} \bar{\omega}_\mu k_{\omega B}) / (k^2 - M^2)$$

$$\int d^4k \, e^{ik \cdot x} \langle 0 | T(\omega_\mu(x) \omega_\mu(0)) | 0 \rangle = -i (g_{\omega A} - (1 - \xi^{-1}) k_{A B} (k^2 - \xi^{-1} M^2) / (k^2 - M^2)$$

$$\int d^4k \, e^{ik \cdot x} \langle 0 | T(A_\mu(x) A_\mu(0)) | 0 \rangle = -i (g_{\omega A} - (1 - \xi^{-1}) k_{A B} k_{\omega B}) / k^2.$$ (18)

From the Lagrangian (1) and the propagators (18) - (20), we see that the theory is renormalizable (for $\xi > 0$) by standard power counting.
In Section 2, we have shown that the couplings of the scalar particles associated with massive 4-vector fields $W^a_{\mu}$ and $Z_{\mu}$ are of renormalizable type. This is precisely what happens in the renormalizable gauge theories.\textsuperscript{2} If the Lagrangian does not have local isospin gauge symmetry (which may be broken by mass terms), then some source terms in the equations for the Lagrangian multiplier fields will not correspond to renormalizable coupling. The situation occurs in the $\xi$-limiting formalism for the massive charge vector mesons.\textsuperscript{1} This indicates that the interaction of the spin zero particle associated with the 4-vector massive field may not be renormalizable. In other words, the parameter $\xi$ (or the mass of the spin zero particle) may not be renormalizable.

Using these rules and n-dimensional regularizations, one can show that the theory is asymptotically free. One has only to calculate the logarithmically divergent terms of order $2$ in $Z_4$ (the self-energy diagram of the photon) because the renormalization-group parameter $\beta(\alpha)$ is\textsuperscript{4}

$$\beta(\alpha) = -\frac{2}{3\ln(\frac{\Lambda}{\alpha})} \frac{Z_4}{Z_2}$$

(21)

where $Z_2$ is equal to $Z_1$ due to Abelian gauge invariance. In the absence of the leptons, one finds that

$$Z_3 = 1 + \frac{6}{5} \frac{e^2}{\Lambda^2} \ln \frac{\Lambda}{M}$$

(22)

where the limit $\xi \to 0$ is taken after the integration is carried out. The $\xi$-dependent parts which become infinite when $\xi \to 0$ are absorbed into counter terms. (cf. K. T. Mahanthappa, Ref. 3). The result (22) implies $\beta(\alpha) \leq 0$ and the theory is asymptotically free. Asymptotic freedom can be maintained provided not too many fermions are introduced, for example, no more than 10 iso-doublets in $SU(2)$.\textsuperscript{4}
5. Symmetry of the Lagrangian

The renormalizability of the theory comes about because the dynamics of interactions and symmetry are intimately related. To see the symmetry of the Lagrangian (1), we write $\mathcal{L}_b$ and $\mathcal{L}_p$ given by (2) and (3) in the following form:

$$
\mathcal{L}_b + \mathcal{L}_p = \mathcal{L}_w - \frac{1}{2} \mathcal{M}^2 \left( (A_0^2 + \bar{A}_0^2) + \frac{1}{2} \mathcal{M}^2 \right) + \frac{1}{2} \left( \mathcal{M}^2 \right) \left( \mathcal{M}^2 + \mathcal{M}^2 \right) - \frac{1}{2} \mathcal{M}^2 \left( \mathcal{M}^2 + \mathcal{M}^2 \right)
$$

$$
\mathcal{L}_w = -\frac{1}{4} \left( \partial_\mu A_\nu + \partial_\nu A_\mu \right) \left( \partial_\nu A_\mu + \partial_\mu A_\nu \right) + \frac{1}{2} \mathcal{M}^2 \left( \mathcal{M}^2 + \mathcal{M}^2 \right)$$

$$
+ \frac{1}{2} \mathcal{M}^2 \left( \mathcal{M}^2 + \mathcal{M}^2 \right) + \frac{1}{2} \mathcal{M}^2 \left( \mathcal{M}^2 + \mathcal{M}^2 \right)
$$

where $g = G \cos \theta$, $g^* = -eG/g$, and

$$
\mathcal{M}_a \equiv \frac{1}{2} \left( \mathcal{M}_a + \mathcal{M}_a \right)
$$

We see that $\mathcal{L}_w$ is invariant under the local isospin gauge transformation. The isospin symmetry of $\mathcal{L}_p + \mathcal{L}_b$ is violated only by the mass terms which involve a constant factor and two vector fields, e.g., $\mathcal{M}^2 (A_0^2 + \bar{A}_0^2)$. The Lagrangian (23) differs from that in the Weinberg theory in that we do not have the Lagrangian for the scalar isodoublet meson (consisting of $\xi^+$ and $\xi^0$):
multiplier field also occur in the case where the vector boson masses are generated from spontaneously broken gauge symmetry due to \( \mathcal{L}_\mu \) given by (26).

6. Genuine Unification and Universal Couplings

The Lagrangian (1) has two different coupling constants for the interactions of physical particles. This has the advantage of easy adjustment with experimental data. However, this does not seem to be in harmony with the theoretical spirit of unifying the weak and the electromagnetic interactions. In the genuine unified theory, the weak and the electromagnetic forces are physically the same thing, namely, they are characterized by one and the same coupling constant. The observed dissimilarities between weak and electromagnetic interactions are purely due to mass differences between the vector bosons. The reason for such a huge difference (< 50 GeV) is not clear so far. It has been suggested that the heavy mass of the intermediate vector bosons might come from their strong cubic self-interactions.\(^5\) Yet, experiment seems to suggest a different picture. All observed particles, which participate in strong interactions, have masses of order 1 GeV. The ratios between these masses range roughly from 1 to 10. On the other hand, the electron and the muon have almost identical interactions and they do not have strong interactions. They have, however, quite different masses: \( m_\mu / m_e \geq 200 \).\(^6\) Similarly, the vector fields \( W^\mu \), \( Z_\mu \), and \( A_\mu \) may have the same interaction but quite different masses. And this may well be because they do not have strong interactions.

In fact, universality of electromagnetism has meant all along that the interaction part of the Lagrangian involves a universal coupling constant for as diverse particles as the electron, the muon, the pion, and the proton. There is no requirement that the particles have to have the same mass or spin. When we refer to the electron–muon universality of their weak and electromagnetic interactions, we do not imply that they have the same mass; the chiral V–A interaction possesses a high degree of symmetry which is not shared by the mass terms and hence by the total Hamiltonian. It is in this spirit that we develop our theory, and our work shows that the high degree of symmetry of the interaction Lagrangian has important dynamical consequences even when symmetry is violated by mass terms.

If one treats all the vector fields \( W^\mu \), \( Z_\mu \), and \( A_\mu \) on equal footing dynamically, their couplings among themselves and to the leptons must be universal. This can be accomplished by setting \( g = g' \) in (1).\(^7\)
\[ L = -\frac{1}{4} \left( \tilde{\psi} \gamma^a W^{a-} - W^{a-} \tilde{\psi} \right)^2 - \frac{1}{4} \left( \tilde{\psi} \gamma^a W^{a+} - W^{a+} \tilde{\psi} \right)^2 + \frac{1}{2} \left( \tilde{\psi} \gamma^a \gamma^5 W^{a+} \tilde{\psi} \right)^2 + \frac{1}{2} \left( \tilde{\psi} \gamma^a \gamma^5 W^{a+} \tilde{\psi} \right)^2 + \frac{1}{2} \left( \tilde{\psi} \gamma^a \gamma^5 W^{a+} \tilde{\psi} \right)^2 + \frac{1}{2} \left( \tilde{\psi} \gamma^a \gamma^5 W^{a+} \tilde{\psi} \right)^2 + \frac{1}{2} \left( \tilde{\psi} \gamma^a \gamma^5 W^{a+} \tilde{\psi} \right)^2 \]

but is related to the charged boson mass. In contrast, in our theory the 2 boson mass is arbitrary. We must therefore deduce it from observations on neutral currents. The data on neutral lepton currents indicate that

\[ M_\nu \approx 65.6 \text{ GeV} . \]

7. Conclusion

The simplicity and elegance of the present unified theory can be best seen by comparing the Lagrangian (27) and that of the Weinberg theory of the leptons. In the Weinberg theory, aside from the vector field \( W^+ \), \( Z^0 \), \( A_\mu \) and the leptons, one also has to introduce scalar field \( \phi \) with the Lagrangian \( L_\phi \) given by (39). Moreover, the dynamics of interactions among physical particles in Weinberg's theory are characterized by five different coupling constants, i.e., \( g, g', G_\epsilon, G_\mu, \alpha \). On the other hand, the present unified theory involves only the vector fields \( W^+ \), \( Z^0 \), \( A_\mu \) and the leptons and, furthermore, their interactions are characterized by one single coupling constant. In view of this, the present theory is much closer to the spirit of unifying apparently different forces in nature.

Historically, the idea of unifying weak and electromagnetic interactions has been discussed previously by Schwinger, Glashow, Salam and Ward, etc. The idea of unifying CP-conserving and CP-violating weak processes has been discussed by
Nishijima and Swank.\textsuperscript{12} A more systematic attempt has been made by Okubo,\textsuperscript{13} and by Marshak and others\textsuperscript{14} on the basis of a Yang-Mills Lagrangian.\textsuperscript{15} These ideas are more in line with the present unified theory because one always desires to postulate fewer field and coupling strengths to explain more phenomena.

The above discussions show that a renormalizable and asymptotically free theory can be constructed involving self-coupled massive vector mesons. All these come about because the dynamics of the Lagrangian is interlocked with gauge symmetry which is violated only by mass terms.\textsuperscript{16} Unitarity is recovered by the \( \bar{g} \)-limiting procedure.\textsuperscript{17,18} In this limit, the characteristics related to renormalizability and unitarity in this theory are the same as those in the Weinberg unified theory with spontaneously broken gauge symmetry. We believe that the present theory is renormalizable and unitary in the same sense as the usual gauge theories. Furthermore, the present theory is much simpler than the Weinberg theory of leptons and possesses the desirable qualities of simplicity, genuine unification and asymptotic freedom.

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References

6. The possibility that the u-e mass difference has its origin in symmetry rather than in dynamics of interactions has been suggested. See, for example, K. Tennakone and S. Pakvasa.
7. The same situation happens in the model considered in Ref. 1.
8. D. H. Perkins, Fifth Hawaii Topical Conference in Particle Physics, August 8-21, 1973 (to be published). We use the new result (Trieste, 1973): \( \sin^2 \theta < 0.35 \).
17. T. D. Lee and C. N. Yang, Phys. Rev. 128, 985 (1962); see also K. T. Mahanthappa, Ref. 3.
18. If the amplitudes due to the source terms in the equations (14) and (15) could be removed from the theory by some Lagrangian involving fictitious particles, the theory will be unitary for any \( \xi > 0 \). However, because of the structure of our Lagrangian (1), this cannot be done in the conventional way. See, for example, J. P. Hsu and E. C. G.
Sudarshan, "Gauge Independence and Unitarity in the Weinberg Theory" CPT Preprint (1974) and Ref. 2.