Lattice QCD and High-Intensity Flavor Physics

(USQCD Collaboration)

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The copious production of heavy quarks (bottom, charm and strange) promises to shed light on particle interactions in a way complementary to the LHC. In many cases, interpretation of experiments relies on hadronic matrix elements that can be computed with lattice QCD. We review recent successes of lattice QCD and outline the role these calculations play, both in determining parameters of the Standard Model and in establishing new-physics effects. We include an assesment of the current status of especially important flavor-physics observables and forecast how errors will diminish during the current decade.

I. INTRODUCTION

During the past decade, lattice QCD has made substantial progress in several areas that influence particle physics, nuclear physics, and astrophysics. Once enough computing and algorithmic power became available to treat virtual quark-antiquark pairs (the "sea" quarks) realistically, the results of lattice-QCD calculations rapidly reproduced a wide variety of hadron properties [1]. The same techniques then enabled genuine predictions of D meson semileptonic form factors, D- and D_s-meson leptonic-decay constants, and the mass of the B_c meson [2]. Lattice QCD now plays an important role in quark flavor physics, yielding indispensible results for neutral meson mixing and leptonic decay rates, and important results for semileptonic form factors [3, 4]. These results not only constrain the Cabibbo-Kobayashi-Maskawa (CKM) matrix but also enable indirect searches for new particles.

The success of lattice QCD is not confined to flavor physics alone. The nucleon mass, one of the original objectives, has been computed with a precision of about 2% [5]. Nambu's ideas of spontaneous chiral symmetry breaking, once strong beliefs, have been verified via direct calculation from the QCD Lagrangian [6]. Connected to these developments are the only *ab initio* determinations of the light-quark masses [7]: the up-, down-, and strange-quark masses turn out to be small—about four, nine, and 180 times the electron mass, respectively. Lattice QCD meanwhile provides the most accurate determinations of the strong coupling α_s [8] and competitive determinations of the bottom- and charm-quark masses. These results connect the QCD probed in high-energy processes with the QCD description of hadrons.

With matrix elements from flavor physics and the nucleon mass under control, a next step is to compute nucleon matrix elements [9]. These are helpful for interpreting experiments on the neutron electric dipole moment, nucleon β decay, and nucleon structure, as well as planning searches for proton decay. Another recent development is the calculation of virtual hadron properties, which influence electroweak parameters. One example is the evolution of QED's fine structure constant from electronic to Z-pole scales. More prominent for the intensity frontier are related calculations of hadronic contributions to the muon's anomalous magnetic moment [10, 11].

There are many other lattice-QCD calculations that are beyond the scope of this document, which we shall mention only briefly. Together with the CKM matrix, the quark masses and α_s constrain speculation about unification of the forces and other physics beyond the reach of accelerators. Calculations of the strangeness content of the nucleon are needed to understand dark-matter detection experiments [12, 13]. Studies of the QCD phase transition with lattice QCD have shown that the quark masses, though small, are just large enough to make the transition a crossover [14, 15]. Previously, research on the early universe assumed the transition was of first order, with phenomena like bubbles of hadrons; we now know that the universe did not cool this way. Calculations of hadron-hadron interactions help us understand the physics of neutron stars, particularly whether neutrons could dissociate into $K\Lambda$ pairs [16]. Lattice gauge theories varying the number of colors and matter content are shedding light on the dynamics of technicolor models of electroweak symmetry breaking [17].

The rest of this document is organized as follows. Section II reviews in detail how lattice QCD plays a key role in heavy-quark physics, including the strange quark. We address both the determination of the CKM matrix and the search for new phenomena. In Sec. III, we present forecasts of future precision that we expect to obtain in flavor physics, and the assumptions underlying the forecast. We conclude with some remarks of the broader role of lattice QCD at the intensity frontier in Sec. IV.

For similar information on nucleon properties and on muon $g-2$, please consult documents submitted to the Nucleons/Nuclei/Atoms and Charged Lepton WGs, respectively.

II. LATTICE QCD AND QUARK FLAVOR PHYSICS

The US lattice-QCD community has a well-established and successful program to calculate the weak matrix elements needed to obtain the elements and phase of the CKM matrix. See Refs. [3, 4, 18–20] for further details. We are now expanding our program to meet the needs of upcoming intensity-frontier experiments such as Belle II, Super-B, NA62, KOTO, the Project X kaon program, and mid-term experiments at Fermilab leading towards Project X. Below we discuss several opportunities where we expect lattice-QCD calculations to play a key role in searches for (and possibly discovery of) new physics in the quark-flavor sector.

A. CKM physics

The elements of the CKM quark-mixing matrix are fundamental parameters of the Standard Model (SM) and are, thus, key to understanding flavor structure within and beyond the SM. Further, the CKM matrix elements are parametric inputs to SM predictions for many flavor-changing processes such as neutral kaon mixing and charged and neutral $K \to \pi\nu\bar{\nu}$ decays; hence they must be known precisely to search for new physics in these channels.

For the past decade, the flavor factories have been pouring out data to pin down the values of the CKM matrix elements and the CP-violating phase. Lattice-QCD calculations of hadronic weak matrix elements are needed, however, to interpret many of their results. Quantities with a single hadron in the initial state and at most one hadron in the final state, where both hadrons are stable (or at least narrow and far from threshold), are most readily calculated with high precision in lattice QCD [1]. By analogy with usage elsewhere in flavor physics, these processes are sometimes called "gold-plated." All CKM matrix elements except $|V_{tb}|$ can be determined with such observables, as shown in Fig. 1. Because they are easiest to compute numerically with standard lattice methods, they are among the most well-studied lattice-QCD quantities. Lattice-QCD calculations currently allow determinations of $|V_{us}|$, $|V_{ub}|$, $|V_{cd}|$, $|V_{cs}|$, and $|V_{cb}|$ that are competitive with the world's best. With lattice QCD, moreover, we have a clear set of tools for reducing uncertainties steadily over the next several years.

Given these results, one can test the unitarity of the first and second rows of the CKM matrix; deviations would indicate the presence of physics beyond-the-Standard Model (BSM). Current lattice-QCD and experimental results are consistent with first-row unitarity at the sub-percent level and with second-row unitarity at the percent-level. This stringent test of first-row unitarity was enabled by precise lattice-QCD determinations of the ratio of leptonic decay constants f_K/f_π and of the $K \to \pi\ell\nu$ semileptonic form factor $f_+^{K\pi}(0)$, and is evidence of the worldwide lattice-QCD community's successful kaon physics program.

Another standard way of searching for new physics in the flavor sector is by overconstraining the angles and sides of the CKM unitarity triangle, as shown in Fig. 2. New quark flavor-changing interactions or CP-violating phases would manifest themselves as apparent inconsistencies between measurements of the apex (ρ, η) that are predicted to be the same within the SM framework. Many constraints on the apex of the CKM unitarity triangle require lattice-QCD calculations of nonperturbative hadronic weak matrix elements.

Over the past decade lattice QCD and the flavor factories have established that the CKM paradigm of CP-violation describes experimental observations at the few-percent level. At the same time, however, experimental measurements and unquenched lattice-QCD calculations have revealed a $\sim 3\sigma$ tension in the CKM unitarity triangle [21–23], see Fig. 2. This may indicate the presence of a BSM source of CP-violation. Current measurements suggest that the most likely scenarios are that the new physics is either in $B \to \tau\nu$ decay or in B_d mixing [24]. Hence ruling out these hypotheses or establishing the presence of new physics with higher significance will require improved lattice-QCD calculations of the B_d and B_s mixing parameters and of the B^+-meson leptonic-decay constant f_B.

Improvements in lattice-QCD calculations of the neutral kaon mixing parameter B_K were crucial in unearthing this tension in the global unitarity triangle fit. In fact, B_K is now only the fourth-largest source of uncertainty in the unitarity-triangle constraint from ϵ_K, below parametric errors from CKM matrix elements and perturbative-QCD errors from the short-distance coefficients [25]. The width of the ϵ_K band now arises principally from the $\sim 10\%$ uncertainty in $|V_{cb}|^4$. Hence, improvements in lattice-QCD calculations of the $B \to D\ell\nu$ and $B \to D^*\ell\nu$ form factors are also needed to investigate this possible indication of new physics.

A more long-standing puzzle in the quark-flavor sector is the persistent 2σ tension between the determinations of $|V_{ub}|$ from inclusive and exclusive semileptonic B-meson decays. This situation has been further muddled by the recent experimental measurements of $\mathrm{BR}(B \to \tau\nu)$ by Belle and BaBar, which lead to a determination of $|V_{ub}|$ that disagrees with both. An elegant solution can be provided by the presence of a right-handed current with coupling $|V_{ub}^R|$. As illustrated in Fig. 3, an admixture of $\sim 15\%$ right-handed current can bring the three $|V_{ub}|$ determinations into agreement and can also reduce the tension between inclusive and exclusive determinations

$$\begin{pmatrix}
\boldsymbol{V_{ud}} & \boldsymbol{V_{us}} & \boldsymbol{V_{ub}} \\
\pi \to \ell\nu & K \to \ell\nu & B \to \ell\nu \\
 & K \to \pi\ell\nu & B \to \pi\ell\nu \\
\boldsymbol{V_{cd}} & \boldsymbol{V_{cs}} & \boldsymbol{V_{cb}} \\
D \to \ell\nu & D_s \to \ell\nu & B \to D\ell\nu \\
D \to \pi\ell\nu & D \to K\ell\nu & B \to D^*\ell\nu \\
\boldsymbol{V_{td}} & \boldsymbol{V_{ts}} & \boldsymbol{V_{tb}} \\
\langle B_d|\bar{B}_d\rangle & \langle B_s|\bar{B}_s\rangle &
\end{pmatrix}$$

FIG. 1: Processes that can be used to obtain each CKM matrix element. Neutral K^0-$\bar{K}^0$ mixing is also pertinent here, giving a constraint on the apex of the unitarity triangle (ρ, η). Lattice-QCD calculations for these processes are mature and can be improved steadily to reduce the theoretical uncertainty in the CKM matrix.

of $|V_{cb}|$ [26]. Establishing or ruling out the presence of such a right-handed current will require improved lattice-QCD calculations of the $B \to \pi\ell\nu$ form factor and the B-meson decay constant f_B, as well as new lattice-QCD calculations of the $B \to \rho\ell\nu$ form factors.

Although the determination of $|V_{ub}|$ is problematic, it is not the source of the tension in the global CKM unitarity trangle fit. Even when $|V_{ub}|$ is removed from the fit, the $\sim 3\sigma$ tension remains. If the "$|V_{ub}|$ puzzle" is resolved, however, the constraint on the CKM unitarity triangle from $|V_{ub}|$ can be immensely helpful in new-physics searches because near the apex it is roughly parallel to the constraints from $\sin 2\beta$ and ϵ_K.

B. Rare decays and other loop processes

Quark flavor-changing processes in which the leading-order SM contribution is at the 1-loop level are particularly good channels in which to search for new physics. First, this is because new-physics contributions may be easier to observe above the suppressed SM background. And second, loop processes are sensitive to physics at much higher scales than can be probed in high-p_T collider experiments, in some cases $\sim$ 1000 TeV.

For example, neutral kaon and B-meson mixing occur via box diagrams. Kaon mixing, in particular, is sensitive to many new-physics scenarios such as SUSY and Randall-Sundrum [27], and currently places the strongest constraints on the possible scale of new physics. The experimental observables ϵ_K, ΔM_d, and ΔM_s have all been measured to sub-percent accuracy, and their constraints on the unitarity triangle are limited, respectively, by the uncertainties in $|V_{cb}|^4$ and the SU$_f$(3)-breaking ratio $\xi \equiv f_{B_s}\sqrt{B_{B_s}}/f_{B_d}\sqrt{B_{B_d}}$. Hence improved lattice-QCD calculations of the $B \to D^{(*)}\ell\nu$ form factors and of the B_s- and B_d-mixing matrix elements are needed.

Rare kaon and B-meson decays may be suppressed in the SM for many reasons: 1) they can be loop-suppressed because they occur via flavor-changing neutral currents, 2) they can be helicity suppressed, and 3) they can be suppressed by CKM and color factors. Hence they are challenging to observe experimentally. LHCb, Belle II, and Super-B will improve measurements of (or discover) many rare $b \to s$ transitions. New particles can enter the loops and significantly modify the decay amplitudes, but the SM branching fractions for many of these processes are limited by theoretical uncertainties in the hadronic form factors. Lattice-QCD calculations of the form factors for the semileptonic decays $B \to K^{(*)}\ell^+\ell^-$, as well as for the radiative decay $B \to K^*\gamma$ are underway.

The "golden" rare kaon decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ enjoy great new-physics discovery potential, because the SM branching ratios are known to a precision unmatched by any other quark FCNC processes. Nevertheless, the branching ratios are still known only to $\sim$ 10% due to the parametric uncertainty in $|V_{cb}|^4$. The lowest-order SM contributions to $K \to \pi\nu\bar{\nu}$ are from QCD and EW penguin diagrams, so this channel is sensitive to many new-physics scenarios such as Little-Higgs Models, warped extra dimensions, and a fourth generation [27]. Furthermore, spectacular deviations from the SM predictions are possible in many of these new-physics models, as shown in Fig. 4.

The upcoming intensity-frontier experiments NA62 and KOTO will, respectively, improve the measurement of the $K^+ \to \pi^+\nu\bar{\nu}$ branching ratio and observe the $K_L \to \pi^0\nu\bar{\nu}$ for the first time. A mid-term initiative at Fermilab proposes to collect $\mathcal{O}(1000)$ events of $K^+ \to \pi^+\nu\bar{\nu}$. The Project X research program will follow with high-precision measurements of both $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ matching the expected theoretical errors of 1–3%. The combination of these two high-precision measurements is a very sensitive probe of BSM physics. To maximize the discovery impact of these experiments, however, improvements in the determination of $|V_{cb}|$ will be needed. As discussed in Sec. III, we expect to halve the error on $|V_{cb}|$ from $B \to D^{(*)}\ell\nu$ by 2014. With this precision, even a 30% deviation from the SM would provide 5σ evidence for new physics. In the longer term,

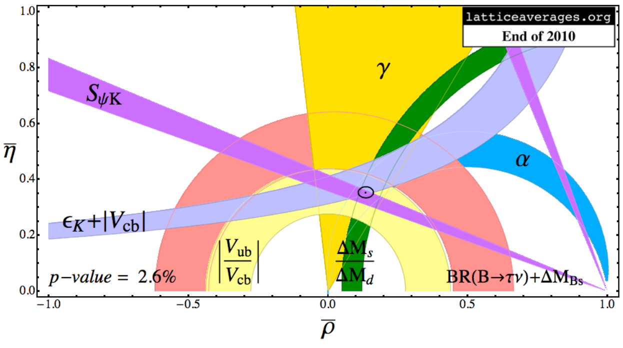

FIG. 2: Global fit of the CKM unitarity triangle [3]. The constraints labeled $\epsilon_K + |V_{cb}|$, $|V_{ub}/V_{cb}|$, $\Delta M_s/\Delta M_d$, and BR$(B \to \tau\nu) + \Delta M_{B_s}$ all require lattice-QCD inputs.

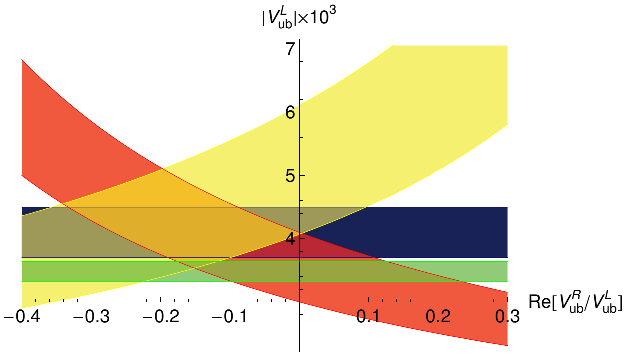

FIG. 3: $|V^L_{ub}|$ obtained from different processes as a function of the amount of right-handed current $|V^R_{ub}|$ [26]. Blue (dark horizontal band): inclusive $B \to X_u\ell\nu$ decays. Red (medium gray curve): $B \to \pi\ell\nu$. Yellow (light gray curve): $B \to \tau\nu$. Green (light gray horizontal band): CKM unitarity.

lattice-QCD calculations of the form factors will reduce the error in $|V_{cb}|$ to $\sim 0.5\%$, corresponding to an error in the SM branching ratio of 2%.

Finally, we note that the CP-violating decay $K_L \to \pi\pi$ is sensitive to some of the same penguin diagrams (and hence new physics) as $K \to \pi\nu\bar{\nu}$. Although ϵ'_K/ϵ_K has been measured experimentally to $\sim 10\%$, its utility for constraining new physics has been limited by the uncertainty of the corresponding hadronic weak matrix elements. Fortunately, significant recent progress should soon allow unquenched lattice-QCD calculations of ϵ'_K/ϵ_K with $\sim 20\%$ precision. With these projected improvements, combining the pattern of experimental results for $K \to \pi\nu\bar{\nu}$ and ϵ'_K/ϵ_K can help to distinguish between new-physics models [29]. Indeed, a very recent calculation [30] of the imaginary part of the $I = 2$, $K \to \pi\pi$ decay amplitude A_2, when combined with the experimental value of ϵ'_K/ϵ_K, determines the poorly known ratio $\mathrm{Im}(A_0)/\mathrm{Re}(A_0)$ to 10% accuracy, a quantity which itself makes an 8% contribution to ϵ_K.

Although we have only focused on the most important lattice-QCD calculations for quark-flavor physics in the next few years, we note that lattice-QCD can also provide supporting calculations to aid in many other experimental measurements. Even when they are not essential to interpret the experimental results, the information that they provide is often complementary to the more traditional methods. For example, lattice-QCD calculations of the ratio of form factors $f_0^{B_s\to D_s\ell\nu}(m_\pi^2)/f_0^{B\to D\ell\nu}(m_K^2)$ enable an independent determination of the fragmentation fraction f_d/f_s, which is needed to normalize the $B_s \to \mu^+\mu^-$ branching fraction at hadron colliders [31, 32]. Similarly, lattice-QCD calculations of nonleptonic $B \to DK(\pi)$ decays may allow a precise extraction of the CKM angle γ free from penguin contamination short of the full Super-B data set [33]. The development of such strategies occurs naturally in a thriving flavor-physics community with ample interaction between theorists and experimentalists.

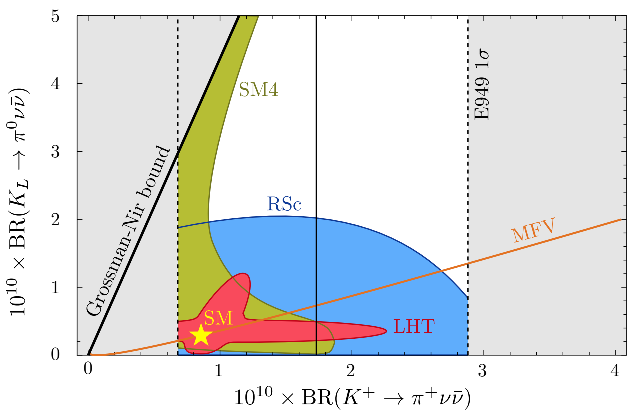

FIG. 4: Correlation between the $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\nu\bar{\nu}$ branching ratios in the Standard Model and in several new-physics scenarios [28].

III. FORECASTS

In order to make forecasts of the precision attainable in future lattice-QCD calculations, one must begin with assumptions about the computing landscape. The discussion here is given from the perspective of the USQCD Collaboration, an umbrella organization that coordinates computing and software for the US lattice-gauge-theory community. Once assumptions about the cost and availability have been spelled out, we can proceed to forecasting the precision that can be attained for the most important flavor-physics matrix elements.

A. Assumptions

The DOE HEP and NP program offices fund dedicated computing resources for lattice gauge theory through the LQCD Infrastructure Project on behalf of the USQCD Collaboration. The Collaboration then allocates these computing resources to its members. This Project has a decade's experience constructing computing clusters based on Intel or AMD chips and has, thus, been able to measure the cost per delivered floating-point operation per second (flop/s). Figure 5 shows measurements up to the start of FY2011 (blue) and Project projections for the next four years (red). The projections make cautious assumptions to guard against vendor delays. The flop/s delivered are based on real computations of quark

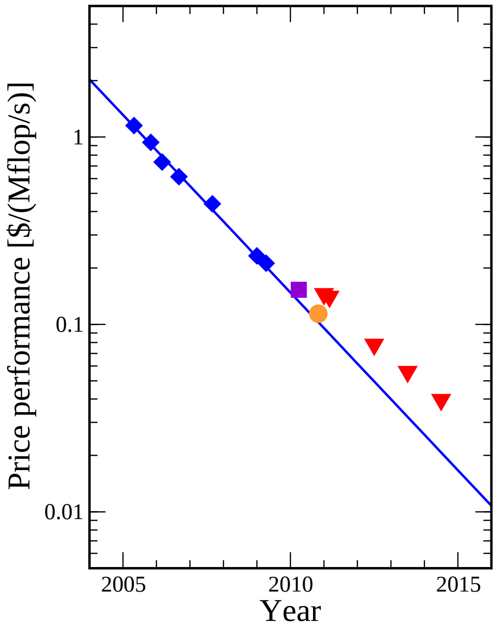

FIG. 5: Price performance in \$/(Mflop/s) of USQCD clusters installed at Fermilab and Jefferson Lab. The FY2010 and FY2011 installations, `10q` at JLab and `Ds` at Fermilab, are shown as a purple square and orange circle, respectively. Earlier clusters in blue diamonds, including fit to measured performance. Cautious projections for FY2011–2014 in red.

propagators with two of the most popular lattice-fermion actions, and the performance relies on a low-latency network; in practice, recent clusters have used Infiniband. As one can see from the magenta and black points, corresponding to the installation of our most recent clusters, `10q` and `Ds`, we remain on the trend line, where costs halve in 1.6 years.

The LQCD Project received \$9.2M for four years, FY2006–2009. The Project is now being extended for another four years with approximately \$4M per annum. For our forecasts below, we assume that the budget will remain flat and that the costs will fall close to the historical trend line. Lattice gauge theory research is broadening, owing to its success, so increases in funding may well support greater breadth rather than greater precision in a narrow set of quantities. One should bear in mind, however, that flavor physics has been a principle focus of USQCD, receiving approximately 50% of the available computing resources.

In addition to its dedicated high-capacity hardware, which is used mostly for physics analysis, USQCD has received significant resources from the DOE's leadership-class computing facilities at Argonne and Oak Ridge National Labs. USQCD has mostly used these high-capability resources to generate the ensembles of lattice gauge fields that underlie all physics projects. Groups within USQCD also receive significant resources on supercomputers at NERSC and at the centers supported through the NSF's XSEDE Program. Here we assume that USQCD and its members will receive a similar fraction of the planned upgrades to these facilities.

A further assumption we must make is for software support. The Moore's Law seen in Fig. 5 has, lately, depended on new CPUs with many cores. In the last two years, a further hardware development has been use of graphics cards (GPUs), which have hundreds of cores. USQCD has been able to respond to these changes thanks to software funding from DOE SciDAC. We assume that this, or equivalent, support will continue. Indeed, if GPUs live up to their early promise, the cost of computing may fall faster than our observed trend; we do not, however, assume such decreases here.

Over the past several years, US-led groups have dominated lattice QCD with bottom and charm physics. With proposals for heavy-flavor factories in Japan and Europe, not to mention the running charm factory BES3 in China, it seems likely that Asian and European groups will resume work in this area with greater vigor. Here we do *not* assume that these groups will do so, however. Instead, we take the perspective that lattice-QCD calculations are still in an era where cross-checks are affordable and desirable. In any case, many systematic uncertainties are correlated among various calculations, and it requires careful study to evaluate the correlations, rather than reducing errors by $1/\sqrt{2}$ or $1/\sqrt{3}$, when two or three calculations become available.

Finally, we assume that funding to support junior researchers—graduate students, postdocs, and junior faculty—does not decrease. In HEP, this may be an optimistic assumption.

B. Forecasts

For most of the matrix elements discussed above, there are already several calculations available in which all sources of error can be estimated. In such cases, one can attempt to forecast how errors will decrease with time, based on the expected growth of computational power. In some cases the largest errors are statistical, in which case forecasting is relatively straightforward, while in others a dominant error is from the need to do a chiral extrapolation to the physical up- and down-quark masses. The latter error will be greatly reduced by simulations directly at the physical quark masses, which are now underway [34–36]. On the other hand, one must then be more careful with finite-volume errors induced by a light pion. For calculations involving heavy-quarks (c and particularly b) there are a variety of approaches (nonrelativistic action, Fermilab interpolating action, extrapolation from relativistic quarks) all with differing dominant systematics. In light of all these factors, forecasting is challenging. Past experience also suggests that unanticipated algorithmic improvements (such as multiscale techniques based on deflation or multigrid) can lead to dramatic jumps in performance that outweigh the gradual improvement due to Moore's law.

Another caveat is that we know that we will need to account for isospin breaking, electromagnetic effects, and charmed sea quarks, once errors reach 0.5–1%. Ensembles for the latter are already being generated, in the US and in Europe. Methods for the other two are being developed, and are likely to be standard five years hence, but it is hard to anticipate how such "new" systematics will help shrink the errors.

With all this in mind, we display in Table I forecasts for key matrix elements. We give estimates for errors in 2014, when widescale computations running at 10s of petaflop/s are expected, and for 2020, when computations will be approaching the exascale. Present lattice results in the table are drawn from two recent efforts to provide averages of lattice results [3, 4].

In light of the discussion in Sec. II, the forecasts for $|V_{cb}|$ and ξ are of particular importance for tightening the constraints on the unitarity triangle. For the former, we expect the error in the relevant quantity $|V_{cb}|^4$ to drop below 2% by 2020. This will reduce the width of the error band in the constraint from ϵ_K by nearly a factor of 4. Although it should be possible to reduce the error on B_K, currently $\sim 3\%$, to $< 1\%$ by 2020, it is probably more promising to formulate the problem with dynamical charm quarks (on and off the lattice), to circumvent the poor convergence of perturbative QCD at the charm-quark scale [25].

The quantity ξ is needed to determine the constraint from the ratio of B-$\bar{B}$ mixing matrix elements, $\Delta M_s/\Delta M_d$. As the table shows, lattice errors are much

TABLE I: Forecasts for future errors in lattice-QCD calculations of selected quantities which can be used to determine elements of the CKM matrix. Where appropriate, errors from non-lattice methods are shown for comparison.

Quantity	CKM element	Present expt. error	Present lattice error	2014 lattice error	2020 lattice error	QCD error w/o lattice	Non-lattice QCD method
f_K/f_π	$\lvert V_{us}\rvert$	0.2%	0.6%	0.3%	0.1%	–	
$f_+^{K\pi}(0)$	$\lvert V_{us}\rvert$	0.2%	0.5%	0.2%	0.1%	1%	ChPT + quark model
$D \to \pi\ell\nu$	$\lvert V_{cd}\rvert$	2.6%	10.5%	4%	1%	–	
$D \to K\ell\nu$	$\lvert V_{cs}\rvert$	1.1%	2.5%	2%	$< 1\%$	5%	$\nu N \to$ charm + pQCD
$B \to D^{(*)}\ell\nu$	$\lvert V_{cb}\rvert$	1.8%	1.8%	0.8%	$< 0.5\%$	$< 2\%$	$B \to X_c\ell\nu$ + OPE + HQE
$B \to \pi\ell\nu$	$\lvert V_{ub}\rvert$	4.1%	8.7%	4%	2%	10%	$B \to X_u\ell\nu$ + OPE + HQE
$B \to \tau\nu$	$\lvert V_{ub}\rvert$	21%	6.4%	2%	$< 1\%$	–	
ξ	$\lvert V_{ts}/V_{td}\rvert$	1.0%	2.5%	1.5%	$< 1\%$	–	

larger than those from the present measurement. By 2020, however, we expect that a lattice result with subpercent level errors is possible.

Lattice-QCD calculations of the matrix elements relevant for determining constraints from the individual mixing rates ΔM_d and ΔM_s are also underway. These have larger errors than in the ratio ξ, because several systematics cancel when taking a ratio. Only one calculation with the full complement of light sea quarks is available, and it has errors of 12% and 10.5%, respectively, for the two matrix elements. We expect that these errors will be roughly halved by 2014 and halved again by 2020.

BSM physics can lead to enhanced mixing in the K, D and B systems. Generically, models lead to four-fermion operators with different Dirac structures than the left-handed operators of the SM. Thus, to determine the form of the constraints on the models of new physics one needs to know the matrix elements of generic operators. These calculations are being done in parallel with those for B_K, ΔM_d, and ΔM_s, and are expected to have error budgets similar to those of the SM matrix elements.

It is important to reduce the uncertainty on $|V_{ub}|$ via both semileptonic and leptonic decays. If the V_{ub} puzzle is resolved without new physics, these decays provide a direct constraint on the apex of the unitarity triangle using tree-level processes. The error on the both the decay constant and the form factor will fall quickly, once lattice spacings are fine enough to exploit methods where the normalization of the currents is automatic. Then these calculations share many features with the analogous calculations for the kaon and $|V_{us}|$. Therefore, we project that the errors will fall to 2% and below 1%, respectively, for semileptonic and leptonic decays.

As noted in Sec. II, another place where lattice calculations can contribute is by determining the SM prediction for ϵ' (the CP-violating contribution to $K \to \pi\pi$ decays), and ΔM_K. Both calculations are very challenging for a variety of reasons. Nevertheless, practical methods have been developed to calculate them, and first results from pilot calculations have appeared. It is reasonable to expect an error of $\sim 30\%$ by 2014, and a $\sim 10\%$ error by 2020.

Finally, we note that lattice calculations of matrix elements needed for some of the rare decays discussed above are in their infancy, so it is hard to forecast future errors.

IV. OUTLOOK

The intensity frontier complements high-p_T physics in at least two ways. Observations discrepant with the Standard Model are discoveries in their own right. More generally, precise measurements offer constraints on the identity of high-mass particles, such as those that may be observed at the LHC.

The interpretation of precise experiments at the intensity frontier requires comparable precision in the corresponding theoretical calculations. In many experiments at the intensity frontier, hadrons are involved in an essential way, leading inevitably to the need to calculate hadronic properties, in particular matrix elements of operators that arise when integrating out short-distance SM or BSM particles. Even in some leptonic observables, the precision is such that virtual hadrons make a significant contribution. Lattice gauge theory provides a set of numerical methods for computing these hadronic properties, within a framework where uncertainties can be systematically reduced.

In the past several years, the right combination of algorithms, computing power and infrastructure, and collaboration structure has come together, leading to a plethora of results. Some of these results are quantitatively impressive and bode well for future experiments at the intensity frontier. Others are qualitatively interesting and connect to the energy and cosmic frontiers. We see special opportunities in quark flavor physics, nucleon matrix elements, and muon $g-2$. With continued support, we look forward to the coming decade's interplay between experiment, theory, and lattice QCD.

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