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electron cyclotron (for materials processing)

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free electron plasma radiation sources

X-ray production (for lithography & materials testing)

rf sources, electron cyclotron, helicon, inductive (e.g., materials processing)

coherent microwave sources

arcs, both DC and pulsed (e.g., steel processing, welding, toxic waste)

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spacecraft charging
rf heating
sheath dynamics
plasma ion implantation
plasma probe interactions
**Plasma-based Devices**

plasma opening switches

high-power switch tubes (thyratrons, ignitrons, klystrons)

pulsed power systems

plasma-based light sources

vacuum electronics

thin panel displays

relativistic electron beams (high-power X-ray sources)

plasma channels for flexible beam control

free electron lasers (tunable electromagnetic radiation)

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dense plasma focus or pinch plasmas for X-rays and beams

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ignition and detonation devices

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plasma accelerators using relativistic space-charge wave
Wave and Beam Interactions in Plasmas

externally driven waves
waves as plasma sources
waves as diagnostics
waves as particle accelerators
beam instabilities (free electron lasers; gyrotrons)
parametric instabilities
solitons
wave-particle interactions
charged particle trapping
rich variety of waves for basic physics research (e.g., electron plasma, upper hybrid, lower hybrid, whistler, Alfven, drift, ion acoustic, ion cyclotron, electron cyclotron)
ionospheric modification; active experiments in space plasmas
light ion beam/plasma interactions (e.g., Li diagnostic beams)
solar power satellite microwaves
nonlinear waves
turbulence, stochasticity and chaos (e.g., MHD, drift, Langmuir)
striation formation and transport
**Numerical Plasmas and Simulations**

realistic particle-in-cell (PIC) simulations of discharge plasmas

PIC simulations of beams in plasmas

MHD and PIC simulations for space and astrophysical plasmas

MHD and PIC models of earth's magnetosphere, solar wind, solar corona

MHD simulations for modeling plasma thrusters

understanding of nonlinear phenomena (e.g., solitons)

numerical modeling of plasma sheaths

modeling of superconducting plasmas

fluid modeling of inductively-driven plasma sources
Plasma Theory

fundamental studies of many-body dynamics

fundamental studies of kinetic theory

fundamental studies of Hamiltonian systems

nonlinear systems; non equilibrium systems

double layers

self-organization and chaos

turbulence

fundamental links of micro-, meso-, to macro scale processes
**Plasma Diagnostics**

ion and neutral beam diagnostics

spectroscopy (mass, photon) and imaging

probe measurements to determine density and temperature

scattering for remote sensing of density and perturbations

laser-induced fluorescence to determine distribution functions

laser transmission diagnostics (e.g., interferometry, polarimetry)

charged-particle spectrometers to determine distribution functions

magnetic field measurements

electric field measurements

neutral particle analysis

diagnostics at one atmosphere pressure
**Industrial Plasmas**

plasma surface treatment

plasma etching

plasma thin film deposition (e.g., synthetic diamond film and high-temperature superconducting film)

ion interaction with solids

synthesis of materials (e.g., arc furnaces in steel fabrication)

destructive plasma chemistry (e.g., toxic waste treatment)

destruction of chemical warfare agents

thermal plasmas

isotope enrichment

electrical breakdown, switchgear, and corona

plasma lighting devices

meat pasteurization

water treatment systems

electron scrubbing of flue gases in coal or solid waste burning

ion beams for fine mirror polishing

plasma surface cleaning

electron beam-driven electrostatic fuel and paint injectors

sterilization of medical instruments

synthetic diamond films for thin-panel television systems

plasma chemistry (produce active species to etch, coat, clean and otherwise modify materials)

- low-energy electron-molecule interactions

- low-pressure discharge plasmas

- production of fullerenes

- plasma polymerization
heavy ion extraction from mixed-mass gas flows

deterioration of insulating gases (e.g., high voltage switches)

one-atmosphere glow discharge plasma reactor for surface treatment of fabrics (enables improved wettability, wickability, printability of polymer fabrics and wool)

laser ablation plasmas; precision laser drilling

plasma cutting, drilling, welding, hardening

ceramic powders from plasma synthesis

impulsive surface heating by ion beams

metal recovery, primary extraction, scrap melting

waste handling in pulp, paper, and cement industries

laser ablation plasmas

laser and plasma wave undulators for femtosecond pulses of X-rays and gamma rays

tunable and chirpable coherent high-frequency radiation from low frequency radiation by rapid plasma creation

DC to AC radiation generation by rapid plasma creation

infrared to soft X-ray tunable free-electron laser (FEL)

optoelectronic microwave and millimeter wave switching

plasma source ion implantation (PSII)
99% of matter is plasma; i.e. electrified gas with atoms dissociated into ions, electrons. Includes stellar interiors and atmospheres, nebulae, interstellar 'gas', Van Allen radiation belt, solar wind, lightening, Aurora Borealis, fluorescent tube, neon sign, rocket exhaust.

1920's plasma oscillations found in the lab, and radio waves were reflected from the ionosphere.

1930 - 1950: foundations of plasma physics created as a byproduct of ionospheric, solar-terrestrial and astrophysical research.

1940's: realization of importance of 'collisionless' and collective processes. Collective behavior means that plasmas behave as fluids.

1960's: Solar wind and Van Allen belts discovered by spacecraft. Fusion research. Linear theory of fluctuations

1970's non linear plasma physics

Saha equation:

\[ \frac{n_i}{n_n} = 2.4 \times 10^{21} \frac{T^{3/2}}{U_i} \exp \left( \frac{U_i}{kT} \right), \]

\( n_i, n_n \) are densities, \( U_i \) is ionization energy, \( T \) is temperature. Air at room temperature: \( n_n \approx 3 \times 10^{25} \text{ m}^{-3}, \) \( T = 300 \text{ K} = 300/(1.16 \times 10^4) \text{ eV}, \) \( U_i = 14.5 \text{ eV (N}_2) \). Then fraction ionization is about 1 part in \( 10^{122} \). Note 1 eV means \( kT = 1 \text{ eV} \), i.e. energy = \( 1.6 \times 10^{19} \text{ J} \), \( T = 1.16 \times 10^4 \text{ K} \). \( 1/n_i \) from recombination, \( T^{3/2} \) term from energy of atoms, exponential term from decreasing number of fast atoms with increasing \( T \) (i.e. available for ionizing).
Various plasmas

<table>
<thead>
<tr>
<th>Plasma</th>
<th>Density ($10^2$ cm$^{-3}$)</th>
<th>Temperature ($10^3$ °K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>interstellar gas</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>gaseous nebula</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>F layer</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>solar corona</td>
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<tr>
<td>tenuous lab plasma</td>
<td>11</td>
<td>4</td>
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<tr>
<td>solar atmosphere</td>
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<td>4</td>
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<tr>
<td>dens lab plasma</td>
<td>15</td>
<td>5</td>
</tr>
<tr>
<td>thermo. plasma</td>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>metal</td>
<td>23</td>
<td>2</td>
</tr>
<tr>
<td>stellar interior</td>
<td>27</td>
<td>7</td>
</tr>
</tbody>
</table>

**Temperature**

A 1D distribution function $f(u)$ is the number of particles m$^{-3}$ with velocities between $u$ and $u + du$. For a Maxwellian

$$f(u) = A e^{-\frac{1}{2}mu^2/(kT)}$$

where $k = 1.38 \times 10^{-23}$ J/°K. The density $n$ is given by

$$n = \int_{-\infty}^{\infty} f(u) du.$$  

Then $n = A \int_{-\infty}^{\infty} e^{-\frac{1}{2}mu^2/(kT)} du$ implies $A = n \left(\frac{m}{2\pi kT}\right)^{1/2}$.

(note $\int e^{-x^2} dx = \frac{\sqrt{\pi} Erf(x)}{2}$; $Erf(x = \infty) = 1$; $Erf(x = -\infty) = -1$)
A 1-D Maxwellian distribution, $f(d)$ versus $u$.

The average kinetic energy $E_{av}$ defines $T$:

$$E_{av} = \frac{\int_{-\infty}^{\infty} \frac{1}{2} m u^2 f(u) du}{\int_{-\infty}^{\infty} f(u) du}$$

Now define

$$v_{th} = \left(\frac{2kT}{m}\right)^{\frac{1}{2}}; \quad y = \frac{u}{v_{th}}$$

Then

$$f(u) = Ae^{-u/v_{th}^2}$$

$$E_{av} = \frac{1}{2} Amv_{th}^3 \int_{-\infty}^{\infty} e^{-y^2} dy = \frac{\left[ \frac{1}{2} e^{-y^2} \right]_{-\infty}^{\infty} - \frac{1}{2} \int_{-\infty}^{\infty} e^{-y^2} dy}{\int_{-\infty}^{\infty} e^{-y^2} dy} = \frac{mv_{th}^2}{4} = \frac{kT}{2}$$

(Integrate by parts $\int adb = [ab] - \int bda$, with

$a = y; \quad db = ye^{-y^2} dy; \quad da = dy; \quad b = \frac{e^{-y^2}}{2}$)

Extension to a 3-D distribution follows:
\[ f(u, v, w) = A_3 e^{-\frac{1}{2}m(u^2 + v^2 + w^2)/(kt)} \]

\[ A_3 = n \left( \frac{m}{2\pi kT} \right)^{3/2} \]

\[ E_{av} = \iiint A_3 \frac{1}{2} m(u^2 + v^2 + w^2) e^{-\frac{1}{2}m(u^2 + v^2 + w^2)/(kt)} \, du \, dv \, dw \]

By symmetry, we can evaluate one term in the numerator and multiply by 3:

\[ E_{av} = \frac{3}{2} \frac{kT}{m} \int_{-\infty}^{\infty} e^{-\frac{1}{2}m(u^2 + v^2)/(kt)} du \int_{-\infty}^{\infty} e^{-\frac{1}{2}m(v^2 + w^2)/(kt)} dv \int_{-\infty}^{\infty} e^{-\frac{1}{2}m(w^2)/(kt)} dw = \frac{3}{2} kT \]

That is, we have found the energy per degree of freedom is kT/2. We usually give T units of energy, specifying the energy associated with 1 kT. e.g. for kT = 1 eV = 1.6x10^{19} J, then

\[ T = \frac{1.6x10^{-19}}{1.38x10^{-23}} = 1.16x10^4 \]

We can allow T_i \neq T_e, and T_{\parallel} \neq T_{\perp}.

**Some notes on electrostatics**

The magnitude of the force between charges is

\[ F = \frac{q_1 q_2}{4\pi \varepsilon_0 r^2} \]

by definition \( \frac{1}{4\pi \varepsilon_0} = 10^{-7} \text{C}^2 = 9x10^9 \text{N-m}^2/\text{C}^2 \) or V-m/C.

The electric field is defined so that the force \( F = qE \). Then the electric field produced by a point charge is,

\[ E = \frac{q}{4\pi \varepsilon_0 r^2} \]

and by multiple charges is
\[ E = \sum \frac{q_i}{4\pi \varepsilon_0 r^3} \]

where \( \vec{r}_i \) is the vector of magnitude \( r_i \).

The electrostatic potential at a point \( p \) is found from the work required to move the charge to a point \( r_p \) from infinity where the potential is 0. For a point charge the work \( d\phi \) required to move the charge a distance \( -E \cdot dr \), i.e.

\[ \phi_p = \int_{0}^{\phi_p} d\phi = -\frac{q}{4\pi \varepsilon_0} \int_{r_0}^{r_p} \frac{dr}{r^2} = -\frac{q}{4\pi \varepsilon_0 \left( \frac{1}{r_p} - \frac{1}{r_0} \right)} = \frac{q}{4\pi \varepsilon_0 r_p} \]

or, for multiple charges

\[ \phi_p = \frac{1}{4\pi \varepsilon_0} \sum \frac{q_i}{r_i} \]

Gauss's theorem is

\[ \int_{S} \vec{A} \cdot \vec{n} dS = \int_{V} \nabla \cdot \vec{A} dV \]

Poisson's equation: let the vector \( \vec{A} = \vec{D} = \varepsilon \vec{E} \), so that

\[ \int_{V} \nabla \cdot \varepsilon \vec{E} dV = \int_{S} \varepsilon \vec{E} \cdot \vec{n} dS = Q = \int_{V} \rho dV \]

where \( \rho \) is charge density. Now let \( dV \) become vanishingly small

\[ -\nabla \cdot \varepsilon \vec{E} = \nabla \cdot (\varepsilon \nabla \phi) = -\rho \]

Note the solution is

\[ \phi = \frac{1}{4\pi \varepsilon_0} \int_{V} \frac{q dV}{r} \]

Application to a sheet charges, e.g. a parallel plate capacitor with each plate area \( A \) and charge \( q \), surface charge density \( s \). See Feynman vol. 2. Applying Gauss's flux theorem

\[ \int_{S} \varepsilon \vec{E} \cdot \vec{n} dS = Q = \int_{V} \rho dV \]

i.e. we have \( E = s/\varepsilon \).
Plasmas shield out applied electric fields. Put in two electrodes (covered with a dielectric to stop recombination at the surface) and connect to a battery. The electrodes would attract charges of the opposite sign to try and cancel the local charges. Without thermal motion (a cold plasma) the cancellation would be perfect. If the temperature is finite, the particles at the edge of the cloud can escape. The edge occurs where the potential energy is approximately equal to kT. i.e. we can expect to find potentials of order kT/e in plasmas.

Let $f$ be the fractional departure of $n_e$ from $n_i$ over a spherical volume of radius $r$. Take as an example a plasma with (very large) density $n = 10^{16} \text{ cm}^{-3}$. Then the electrostatic field is

$$E = \frac{4\pi r^3}{3} fne \frac{1}{r^3} = \frac{4\pi r fne}{3}$$

i.e. if $f = 10^{-6}$, then $E = 6 \text{ kV/cm}$, and proportional to the radius of the charged space. This electrostatic field operates to restore equilibrium, i.e. neutrality. Any departure from neutrality
tends to screen the original field; thermal agitation tends to disrupt neutrality, charged particle density tends to maintain neutrality.

Potential distribution near a grid in a plasma. \( \lambda_D = 1, \phi_0 = 1. \)

Consider a transparent grid at \( x = 0 \), held at a potential \( \phi_0 \). Assume ions so heavy they cannot move, but rather they create a uniform background of positive charge. Poisson's equation in 1-D gives

\[
\varepsilon_0 \nabla^2 \phi = \varepsilon_0 \frac{d^2 \phi}{d\phi^2} = -e (n_i - n_e) \quad \text{for} \ Z = 1
\]

Far away the density is \( n_\infty \) i.e.

\[ n_i = n_\infty \]

Electron distribution function is

\[ f(u) = Ae^{\left(-\frac{1}{2} \frac{m_u^2}{q} \phi \right) / (kT_e)} \]

(fewer particles where P.E. is high). Integrate over \( u \), setting \( q = -e \), with \( n_e(\phi \to 0) = n_\infty \)

\[ n_e = A \int_{-\infty}^{\infty} e^{\left(-\frac{1}{2} \frac{m_u^2}{q} \phi \right) / (kT_e)} \, du = n_\infty e^{-\phi / (kT_e)} \]

Then Poisson's equation becomes, for small potentials,

\[
\varepsilon_0 \frac{d^2 \phi}{d\phi^2} = en_\infty \left(e^{-\phi / (kT_e)} - 1\right) = en_\infty \left(\frac{\phi}{kT_e} + \frac{1}{2} \left(\frac{\phi}{kT_e}\right)^2 + \ldots\right)
\]
\[ \varepsilon_0 \frac{d^2 \phi}{d \phi^2} = \frac{e^2 n_e}{kT_e} \phi \]

i.e. \[ \phi = \phi_0 e^{-\frac{1}{\lambda_D}} \]

\[ \lambda_D = \left( \frac{\varepsilon_0 kT_e}{ne^2} \right)^{\frac{1}{2}} \]

\[ \lambda_D = 7430 \left( \frac{kT_e}{n} \right)^{\frac{1}{2}} \text{ in m; } kT_e \text{ in eV} \]

where \( n \) is used for \( n_\infty \).

Quasi neutral means that, for a system with size \( L \gg \lambda_D \), any concentrations of charge or external potentials are shorted out. Outside the sheath \( n_e = n_i \) to typically better than 1 part in \( 10^6 \). Plasma means \( L \gg \lambda_D \). Outside \( \lambda_D \) the effective potential is very small. For distances \( \ll \lambda_D \) the effective potential is equivalent to the bare Coulomb charge.

Plasma parameter: Debye shielding only valid if enough particles in cloud. Number of particles in a Debye sphere is

\[ N_D = n \frac{4}{3} \pi \lambda_D^3 \]

Let's do the above for a point charge \( q_0 \) located at the origin, which produces a potential field

\[ \phi_0 = \frac{q_0}{4 \pi \varepsilon_0 r} \]

Consider an electron gas with density \( n_e \) and temperature \( T_e \). Poisson's equation is

\[ \varepsilon_0 \nabla^2 \phi = -q_0 \delta(r) - e(n_i - n_e) \]

where \( \delta(r) \) is the 3-D delta function. Note spherical cord system - we have

\[ \varepsilon \nabla^2 \equiv \varepsilon \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \phi}{\partial r} \right) \]

As before, \( n_e - n_i = n_i e^{\phi / kT_e} - n_i = n_i e \phi / T_e \), and

\[ \varepsilon_0 \nabla^2 \phi = -q_0 \delta(r) + \frac{ne^2}{kT_e} \phi \]

with a solution
If the ions can participate (quasi static conditions) then we can assume a M-B distribution for both ions and electrons, and assuming \( n_e = n_i = n_\infty = n \) at infinity,

\[
n_e = n_\infty e^{-\phi/(kT_e)} \quad n_i = n_\infty e^{-\phi/(kT_i)}
\]

Poisson's equation becomes

\[
\varepsilon_0 \nabla^2 \phi = -q_0 \delta(r) - e(n_i - n_e) = -q_0 \delta(r) - en\left( e^{\phi/(kT_e)} - e^{-\phi/(kT_e)} \right)
\]

\[
\approx -q_0 \delta(r) + e^2 n_\infty \left( \frac{1}{kT_i} + \frac{1}{kT_e} \right)
\]

In this case the Debye length is given by

\[
\lambda_D = \left( \frac{\varepsilon_0 k}{ne^2 (1/T_e + 1/T_i)} \right)^{\frac{1}{2}}
\]

In the expansion of the Exp function we assumed that the potential energy \( e\phi \ll \) the kinetic energy. We return to this later.

**Plasma Oscillations.**

Consider a dynamic situation. When electrons attempt to screen out a net positive charge they will overshoot, because they have a mass and thus a momentum. The electrons can oscillate, with the Coulomb force acting as the restoring force and the mass of the electron as the inertia.
Model for plasma oscillations

Assume a 1-D case with \( n_e = n_i = n \). Consider the displacement of a portion of the electron gas of thickness \( \xi \). Two charge layers appear at the boundaries with charge densities \( n_e \xi \). The electric field produced is given by

\[
\varepsilon_0 \nabla \cdot E = \varepsilon_0 \frac{\partial E}{\partial \xi} = en; \quad E = \frac{en\xi}{\varepsilon_0}
\]

Using this in the equation of motion

\[
m \ddot{\xi} = -eE
\]

gives

\[
\ddot{\xi} + \frac{ne^2}{\varepsilon_0 m} \xi = 0
\]

This describes a longitudinal oscillation with angular frequency

\[
\omega_p = \left( \frac{ne^2}{\varepsilon_0 m} \right)^{\frac{1}{2}}
\]
For a multi component plasma we have

$$\omega_p = \sqrt{\sum \left( \frac{n_i q_i^2}{\varepsilon_0 m_i} \right)^2}$$

We can also consider other modes of oscillation because there are additional degrees of freedom corresponding to relative motion between the different kind of particles. For an electron - ion plasma with $n_i = n_e$ the above equation corresponds to an out of phase oscillation between electrons and ions, which may be viewed as an optical mode. The other branch may be viewed as an acoustic mode.

Find radio waves will only propagate with frequencies > plasma frequency.

But use lower frequencies so they reflect back to earth.

In metals we have contained ions and free electrons. i.e. should be able to observe oscillations. According to quantum mechanics there should be energy levels separated by $\hbar \omega_p/(2\pi)$. Pass electrons through a foil, find electrons sometimes lose energy $\hbar \omega_p/(2\pi)$ to foil.

**Discreteness**

$\lambda_D$ and $\omega_p$ are computed from 4 parameters, $m$, $e$, $1/n$ and $T$. These are the basic quantities characterizing a classical plasma; they are related to the discrete nature of the particles. They describe the mass, charge, average volume and average kinetic energy per particle. Introduce a theoretical limit in which the individuality of the particles is suppressed, and the plasma takes on a medium like property. Cut each particle into finer and finer pieces, so that the discrete parameters all approach 0. In doing this we keep certain fluid like parameters constant; the mass density $n_m$, the charge density $n_e$, the kinetic energy density $nT$.

Look for a dimensionless parameter which can be constructed from these 4 discrete parameters. Write

$$\left[ m^{\alpha} (1/n)^{\beta} T^\gamma e \right] = 1$$

where we are considering only dimensions. $e$ has dimensions $[M]^{1/2} [L]^{3/2} [T^{-1}]$, so that

$$\alpha + \gamma + 1/2 = 0; \quad 3\beta + 2\gamma + 3/2 = 0; \quad -2\gamma - 1 = 0$$

i.e. $\alpha = 0; \quad \beta = -1/6; \quad \gamma = -1/2$
this is the only solution. That is, the discreteness parameter is any power of the function

\[ en^{1/6}T^{-1/2} \]

taking the 3rd power gives us the dimensionless parameter

\[ g = \left( en^{1/6}T^{-1/2} \right)^3 = 1/(n\lambda_p^3) \approx 1/N_p \]

g is called the plasma parameter, which measures the ratio between potential and kinetic energies. It is small, and used as an expansion parameter.

**Collective versus individual behavior**

Consider the equation of motion for a density fluctuation in an electron gas. Deal with point particles and charges, so that the density field is given by the superposition of 3-D delta functions

\[ \rho(\vec{r}) = \sum_{i=1}^n \delta(\vec{r} - \vec{r}_i) \]

where \( \vec{r}_i \) is the position of the \( i \)-th electron. Consider a cube of unit volume with periodic boundary conditions, so \( n \) is the number density. The spatial Fourier components of any fluctuations is

\[ \rho_\mathbf{k} = \int \rho(\vec{r})e^{-i\mathbf{k} \cdot \vec{r}} d\vec{r} = \sum_i e^{-i\mathbf{k} \cdot \vec{r}_i} \]

Differentiate twice with respect to time

\[ \ddot{\rho}_\mathbf{k} = -\sum_i \left[ (\mathbf{k} \cdot \mathbf{v}_i)^2 + i\mathbf{k} \cdot \dot{\mathbf{v}}_i \right] e^{-i\mathbf{k} \cdot \vec{r}_i} \]

\( \mathbf{v}_i \) is the velocity of the \( i \)-th electron. The acceleration \( \dot{\mathbf{v}}_i \) is calculated from the force acting on the electron from all other electrons and the positive charge background

\[ \dot{\mathbf{v}}_i = -\frac{1}{m} \frac{\partial}{\partial \vec{r}_i} \sum_{j \neq i} \frac{e^2}{|\vec{r}_i - \vec{r}_j|} \]

We want to write this in a different form, so note that

\[ \nabla^2 \frac{1}{|\vec{r} - \vec{r}'|} = -4\pi \delta(\vec{r} - \vec{r}') \]
\[
\frac{\partial}{\partial \mathbf{r}} \frac{1}{|\mathbf{r} - \mathbf{r}'|} = -4\pi \int \delta(\mathbf{r} - \mathbf{r}')
\]

Now an identity is
\[
\delta(\mathbf{r} - \mathbf{r}') = \sum_k e^{i \mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')}
\]
so that
\[
\frac{\partial}{\partial \mathbf{r}} \frac{1}{|\mathbf{r} - \mathbf{r}'|} = -4\pi \sum_k \frac{\mathbf{k}}{|\mathbf{k}|^2} e^{i \mathbf{k} \cdot (\mathbf{r} - \mathbf{r}')}
\]
Therefore
\[
\dot{\mathbf{v}}_i = -i \frac{4\pi e^2}{m} \sum_{k \neq i} \sum_k \frac{\mathbf{k}}{|\mathbf{k}|^2} e^{i \mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} = -i \frac{4\pi e^2}{m} \sum_k \mathbf{k} \cdot \mathbf{v}_i e^{i \mathbf{k} \cdot \mathbf{r}_i}
\]
Substituting this into the expression for the acceleration gives
\[
\ddot{\rho}_k = -\sum_i \left[ (\mathbf{k} \cdot \mathbf{v}_i)^2 e^{-i \mathbf{k} \cdot \mathbf{r}_i} - \frac{4\pi e^2}{m} \sum_{q \neq k} \frac{\mathbf{k} \cdot \mathbf{q}}{|\mathbf{q}|^2} \rho_{k-q} \rho_q \right]
\]
rewrite this by separating out the term with \(k = q\) on the HS to give
\[
\ddot{\rho}_k + \omega_p^2 \rho_k = -\sum_i \left[ (\mathbf{k} \cdot \mathbf{v}_i)^2 e^{-i \mathbf{k} \cdot \mathbf{r}_i} - \frac{4\pi e^2}{m} \sum_{q \neq k} \frac{\mathbf{k} \cdot \mathbf{q}}{|\mathbf{q}|^2} \rho_{k-q} \rho_q \right]
\]
i.e. fluctuations occur at the plasma frequency as long as the RHS terms can be neglected.
Consider a Maxwellian distribution. Then the first term gives
\[
\sum_i \left[ (\mathbf{k} \cdot \mathbf{v}_i)^2 e^{-i \mathbf{k} \cdot \mathbf{r}_i} \right] = \sum_i e^{-i \mathbf{k} \cdot \mathbf{r}_i} \int (\mathbf{k} \cdot \mathbf{v})^2 f(v) dv = k^2 \frac{T}{m} \rho_k
\]
The second term involves the product of two density fluctuations. The density fluctuation for \(q \neq 0\) is the sum of exponential terms with randomly varying phases, his term can be ignored.
Then the condition for collective oscillation becomes
\[
k < \left( \frac{4\pi n e^2}{T} \right)^{\frac{1}{2}} = \frac{1}{\lambda_D}
\]
For short wavelengths, however, the plasma behaves as a system of individual charges., and the first term on the RHS determines what happens
Notes on identities

Use spherical coordinates. Then

\[ \nabla f(r) = \frac{\partial f / \partial r}{r} \hat{r} = \frac{f}{r} \hat{r} \]

\[ \nabla^2 f(r) = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f(r)}{\partial r} \right) \]

\[ = f''(r) + \frac{2}{r} f(r) \]

For \( f(r) = \frac{1}{r} \) this gives 0, i.e. \( 1/r \) is a solution of the Laplace equation. However, at the origin there is a discontinuity, and we must use

\[ \nabla^2 \left( \frac{1}{r} \right) \bigg|_{r \to 0} = -4\pi \delta \]

If this is correct, then the integral over volume = 4\( \pi \), because the definition of the delta function is that its integral is 1. Therefore note

\[ \int \nabla^2 \left( \frac{1}{r} \right) dV = \int \nabla \cdot \left( \nabla \left( \frac{1}{r} \right) \right) dV = \int \left( \nabla \left( \frac{1}{r} \right) \right) \cdot \hat{n} dS \]

\[ = \int \int_{0}^{\pi/2} 1 \sin(\theta) d\theta d\phi = 8 \int_{0}^{\pi/2} \sin(\theta) d\theta = 8 \left[ -\cos(\theta) \right]_{0}^{\pi/2} d\phi \]

\[ = 8 \int_{0}^{\pi/2} d\phi = 4\pi \]
\[ dV = (r \sin \theta \, d\phi) \, (r \, d\theta) \, (dr) = r^2 \sin \theta \, dr \, d\theta \, d\phi \]
Applications of Plasma Physics

Characterize by n, T, or better N. n from $10^6$ to $10^{34}$, T from 0.1 to 10$^6$ eV. This is a large range, e.g., the density of a neutron star is $10^{15}$ times water.

Gas Discharges

Langmuir, Tonks. Originally researching vacuum tubes. Weakly ionized glows and positive columns. \( T \approx 2 \text{ eV}, \ 10^{14} < n < 10^{18} \text{ m}^{-3}\). Discovered shielding, (sheath as a dark layer). Now applicable as mercury rectifiers, thyratrons, ignitrons, spark gaps, welding arcs, neon lights, lightning.

Controlled Thermonuclear Fusion

1929: Atkinson and Houterman proposed that Fusion might explain the energy of stars. Beam target interactions demonstrated reality, but \( E_{\text{in}} \gg E_{\text{out}} \) (Rutherford: Fusion Energy is ‘Moonshine’).

early 1940’s: discussions of possible laboratory experiments.
late 1940’s: possible geometry discussed.

early 1950’s: H bomb.

1951: Peron claimed Richter solved problem.

< 1958: Classified programs by USA, USSR, UK (because copious neutrons might be used to create fissile material for bombs).

1957: Lawson’s criterion for useful energy production (and a yardstick of our progress):

for D-T

\[
\begin{align*}
T &\approx 20 \text{ keV} \ (2 \times 10^8 \text{ 0K}), \\
n\tau &\approx 2 \times 10^{14} \text{ cm}^{-3} \text{ s}.
\end{align*}
\]

for D-D

\[
\begin{align*}
T &\approx 50 \text{ keV} \ (5 \times 10^8 \text{ 0K}), \\
n\tau &\approx 6 \times 10^{15} \text{ cm}^{-3} \text{ s}.
\end{align*}
\]

late 1950’s: Mirror machines (didn’t work).

1959: The Harwell conference.

1960’s: Toroidal pinches, stellarators.

1970’s: Success of tokamaks.

1980’s: TFTR and JET.

1990’s: First D-T experiments, and the design of ITER.
December 1993: 6 MW of power from TFTR.

FROM THE DEBATE ON THE JET NUCLEAR FUSION PROJECT, THE HOUSE OF LORDS, 1987

Earl Ferrers:

My Lords, what kind of thermometer reads a temperature of 140 million degrees centigrade without melting?

Viscount Davidson:

My Lords, I should think a rather large one.
Space Physics

i.e. the earth's environment in space. The solar wind has \( n = 10^7 \text{ m}^{-3}, T_i = 10 \text{ eV}, T_e = 50 \text{ eV}, B = 5 \text{ nT}, v = 300 \text{ km-s}^{-1} \). The ionosphere from 50 km to 10 RE, is weakly ionized with \( n = 10^{12} \text{ m}^{-3}, T = 0.1 \text{ eV} \). The Van Allen belts have \( n = 10^9 \text{ m}^{-3}, T_i = 1 \text{ eV}, T_e = 1 \text{ keV}, B = 500 \text{ nT} \), with an additional hot component with \( n = 10^3 \text{ m}^{-3}, T_e = 40 \text{ keV} \).

Astrophysics

For example stellar interiors and atmospheres, such as the sun with a core \( T = 2 \text{ keV} \) and the solar corona with \( T 200 \text{ eV} \). Interstellar medium of hydrogen with \( n = 10^6 \text{ m}^{-3} \). Stars in galaxies can be considered like particles in a plasma. Pulsars can be considered as plasmas.

MHD energy conversion and ion propulsion

Energy conversion: Use plasma jet across a magnetic field. Lorentz force \( F = qv \times B \) (v is jet velocity) causes ions to drift up and electrons to drift down. Two sets of electrodes are charged, from which a current can be drawn. The reverse principle produces an 'ion thruster'. Here the force pushes plasma out of the source (e.g. a rocket), and the ensuing reaction accelerates the source.

\[ x \times B \]

An MHD generator
An Ion Thruster

**Solid State**

Free electrons and holes constitute the same sort of oscillations and instabilities as in a gaseous plasma. Lattice effects are equivalent to reducing the collision frequency from that expected in a solid \( n \approx 10^{29} \text{ m}^{-3} \). The holes have a very low effective mass.

**Gas lasers**

Use a gas discharge to pump, i.e. to invert the population of states.

**The brain**
### FUNDAMENTAL PHYSICS STUDIES at TEXAS

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Note that plasma is the most commonly occurring state in the universe.
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Note the excellent non-perturbing plasma diagnostics available.
**APPLIED PHYSICS and ENGINEERING STUDIES at TEXAS**

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Research at the Fusion Research Center

Research opportunities at the FRC can be found at our home page (http://w3fusion.ph.utexas.edu/frc/). Below are some details.

1. The linear machine

We are writing a proposal to NSF to try and obtain funding for this. We propose to build an experiment to investigate the nonlinear regimes of kinetic instabilities. We will study the transitions from the instability onset near its threshold to the coherent nonlinear states that enhance plasma transport. We intend to verify if a recently developed theory of weakly unstable kinetic modes in a driven system can explain different responses that can arise in the experiment. This study is of interest as a precursor to the investigation of fully developed turbulence. We will attempt to determine if an intrinsic kinetic description is needed to describe the observations or whether an averaged fluid description is applicable in a collisionless plasma. For example, it is now known that the collisionality, often assumed to be unimportant, is enhanced for the small group of particles at the resonance and can effect the character of the response.

The proposed initial experiments will study the effect of drive and damping strength on mode behavior, looking for either steady state or bursting behavior. The modes to be studied are the hot electron interchange and the whistler instability. These are chosen because there is evidence in the literature for unexplained phenomena. A small magnetic mirror will be built (see Figure 1), using mostly already available equipment. Electron cyclotron resonance heating of different gases at different pressures will be used to create and control the necessary hot electrons for the drive. Existing unique diagnostics (e.g. heavy ion beam probe, phase contrast imaging system) will be employed to measure parameters.

The apparatus (including its construction) will be used for graduate and undergraduate training for plasma physics and engineering students at the University of Texas. As such it will be used both for basic research, and as a device for students who intend to participate in our off-campus collaborative research programs to learn experimental techniques and develop diagnostics.
2. Plasma Propulsion

We are working with NASA to obtain funding for students to work in Houston at the Sonny Carter Center on a mirror machine. One of the most important issues being addressed today in connection with human interplanetary travel is the crew's prolonged exposure to weightlessness as well as the high radiation dosage which accrues during these long voyages. From this point of view, it becomes crucial to achieve a minimum trip-time as well as to extend the ship's acceleration schedule consistently with human and power plant limitations. However, flexibility in these well-known "trajectory variables" remains limited by the capabilities of conventional (constant specific impulse $I_{sp}$) chemical engines. Trip-times remain "high" and severely restricted by payload and fuel constraints while the acceleration time is negligibly short compared to the total trip. The optimum rocket system would be one in which both thrust and specific impulse are allowed to vary continuously depending on the conditions of flight.
With these observations in mind, the variable thrust/$I_{sp}$, RF-heated electro-thermal rocket, based on the technology of Tandem Mirrors developed for the thermonuclear fusion program, is being pursued. An experiment is currently underway, housed at the Sonny Carter Training Facility in Houston, TX. A schematic of the concept is shown in Figure 2. The particular approach being considered, not being a fusion concept itself, has permitted a substantial relaxation in the physics requirements on plasma density and temperature as compared with its fusion counterpart. At the same time, it has benefited from many of the advances in plasma heating, control and confinement achieved in previous years. The Tandem itself, an open-ended linear device suffering from end-loss limitations in fusion, becomes particularly well suited as a variable $I_{sp}$ rocket by virtue of such innate "leakage". Moreover, the experiments performed in the closing years of the U.S. mirror program reveal an intrinsic axial asymmetry and plasma flow which we seek to exploit.

The device being studied comprises two connected plasmas. The first is the central cell plasma, and the second is the plasma exhaust. Both plasmas must be characterized for stability and equilibrium parameters. We propose to perform these measurements using reciprocating Langmuir probes, systems which we have routinely used on many magnetic fusion devices throughout the United States. All the required major hardware, power supplies and data acquisition systems already exist at the Fusion Research Center, the University of Texas at
Austin. Therefore data can be obtained on a very short time, less than three months from project initiation.

3. EPEIUS

The Fusion Research Center at the University of Texas proposes to design, build, and operate, in collaboration with ASIPP (Hefei, The People's Republic of China), EPEIUS, a low-aspect-ratio torsatron-tokamak hybrid with major radius $R_0 \approx 0.5$ m, aspect ratio $A = R_0/a \approx 3$, and magnetic field $B \approx 1$ T. Here $a$ is an average minor radius and $R_0$ is an average major radius. Low-aspect-ratio torsatrons and stellarators, or in general three-dimensional systems, are being studied for their possible advantages as thermal reactors. In particular, small aspect ratio tokamak-torsatron hybrids (SMARTH's) have recently been proposed. EPEIUS is such a configuration, intended to access low collisionality plasmas, with plasma properties dominated by the physics of low aspect ratio, with an external (i.e. imposed by external coils) rotational transform $\tau_{ext}$ which decreases with increasing minor radius. EPEIUS will be devoted to the study of electric fields $E_r$, (which are central to the success of the SMARTH concept as a fusion device), magnetic surface resilience, stability, global and local confinement and turbulence.
properties. The research program will be carried out in collaboration with other laboratories and universities. To minimize start-up costs, the research is segmented into logical phases, with funding for each phase dependent on the success of the previous phase. We are presently requesting funding only for Phase I, but a discussion of subsequent phases is included to indicate our long term planning.

4. Alcator C-MOD

The FRC is presently involved in a collaboration with the MIT Plasma Science Fusion Center (PSFC) in the study of turbulence and transport in tokamaks. An FRC staff member is presently on site at MIT and we are hiring a post-doctoral fellow to assist, also on site. Most of the other FRC staff are involved in the collaboration to varying degrees, either periodically visiting the PSFC or participating in experiments and data analysis via remote access and video-conferencing over the Internet.

We are adding turbulence diagnostics to the set of C-Mod profile diagnostics to attempt to connect turbulence with transport. The new diagnostic systems include:

- a diagnostic neutral beam, which enables measurements of
  - ion density fluctuations via beam-emission spectroscopy (BES),
  - profiles of impurity density, temperature, and rotation velocities via charge-exchange recombination spectroscopy (CXRS),
  - toroidal current-density profile via the motional Stark effect (MSE).
- an electron-cyclotron-emission (ECE) heterodyne radiometer to measure profiles and fluctuations of electron temperature,
- electrostatic probes for the measurement of fluctuating parameters in the edge plasma,
- an optical diagnostic to measure fluctuations in otherwise inaccessible regions of the edge plasma.

In addition, we are assisting in the development of a phase-contrast imaging (PCI) system for the study of ion-cyclotron waves and perhaps modifying the system for measurements of plasma turbulence as well.
Figure 4. A view of a reconstructed plasma in the C-MOD experiment at MIT.
The long-term goals of our research are to understand the source of free energy (the "drive") of the turbulent fluctuations and to contribute to development of predictive theoretical models. The diagnostics listed above will help us test various existing theoretical models of turbulent transport, most notably the IFS/PPPL model, which has recently been featured prominently in the media (Dec. 6 issue of Science, subsequent articles in the N. Y. Times, etc.). In the edge plasma, we wish to test the model that magnetic curvature serves as a drive of the turbulence. Because C-Mod discharges are at especially high density and toroidal magnetic field, turbulence measurements in C-Mod will also extend the empirical description of tokamak turbulence to new plasma regimes.

5. DIII-D.

The FRC is also participating in turbulence and transport studies on the DIII-D tokamak at General Atomics Co. in San Diego, CA. These involve i) collaborating with UCLA on ECE fluctuation measurements and analysis, ii) collaborating with the University of Wisconsin, Madison (UWM) on BES measurements and analysis, and iii) proposing and carrying out experiments aimed at understanding the role of turbulence in tokamak transport. We also are involved in high-resolution (both space and time) ECE measurements of electron temperature. An FRC staff member, primarily involved in the ECE measurements, is on site at GA full time. Like at C-Mod, other FRC staff either periodically visit GA during experiments, or participate via remote access and video-conferencing.

Our physics program on DIII-D is similar to that on Alcator C-Mod, namely, understanding the relevance of turbulence to transport by testing theoretical models of turbulent transport. The FRC is therefore in a unique position of being involved in the same measurements (BES and ECE) on the (soon-to-be) two remaining US tokamaks. Since the two machines are considerably different (DIII-D is much larger, but of much lower density and toroidal field; DIII-D employs uni-directional neutral beams for heating while C-Mod uses ion-cyclotron resonance heating), these parallel measurements (and experiments) are not redundant, but allow us to efficiently compare and contrast results in disparate plasmas, thereby gaining greater insights into the fundamental mechanisms of the turbulence and transport.
Figure 5. A view of the DIII-D tokamak.