CPT 225

ORO-3992-181

Unified Theory of Weak and Electromagnetic Interactions With One Single Coupling Constant

J. P. Hsu and E. C. G. Sudarshan

Center for Particle Theory
The University of Texas at Austin
Austin, Texas 78712

*Work supported in part by the U.S. Atomic Energy Commission.

Summary

We discuss a unified theory of weak and electromagnetic interactions. The theory involves the 4-vector fields W_μ, Z_μ, A_μ and the experimentally established leptons. The dynamics of interactions among these physical particles is characterized by one single coupling constant. The theory is covariant, renormalizable, unitary and "gauge independent." All these come about because dynamics and symmetry are interlocked in a simple way. These are to be contrasted with those in the Weinberg unified theory for leptons in which there are, in addition, the Higgs scalars with complicated interactions, and the dynamics of interactions among physical particles is characterized by five different coupling constants.

206

25 206

1. Introduction

The machinery in this paper is essentially based on
ıe following properties of the Lagrangian: (a) It involves the
ıgrange multiplier field χ_d $(d = 1,2,\ldots)$ for the massive 4-
ctor fields; (b) Aside from mass terms, it is (local) isospin
uge invariant. The property (a) makes the theory manifestly
variant and the massive vector boson propagator $\propto k^{-2}$ as $k \to \infty$;
ile (b) establishes deep connection between dynamics and sym-
try, and ensures the couplings of χ_d to be renormalizable.[1] The
ual field-theoretic definition of the S matrix is not unitary
χ_d does not obey the free equation. The interactions of χ_d
ll contribute an extra amplitude and upset unitarity.[2]
the definition of the Lagrangian of the interacting fields
have two independent free parameters ζ, and ξ; the pro-
gators and the vertices do depend on these parameters. The
ameter ζ corresponds to a gauge change in the photon pro-
ator; and ξ is the ratio of the square of the mass of the
rged intermediate boson to that of its scalar counterpart.
limit $\xi \to 0$ corresponds to infinity heavy scalar quanta
a manifestly unitary S-matrix.

In this paper, we consider a unified theory in which there is massive neutral vector field $Z_\mu(x)$ in addition to $W^\pm_\mu(x)$ and $A_\mu(x)$ discussed in the previous paper.[1] In this theory, we need not assume heavy leptons and all the leptons are experimentally observed. Aside from the structure of coupling in the Lagrangian, this theory differs from Weinberg theory[3] in two important aspects: (i) This theory has no Higgs mesons and is asymptotically free; (ii) The masses are free parameters, e.g., the Z_μ mass is independent of the W_μ mass. Consequently, we can construct a unified theory of leptons in which the weak and the electromagnetic forces are characterized by one and the same coupling constant without contradicting experimental data on neutral currents. This is to be contrasted with the Weinberg theory of leptons, in which there are five different coupling constants for the interactions of physical particles. (See Section 5)

2. Lagrangian and Field Equations

Let us consider a physical system involving $W^\pm_\alpha(x)$, $A_\alpha(x)$, $Z_\alpha(x)$, $\mu(x)$ and $\nu(x)$ $(\equiv \nu_\mu(x))$ in interactions. The Lagrangian of the system is[3,1]

$$\mathscr{L} = \mathscr{L}_b + \mathscr{L}_\ell + \mathscr{L}_L \ , \tag{1}$$

$$\mathcal{L}_b = -\frac{1}{2} W^+_{\alpha\beta} W^{-\alpha\beta} + M^2 W^+_\alpha W^{-\alpha} - \frac{1}{4} \left| F_{\alpha\beta} - ie(W^+_\alpha W^-_\beta - W^+_\beta W^-_\alpha) \right|^2 - \frac{1}{4} \left| Z_{\alpha\beta} + iG\cos^2\theta (W^+_\alpha W^-_\beta - W^+_\beta W^+_\alpha) \right|^2 + \frac{1}{2} M_Z^2 Z_\alpha^2 , \quad (2)$$

$$\mathcal{L}_\ell = \bar{\nu}_L i\partial_\lambda \gamma^\lambda \nu_L + \bar{\mu}(i\gamma^\alpha \partial_\alpha - m_\mu)\mu - e\bar{\mu}\gamma^\lambda \mu A_\lambda - (2^{-1/2} G\cos\theta)(\bar{\nu}_L \gamma^\lambda \mu W^+_\lambda + \bar{\mu}\gamma^\lambda \nu_L W^-_\lambda) - \frac{1}{2} G \bar{\nu}_L \gamma^\lambda \nu_L Z_\lambda + G\bar{\mu}\gamma^\lambda [\frac{1}{2}\cos 2\theta (1+\gamma_5)/2 - \sin^2\theta (1-\gamma_5)/2]\mu Z_\lambda , \quad (3)$$

$$\mathcal{L}_L = M\chi_0 \partial_\lambda A^\lambda + \frac{1}{2}\zeta^{-1} M^2 \chi_0^2 + M_Z \chi_Z \partial_\alpha Z^\alpha + \frac{1}{2}\xi^{-1} M_Z^2 \chi_Z^2 + M\chi^-(\partial_\alpha - ieA_\alpha + iG\cos^2\theta\, Z_\alpha)W^{+\alpha} + M\chi^+(\partial_\alpha + ieA_\alpha - iG\cos^2\theta\, Z_\alpha)W^{-\alpha} + \xi^{-1} M^2 \chi^+ \chi^- , \quad (4)$$

where

$$W^+_{\alpha\beta} = \partial_\alpha W^+_\beta - \partial_\beta W^+_\alpha + ie(W^+_\alpha A_\beta - W^+_\beta A_\alpha) - iG\cos^2\theta (W^+_\alpha Z_\beta - W^+_\beta Z_\alpha),$$

$$F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha , \qquad Z_{\alpha\beta} = \partial_\alpha Z_\beta - \partial_\beta Z_\alpha , \quad (5)$$

$$\nu_{R,L} = \frac{1}{2}(1 \mp \gamma_5)\nu . \qquad e = -G\sin\theta\cos\theta .$$

The parameters e, G, ζ, ξ, and θ are real. Note that M_Z is independent of M. The Lagrangian $\mathcal{L}_L$ in (1) could be replaced by[1,2]

$$\mathcal{L}_L^\xi = -\frac{1}{2}\zeta(\partial_\lambda A^\lambda)^2 - \frac{1}{2}\xi(\partial_\alpha Z^\alpha)^2 - \xi \left| (\partial_\alpha - ieA_\alpha + iG\cos^2\theta\, Z_\alpha) W^{+\alpha} \right| \quad (6)$$

The presence of $\mathcal{L}_L$ in (1) implies that the massive 4-vector fields contain both the physical spin 1 particles and the unphysical spin 0 particles having negative metric. By appealing to the action principle, the Lagrangian (1) gives the variational equations

$$-\partial^\alpha W^+_{\alpha\beta} - M^2 W^+_\beta + M(\partial_\beta - ieA_\beta + iG\cos^2\theta\, Z_\beta)\chi^+ + W^+_{\alpha\beta}(ieA^\alpha - iG\cos^2\theta\, Z^\alpha) - ieW^{+\alpha}[F_{\alpha\beta} - ie(W^+_\alpha W^-_\beta - W^+_\beta W^-_\alpha)] + iG\cos^2\theta\, W^{+\alpha}[Z_{\alpha\beta} + iG\cos^2\theta (W^+_\alpha W^-_\beta - W^+_\beta W^-_\alpha)] + 2^{-1/2} G\cos\theta\, \bar{\mu}\gamma_\beta \nu_L = 0 , \quad (7)$$

$$(\partial_\alpha - ieA_\alpha + iG\cos^2\theta\, Z_\alpha)\, W^{+\alpha} + \xi^{-1} M\chi^+ = 0 , \quad (8)$$

$$-\partial^\alpha [F_{\alpha\beta} - ie(W^+_\alpha W^-_\beta - W^+_\beta W^-_\alpha)] + M(\partial_\beta \chi_0 + ieW^+_\beta \chi^- - ieW^-_\beta \chi^+) + ieW^{+\alpha} W^-_{\alpha\beta} - ieW^{-\alpha} W^+_{\alpha\beta} + e\bar{\mu}\gamma_\beta \mu = 0 , \quad (9)$$

$$\partial_\lambda A^\lambda + \zeta^{-1} M\chi_0 = 0 , \quad (10)$$

$$-\partial^\alpha [Z_{\alpha\beta} + iG\cos^2\theta (W^+_\alpha W^-_\beta - W^+_\beta W^-_\alpha)] - M_Z^2 Z_\beta + M_Z \partial_\beta \chi_Z - iMG\cos^2\theta (W^+_\beta \chi^- - W^-_\beta \chi^+) - iG\cos^2\theta (W^{+\alpha} W^-_{\alpha\beta} - W^{-\alpha} W^+_{\alpha\beta}) + \frac{1}{2} G\bar{\nu}_L \gamma_\beta \nu_L - G\bar{\mu}\gamma_\beta [\frac{1}{2}\cos 2\theta (1+\gamma_5)/2 - \sin^2\theta (1-\gamma_5/2]\mu = 0. \quad (11)$$

$$\partial^\alpha Z_\alpha + \xi^{-1} M_Z \chi_Z = 0 \; . \qquad (12)$$

The divergence of (9) together with (10), (7) and its adjoint give us

$$\Box \chi_o = 0 \; . \qquad (13)$$

Similarly, we have

$$(\Box + \eta^{-1} M_Z^2)\ _Z + \frac{1}{2} iGm_\mu M_Z^{-1}\ \bar{\mu}\gamma_5\mu = 0 \qquad (14)$$

$$(\Box + \xi^{-1} M^2)\chi^+ - 2ieA_\lambda \partial^\lambda \chi^+ + iG\cos^2\theta\ \chi^+ \partial^\alpha Z_\alpha + 2iG\cos^2\theta\ Z_\alpha \partial^\alpha \chi^+$$
$$+ (ieA_\lambda - iG\cos^2\theta\ Z_\lambda)^2 \chi^+ - G^2\cos^2\theta (W^{+\alpha} W_\alpha^+ \chi^- + W^{+\alpha} W_\alpha^- \chi^+)$$
$$- iG\cos^2\theta\ W^{+\alpha} \partial_\alpha \chi_Z + ig\cos^2\theta\ M_Z^2 M^{-1}\ W^{+\alpha} Z_\alpha$$
$$+ 2^{-\frac{1}{2}} iG\cos\theta\ m\ M^{-1}\ \bar{\mu}\nu_L = 0 \qquad (15)$$

by a straightforward but tedious calculation. In equation (15) we have set $\chi_o = 0$, which is consistent with (13). The equations (13) - (15) for the Lagrange multiplier fields are important in our approach. We see that all the source terms correspond to renormalizable coupling. The significance of this fact will be discussed in Section 4.

We may quantize the theory based on (1) according to the usual canonical quantization procedure. We find that χ_o is a zero-norm field while χ_Z and $\chi^\pm$ are negative metric fields.[2] Thus, they are unphysical. Since χ_o is a free field, it has no physical effect. The fields χ_Z and $\chi^\pm$ are not free. So, the problem is how to treat their interactions which upset unitarity. This will be discussed later. For the free fields $W_\mu^\pm$, Z_μ and A_μ, we have the following general commutators:

$$[W_\alpha^+(x), W_\beta^-(y)] = -i(g_{\alpha\beta} + M^{-2}\partial_\alpha^x \partial_\beta^x)\Delta(x-y, M^2)$$
$$+ iM^{-2}\ \partial_\alpha^x \partial_\beta^x \Delta(x-y, \xi^{-1}M^2)\ , \quad \partial_\alpha^x = \partial/\partial x^\alpha\ ,$$

$$[Z_\alpha(x), Z_\beta(y)] = -i(g_{\alpha\beta} + M_Z^{-2}\partial_\alpha^x \partial_\beta^x)\Delta(x-y, M_Z^2)$$
$$+ iM_Z^{-2}\ \partial_\alpha^x \partial_\beta^y \Delta(x-y, \xi^{-1}M^2)\ ,$$

$$[A_\alpha(x), A_\beta(y)] = -ig_{\alpha\beta}\Delta(x-y, 0) + i(1-\zeta^{-1})\partial_\alpha^x \partial_\beta^y E(x-y)\ ,$$

$$\Delta(x,\rho^2) = -i(2\pi)^{-3}\int d^4p\,\varepsilon(p_o)\delta(p^2-\rho^2)e^{-ip\cdot x}, \quad \rho^2 = M^2, M_Z, \text{ etc.}$$

$$E(x) = -\frac{1}{8\pi}\varepsilon(x_o)\theta(x^2) = -i(2\pi)^{-3}\int d^4p\,\varepsilon(p_o)\delta'(p^2)e^{-ip\cdot x},$$

$$\Box E(x) = \Delta(x,0)\ .$$

3. The Unitary S Matrix

Let us consider the physical processes in which the initial and final states involve only physical particles. According to the usual field-theoretic definition, the amplitudes for these processes could be expressed in terms of the path integral[1,2]

$$A = \int \exp\{i \int d^4x(\mathcal{L} + \mathcal{L}_s)\} d[\phi, \bar{\chi}], \tag{16}$$

$\mathcal{L}_s$ = external source terms,

$[\phi] \equiv [W_\mu^\pm, Z_\nu, A_\lambda, \mu, \bar{\mu}, \nu, \bar{\nu}]$,

$[\bar{\chi}] \equiv [\chi_o, \chi_Z, \chi^\pm]$,

with external physical particles. Integrating over $[\bar{\chi}]$, we have

$$A = \int \exp\{i \int d^4x(\mathcal{L}_b + \mathcal{L}_\ell + \mathcal{L}_L^\xi + \mathcal{L}_s)\ d[\phi] , \tag{17}$$

where $\mathcal{L}_L^\xi$ is given by (6). The amplitude (16) or (17) is not unitary because the Lagrangian involves the unphysical spin 0 part of the massive 4-vector fields which could be produced in the intermediate states and contribute unwanted amplitudes.

One way to restore unitarity of the amplitude (17) is to take the limit $\xi \to 0$. In this limit, the masses of the scalar quanta associated with the 4-vector field $W_\mu^\pm$ and Z_μ become infinite. These unphysical particles cannot be present in initial and final state. Nor can it contribute in the intermediate states of the physical processes. The unphysical particles associated with A_μ do not, as usual, affect physics because they are free particles. Therefore, the amplitude (17) is unitary for arbitrary gauge parameter ζ in this limit.

4. Renormalizability and Asymptotic Freedom

The propagators for the free fields $W_\mu^\pm$, Z_μ and A_μ are respectively given by

$$\int d^4x\ e^{ik\cdot x} <0|T(W_\alpha^+(x)W_\beta^-(0))|0> = \frac{-i(g_{\alpha\beta} - M^{-2}k_\alpha k_\beta)}{k^2 - M^2} - \frac{ik_\alpha k_\beta}{M^2(k^2 - \xi^{-1}M^2)}$$

$$= -i[g_{\alpha\beta} - (1-\xi^{-1})k_\alpha k_\beta(k^2 - \xi^{-1}M^2)^{-1}]/(k^2 - M^2) \tag{18}$$

$$\int d^4x\ e^{ik\cdot x} <0|T(Z_\alpha(x)Z_\beta(0))|0> = \frac{-i[g_{\alpha\beta} - (1-\xi^{-1})k_\alpha k_\beta(k^2 - \xi^{-1}M_Z^2)^{-1}]}{(k^2 - M_Z^2)} \tag{19}$$

$$\int d^4x\ e^{ik\cdot x} <0|T(A_\alpha(x)A_\beta(0))|0> = -i[g_{\alpha\beta} - (1-\zeta^{-1})k_\alpha k_\beta k^{-2}]/k^2 . \tag{20}$$

From the Lagrangian (1) and the propagators (18) - (20), we see that the theory is renormalizable (for $\xi > 0$) by standard power counting.

In Section 2, we have shown that the couplings of the scalar particles associated with massive 4-vector fields $W_\mu^\pm$ and Z_μ are of renormlizable type. This is precisely what happens in the renormalizable gauge theories.[2] If the Lagrangian does not have local isospin gauge symmetry (which may be broken by mass terms), then some source terms in the equations for the Lagrangian multiplier fields will not correspond to renormalizable coupling. The situation occurs in the ξ-limiting formalism for the massive charge vector mesons.[1] This indicates that the interaction of the spin zero particle associated with the 4-vector massive field may not be renormalizable. In other words, the parameter ξ (or the mass of the spin zero particle) may not be renormalizable.

Using these rules and n-dimensional regularizations, one an show that the theory is asymptotically free. One has nly to calculate the logarithmically divergent terms of order ^{2} in Z_3 (the self-energy diagram of the photon) because the enormlization - group parameter $\beta(e)$ is[4]

$$\beta(e) = -e \frac{\partial}{\partial \ln(\frac{\Lambda}{M})} \frac{Z_3^{1/2} Z_2}{Z_1} \qquad (21)$$

ere Z_2 is equal to Z_1 due to Abelian gauge invariance. In e absence of the leptons, one finds that

$$Z_3 = 1 + 7\left(\frac{e}{4\pi}\right)^2 \ln \frac{\Lambda^2}{M^2} \qquad (22)$$

where the limit $\xi \to 0$ is taken after the integration is carried out. The ξ-dependent parts which become infinite when $\xi \to 0$ are absorbed into counter terms. (cf. K. T. Mahanthappa, Ref. 3). The result (22) implies $\beta(e) \leq 0$ and the theory is asymptotically free. Asymptotic freedom can be maintained provided not too many fermions are introduced, for example, no more than 10 iso-doublets in SU(2).[4]

5. Symmetry of the Lagrangian

The renormalizability of the theory comes about because the dynamics of interactions and symmetry are intimately related. To see the symmetry of the Lagrangian (1), we write $\mathscr{L}_b$ and $\mathscr{L}_\ell$ given by (2) and (3) in the following form:

$$\mathscr{L}_b + \mathscr{L}_\ell = \mathscr{L}_W - m_\mu \bar{\mu}\mu - \frac{1}{2} M^2[(A^1_\alpha)^2 + (A^2_\alpha)^2]$$
$$- \frac{1}{2} M_Z^2 (g^2 + g'^2)^{-\frac{1}{2}} (gA^3_\alpha + g'B_\alpha)^2 , \qquad (23)$$

$$\mathscr{L}_W = - \frac{1}{4} (\partial_\alpha \vec{A}_\beta - \partial_\alpha \vec{A}_\beta + g\vec{A}_\alpha x \vec{A}_\beta)^2 - \frac{1}{4}(\partial_\alpha B_\beta - \partial_\alpha B_\beta)^2$$
$$+ \bar{R}\gamma^\alpha (i\partial_\alpha + g'B_\alpha)R + \bar{L}\gamma^\alpha (i\partial_\alpha + \frac{1}{2} g\vec{\tau}\cdot\vec{A}_\alpha + \frac{1}{2} g'B_\alpha)L, \qquad (24)$$

where $g = G \cos \theta$, $g' = - eG/g$, and

$$A^1_\alpha = 2^{-\frac{1}{2}}(W^+_\alpha + W^-_\alpha), \qquad A^2_\alpha = i\, 2^{-\frac{1}{2}}(W^+_\alpha - W^-_\alpha)$$
$$A^3_\alpha = (g^2 + g'^2)^{-\frac{1}{2}}(gZ_\alpha - g'A_\alpha), \quad B_\alpha = (g^2 + g'^2)^{-\frac{1}{2}}(g'Z_\alpha + gA_\alpha) \qquad (25)$$
$$L = \frac{1}{2}(1 + \gamma_5)\binom{\nu}{\mu}, \qquad R = \frac{1}{2}(1 - \gamma_5)\mu .$$

We see that $\mathscr{L}_W$ is invariant under the local isospin gauge transformation. The isospin symmetry of $\mathscr{L}_\ell + \mathscr{L}_b$ is violated only by the mass terms which involve a constant factor and two vector fields, e.g., $M^2(A^1_\alpha)^2$, $M_Z^2(A^3_\alpha)B^\alpha$, etc. The Lagrangian (23) differs from that in the Weinberg theory in that we do not have the Lagrangian for the scalar isodoublet meson ϕ (consisting of ϕ^+ and ϕ^0):

$$\mathscr{L}_\phi = |\partial_\lambda \phi - i\frac{1}{2}g\, \vec{\tau}\cdot\vec{A}_\lambda \phi + i\, \frac{1}{2}g'\, B_\lambda \phi|^2 - M_1^2 \phi^\dagger \phi + h(\phi^\dagger \phi)^2$$
$$- G_\mu (\bar{L}\phi R + \bar{R}\phi^\dagger L). \qquad (26)$$

In the Weinberg-Salam type models such a Lagrangian term serves to induce spontaneous symmetry breaking to generate mass terms corresponding to the ones we have in (23).

In the Lagrange multiplier formalism, the propagators for massive vector field have a nice asymptotic behavior k^{-2} as $k\to\infty$. So the couplings in the Lagrangian (23) are renormalizable by standard power counting. Now the renormalizability of the theory completely depends on the couplings of the Lagrange multiplier fields, namely, if the couplings of the Lagrange multiplier fields are renormalizable, then the whole theory is renormalizable. These couplings cannot be seen directly from the Lagrangian (23) or (1). They can be seen from the effective Lagrangian corresponding to the field equations of the Lagrange multiplier. In deriving these field equations, we see fantastic and complete cancellation of the non-renormalizable couplings of the Lagrange multiplier, just like the cancellation which happens in a gauge invariant Lagrangian (without mass terms). This indicates that the dynamics of interactions interlocked with the "gauge symmetry" is essentially unaltered by the mass terms which break the symmetry no matter where the mass terms come from. Thus, it is not surprising that the complete cancellation of the nonrenormalizable couplings of the Lagrange multiplier field also occur in the case where the vector boson masses are generated from spontaneously broken gauge symmetry due to $\mathcal{L}_\phi$ given by (26).

6. Genuine Unification and Universal Couplings

The Lagrangian (1) has two different coupling constants for the interactions of physical particles. This has the advantage of easy adjustment with experimental data. However, this does not seem to be in harmony with the theoretical spirit of unifying the weak and the electromagnetic interactions. In the genuine unified theory, the weak and the electromagnetic forces are physically the same thing, namely, they are characterized by one and the same coupling constant. The observed dissimilarities between weak and electromagnetic interactions are purely due to mass differences between the vector bosons. The reason for such a huge difference ($\sim$ 50 GeV) is not clear so far. It has been suggested that the heavy mass of the intermediate vector bosons might come from their strong cubic self-interactions.[5] Yet, experiment seems to suggest a different picture. All observed particles, which participate in strong interactions, have masses of order 1 GeV. The ratios between these masses range roughly from 1 to 10. On the other hand,

the electron and the muon have almost identical interactions and they do not have strong interactions. They have, however, quite different masses: $m_\mu/m_e \gtrsim 200$.[6] Similarly, the vector fields $W_\mu^\pm$, Z_μ and A_μ may have the same interaction but quite different masses. And this may well be because they do not have strong interactions.

In fact, universality of electromagnetism has meant all along that the interaction part of the Lagrangian involved a universal coupling constant for as diverse particles as the electron, the muon, the pion, and the proton. There is no requirement that the particles have to have the same mass or spin. When we refer to the electron-muon universality of their weak and electromagnetic interactions, we do not imply that they have the same mass; the chiral V-A interaction possess a high degree of symmetry which is not shared by the mass terms and hence by the total Hamiltonian. It is in this spirit that we develop our theory, and our work shows that the high degree of symmetry of the interaction Lagrangian has important dynamical consequences even when symmetry is violated by mass terms.

If one treats all the vector fields $W_\mu^\pm$, Z_μ and A_μ on equal footing dynamically, their couplings among themselves and to the leptons must be universal. This can be accomplished by setting $\theta = \pi/4$ in (1):[7]

$$
\begin{aligned}
\mathscr{L} = & -\frac{1}{2} W^{+}_{\alpha\beta} W^{-\alpha\beta} + M^2 W^{+}_{\alpha} W^{-\alpha} - \frac{1}{4} \left| F_{\alpha\beta} - ie(W^{+}_{\alpha} W^{-}_{\beta} - W^{+}_{\beta} W^{-}_{\alpha}) \right|^2 \\
& - \frac{1}{4} \left| Z_{\alpha\beta} + ie(W^{+}_{\alpha} W^{-}_{\beta} - W^{+}_{\beta} W^{-}_{\alpha}) \right|^2 + \frac{1}{2} M_Z^2 Z_\alpha^2 \\
& + \bar{\nu}_L i \partial_\alpha \gamma^\alpha \nu_L + \bar{\mu}(i\gamma^\alpha \partial_\alpha - m_\mu)\mu - e\bar{\mu}\gamma^\alpha \mu A_\alpha \\
& - e(\bar{\nu}_L \gamma^\lambda \mu W^{+}_{\lambda} + \bar{\mu}\gamma^\lambda \nu_L W^{-}_{\lambda}) - e\bar{\nu}_L \gamma^\alpha \nu_L Z_\alpha - e\bar{\mu}_R \gamma^\alpha \mu_R Z_\alpha \\
& + M\chi_o \partial_\lambda A^\lambda + \frac{1}{2}\xi^{-1} M^2 \chi_o^2 + M_Z \chi_Z \partial_\alpha Z^\alpha + \frac{1}{2}\xi^{-1} M_Z^2 \chi_Z^2 + \xi^{-1} M^2 \chi^+ \chi^- \\
& + M\chi^-(\partial_\alpha - ieA_\alpha + ieZ_\alpha)W^{+\alpha} + M\chi^+(\partial_\alpha + ieA_\alpha - ieZ_\alpha)W^{-\alpha},
\end{aligned} \tag{27}
$$

$$
W^{+}_{\alpha\beta} = \partial_\alpha W^{+}_{\beta} - \partial_\beta W^{+}_{\alpha} + ie(W^{+}_{\alpha} A_\beta - W^{+}_{\beta} A_\alpha) - ie(W^{+}_{\alpha} A_\beta - W^{+}_{\beta} Z_\alpha) .
$$

The Fermi coupling constant G_F for the usual weak interaction is given by

$$
2^{-\frac{1}{2}} G_F = e^2/(4M^2) , \tag{28}
$$

and, therefore, we have

$$
M = 52.4 \text{ GeV} . \tag{29}
$$

This is to be contrasted with the model proposed by Weinberg where the charged weak boson mass is not uniquely predicted but need only satisfy the inequality

$$
M > 32.7 \text{ GeV}.
$$

In the model of Weinberg, the Z boson's mass is also not unique

but is related to the charged boson mass. In contrast, in our theory the Z boson mass is arbitrary. We must therefore deduce it from observations on neutral currents. The data[8] on neutral lepton currents indicate that

$$
M_Z \gtrsim 65.6 \text{ GeV} . \tag{30}
$$

7. Conclusion

The simplicity and elegance of the present unified theory can be best seen by comparing the Lagrangian (27) and that of the Weinberg theory of the leptons. In the Weinberg theory, aside from the vector field $W^{\pm}_{\mu}$, Z_μ, A_μ and the leptons, one also has to introduce scalar field ϕ with the Lagrangian $\mathscr{L}_\phi$ given by (39). Moreover, the dynamics of interactions among physical particles in Weinberg's theory are characterized by <u>five</u> different coupling constants, i.e., g, g', G_e, G_μ, h.[3] On the other hand, the present unified theory involves only the vector fields $W^{\pm}_{\mu}$, Z_μ, A_μ and the leptons and, furthermore, their interactions are characterized by <u>one</u> single coupling constant. In view of this, the present theory is much closer to the spirit of unifying apparently different forces in nature.

Historically, the idea of unifying weak and electromagnetic interactions has been discussed previously by Schwinger,[9] Glashow,[10] Salam and Ward,[11] etc. The idea of unifying CP-conserving and CP-violating weak processes has been discussed by

Nishijima and Swank.[12] A more systematic attempt has been made by Okubo,[13] and by Marshak and others[14] on the basis of a Yang-Mills Lagrangian.[15] These ideas are more in line with the present unified theory because one always desires to postulate fewer field and coupling strengths to explain more phenomena.

The above discussions show that a renormalizable and asymptotically free theory can be constructed involving self-coupled massive vector mesons. All these come about because the dynamics of the Lagrangian is interlocked with gauge symmetry which is violated only by mass terms.[16] Unitarity is recovered by the ξ-limiting procedure.[17,18] In this limit, the characteristics related to renormalizability and unitarity in this theory are the same as those in the Weinberg unified theory with spontaneously broken gauge symmetry. We believe that the present theory is renormalizable and unitary in the same sense as the usual gauge theories. Furthermore, the present theory is much simpler than the Weinberg theory of leptons and possesses the desirable qualities of simplicity, genuine unification and asymptotic freedom.

We are pleased to thank D. A. Dicus and N. G. Deshpande for discussion. In particular, we indebted to Duane Dicus for communicating his result of the one-loop photon self-energy calculation (for the model discussed in Ref. 1.) to us.

References

1. J. P. Hsu and E. C. G. Sudarshan, preprint, CPT 202 (1974).
2. J. P. Hsu, Phys. Rev. D8, 2609 (1973); ibid. (Feb. 1974); J. P. Hsu and E. C. G. Sudarshan, ibid. (March, 1974).
3. S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); 27, 1688 (1971). A. Salam, Proceedings of the Eighth Nobel Symposium (John Wiley and Sons, N.Y., 1968). For comprehensive review, see K. T. Mahanthappa, "Lectures on Spontaneously Broken Gauge Theories of Weak and Electromagnetic Interactions," CPT preprint (1973).
4. See, for example, D. J. Gross and F. Wilczek, Phys. Rev. Letters 30, 1343(1973) and Phys. Rev. D8, 3663(1973); H. D. Politzer, Phys. Rev. Letters, 1346(1973); J. P. Hsu, CPT Preprint 219(1974); N. G. Deshpande, D. A. Dicus, J. P. Hsu and E. C. G. Sudarshan, Preprint (1974). For discussion of probable reason for asymptotic freedom, see Ref. 1.
5. C. Ryan, S. Okubo and R. E. Marshak, Nuovo Cimento 34, 753 (1964); S. V. Pepper, C. Ryan, S. Okubo and R. E. Marshak, Phys. Rev. 137, B 1259 (1965).
6. The possibility that the μ-e mass difference has its origin in symmetry rather than in dynamics of interactions has been suggested. See, for example, K. Tennakone and S. Pakvasa,

Phys. Rev. Lett. 27, 757 (1971); J.P. Hsu, Phys. Rev. D5, 1161 (1972).

7. The same situation happens in the model considered in Ref. 1.
8. D. H. Perkins, Fifth Hawaii Topical Conference in Particle Physics, August 8 - 21, 1973 (to be published). We use the new result (Trieste, 1973): $\sin^2\theta < 0.35$.
9. J. Schwinger, Ann. Phys. (N.Y.) 2, 407 (1957).
10. S. L. Glashow, Nucl. Phys. 22, 579 (1961).
11. A. Salam and J. Ward, Phys. Letters 13, 168 (1964)
12. K. Nishijima and L. J. Swank, Phys. Rev. 146, 1161 (1966).
13. S. Okubo, Nuovo Cimento 491 (1968); Ann. Phys. (N.Y.) 49, 219 (1968).
14. R. E. Marshak, R. Mohapatra, S. Okubo and J. Rao, Nucl. Phys. B11, 253 (1969); R. E. Marshak, Y. W. Yang and J. S. Rao, Phys. Rev. D3, 1640 (1971).
15. C. N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1954).
16. J. P. Hsu, CPT Preprints 219 and 221(1974).
17. T. D. Lee and C. N. Yang, Phys. Rev. 128, 885(1962); see also K. T. Mahanthappa, Ref. 3.
18. If the amplitudes due to the source terms in the equations (14) and (15) could be removed from the theory by some Lagrangian involving fictitious particles, the theory will be unitary for any $\xi > 0$. However, because of the structure of our Lagrangian (1), this cannot be done in the conventional way. See, for example, J. P. Hsu and E. C. G.

Sudarshan, "Gauge Independence and Unitarity in the Weinberg Theory" CPT Preprint (1974) and Ref. 2.