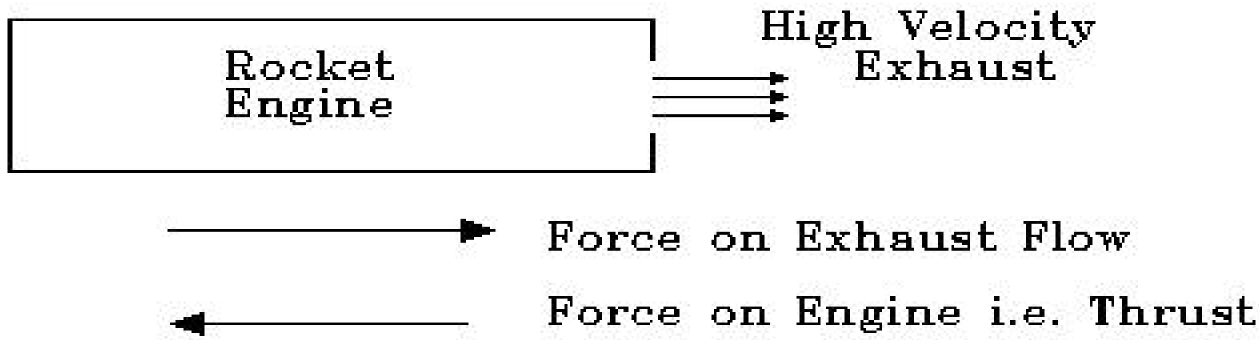
Rockets

Contents

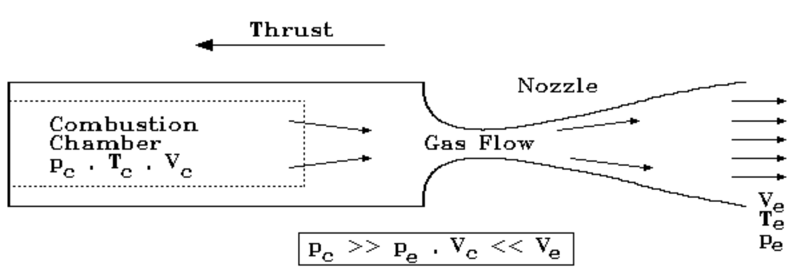
Basics

A rocket engine is an engine that produces a force, (a thrust) by creating a high velocity output without using any of the constituents of the "atmosphere" in which the rocket is operating. The thrust is produced because the exhaust from the rocket has a high velocity and therefore a high momentum. The rocket engine must, therefore, have exerted a force on exhaust material and an equal and opposite force, the thrust, is, therefore, exerted on the rocket.



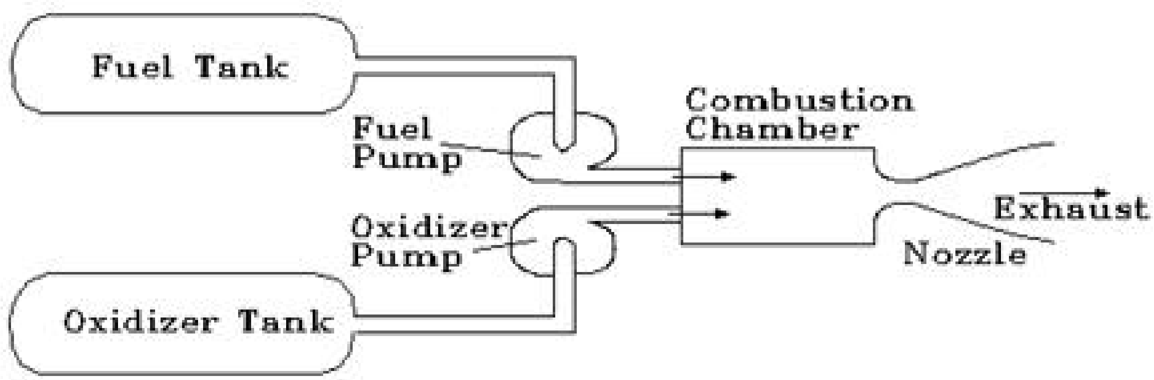
The fact that the rocket engine does not use any constituent of the surrounding atmosphere means that it can operate in any part of the atmosphere and outside the atmosphere which makes it ideal for space propulsion. There are two basic types of rocket engine: Chemical Rockets and Non-chemical Rockets .

In a chemical rocket, a fuel and an oxidizer are usually supplied to the combustion chamber of the rocket. The chemical reaction between the fuel and the oxidizer produces a high pressure and temperature in the combustion chamber and the gaseous products of combustion can be expanded down to the ambient pressure, which is much lower than the combustion chamber pressure, giving a high velocity gaseous efflux from the rocket engine.



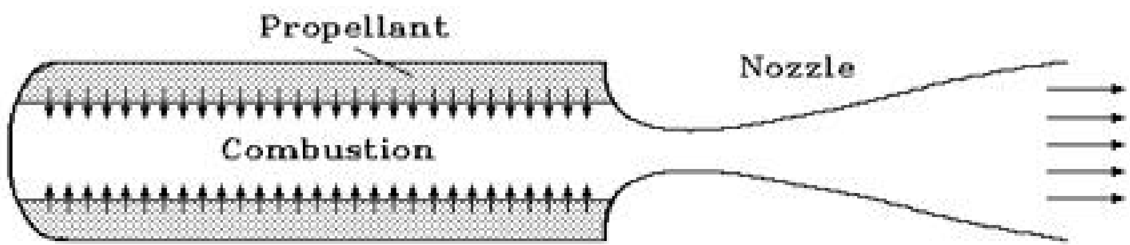
Chemical Rocket

There are two types of chemical rockets, Liquid Propellant Rockets and Solid Propellant Rockets. In a liquid propellant rocket, the fuel and the oxidizer are stored in the rocket in liquid form and pumped into the combustion chamber.



Basic Arrangement of Liquid Propellant Rocket

In a solid propellant rocket, the fuel and the oxidizer are in solid form and they are usually mixed together to form the propellant. This propellant is carried within the combustion chamber. The arrangement of a solid propellant rocket is shown.



Basic Arrangement of a Solid Propellant Rocket.

In a non-chemical rocket, the high efflux velocity from the rocket is generated without any chemical reaction taking place. For example, a gas could be heated to a high pressure and temperature by passing it through a nuclear reactor and it could then be expanded through a nozzle to give a high efflux velocity.

The term rocket has frequently been used to describe both the thrust producing device, i.e. the engine, and the whole rocket powered vehicle. To avoid confusion, especially in the case of large vehicles such as space launch vehicles, the propulsion device is now usually referred to as a rocket engine.

The advantages of liquid-fueled rockets are that they provide

1. Higher exhaust velocity (specific impulse),
2. Controllable thrust (throttle capability),
3. Restart capability, and
4. Termination control.

The advantages of solid-fueled rockets are that they give

1. Reliability (fewer moving parts),
2. Higher mass fractions (higher density implies lower tankage),
3. Operational simplicity.

Liquid propellants

Fuel	**Oxidizer**	**Isp (s)**
Hydrogen(LH_2)	Oxygen (LOX)	450
Kerosene	LOX	260
Monomethyl hydrazine	(MMH)Nitrogen tetroxide (N_2O_4)	310

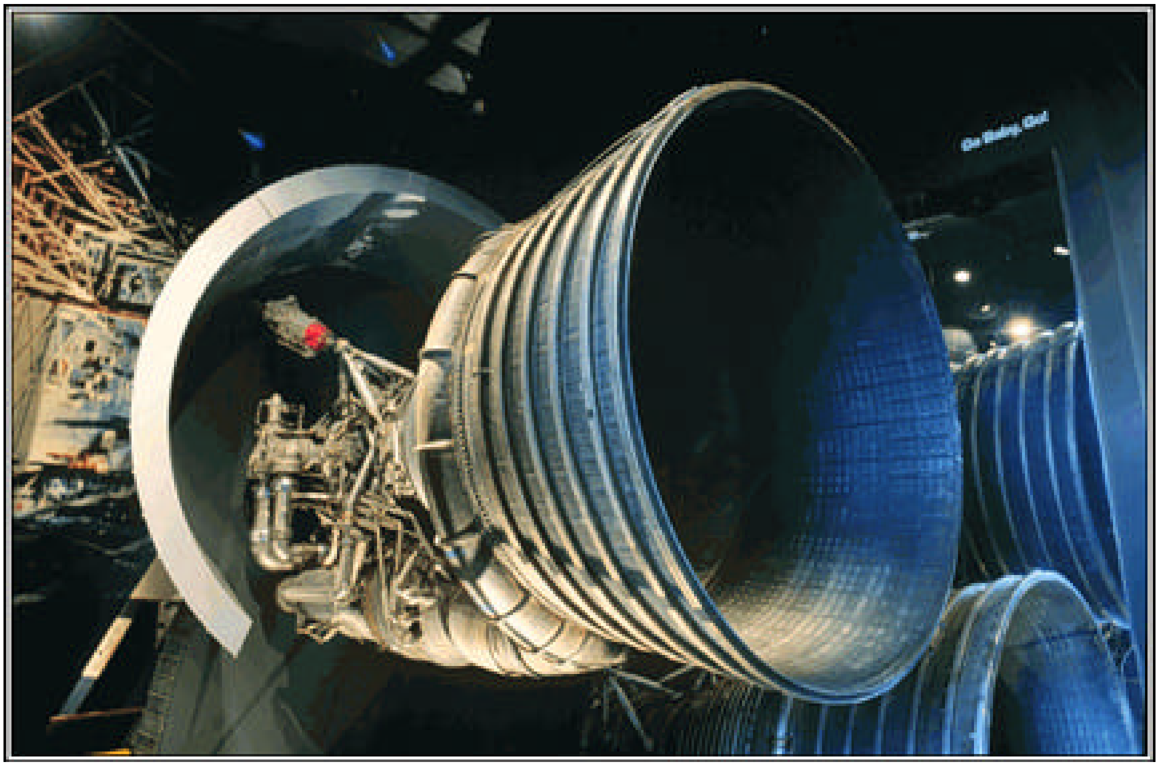
Solid propellants

Fuel	**Oxidizer**	**Isp (s)**
Powdered Al	Ammonium perchlorate	270

Year	Event
300 BC	Gunpowder-filled bamboo tubes used for fireworks in China
1045	Military rockets in use in China
1895	Konstantin Tsiolkovsky derives the fundamental rocket equation
1926	Robert Goddard launches first liquid-fueled rocket
1942	Wernher von Braun's team launches first successful A4 (V2)
1957	Sputnik launch
1958	Explorer I launch
1967	Saturn V first launch
1969	Apollo 11 Moon launch

Engine design

Chemical-rocket engines combine knowledge of physics, chemistry, materials, heat transfer, and many other fields in a complicated, integrated system. The F-1 engine used in the first stage of the Saturn V rockets that launched the Apollo missions appears.



Issues:

Heat transfer (Radiative cooling: radiating heat to space or conducting it to the atmosphere. Regenerative cooling: running cold propellant through the engine before exhausting it. Boundary-layer cooling: aiming some cool propellant at the combustion chamber walls. Transpiration cooling: diffusing coolant through porous walls).

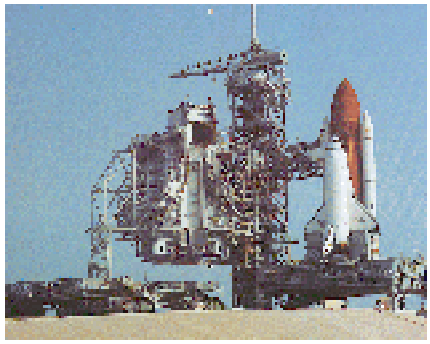
Nozzles (Rocket nozzles are usually of an expansion-deflection design. This allows better handling of the transition from subsonic flow within the combustion chamber to supersonic flow as the propellant expands out the end of the nozzle and produces thrust. Many nozzle variations exist. The governing equation for the magnitude of the thrust, in its simplest form, is $F = v_{ex}\, dm/dt + (p_{ex} - p_a)\, A_{ex} = v_{eq}\, dm/dt$, where v_{ex} is the exhaust velocity, dm/dt is the propellant mass flow rate, p_{ex} is the pressure of the exhaust gases, p_a is the pressure of the atmosphere, A_{ex} is the area of the nozzle at the exit, and v_{eq} is the equivalent exhaust velocity (that is, corrected for the pressure terms). If the pressure inside the chamber is too low, the flow will stagnate, while too high a pressure will give a turbulent exhaust--resulting in power wasted to transverse flow.

Saturn V

Wernher von Braun's team at Marshall Space Flight Center developed the three-stage Saturn V rocket, shown at right. The Saturn V served as the workhorse of the Apollo Moon launches. Its first stage developed over 30 MN (7.5 million lbs) of thrust and burned about 14 tonnes of propellant per second for 2.5 minutes.

Space Shuttle

See NASA's Space Shuttle home page.



Space Shuttle Parameters

Total length 56 m

Total height 23 m

Wingspan 24 m

Mass at liftoff $2x10^6$ kg

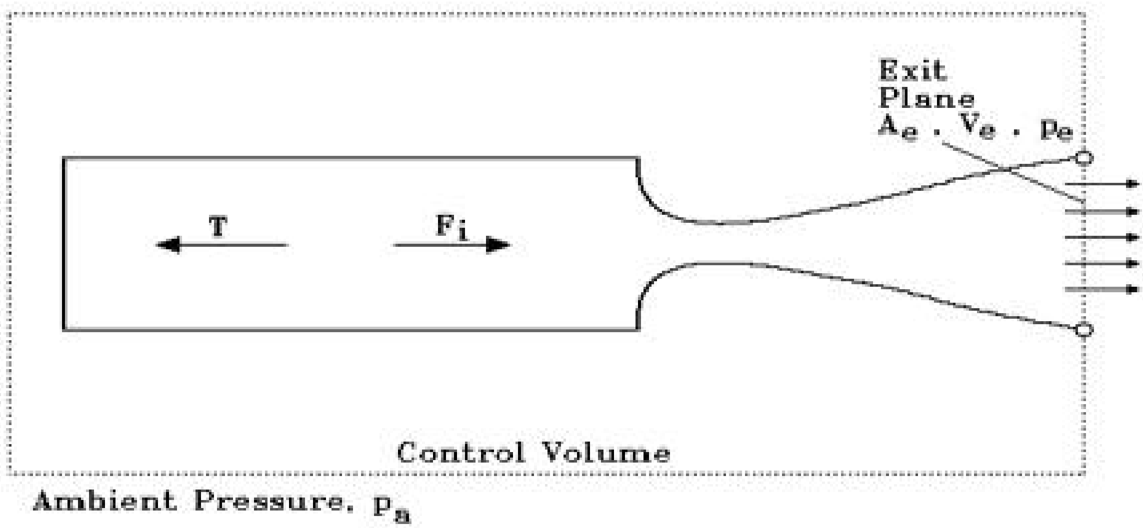
Orbiter dry mass 79,000--82,000 kg

Solid-rocket booster thrust, each of 2 15,000,000 N

SSME (main engine), each of 3 1,750,000 N

Thrust

Consider a control volume surrounding a rocket engine as shown



The net force, F, exerted on the control volume must be equal to the rate of increase of momentum through the control volume. But no momentum enters the control volume so Net force on control volume = Rate at which momentum leaves control volume

$$F = \dot{m} V_e \quad 1)$$

where

$$\dot{m} = -\frac{dM_F}{dt} \quad 2)$$

is rate at which mass leaves the control volume which, as indicated in eq. (2), is equal to the rate at which the mass of propellant, M_F, is decreasing with time. V_e is the discharge velocity from the nozzle.

The force, F, on the control volume arises due to the pressure force exerted on the gases within the rocket engine and due to the difference in pressure over the surface of the control volume. Now, the pressure is the same everywhere on the surface of the control volume except on the nozzle exit plane which has area A_e. Hence the net force on the control volume is given by:

$$F = F_i - (p_e - p_a) A_e \qquad 3)$$

But the thrust is equal in magnitude to the force acting on the rocket, F_i, so the magnitude of the thrust is given by eqs (1) and (3) as:

$$T = \dot{m} V_e + (p_e - p_a) A_e \qquad 4)$$

This can be written as follows by using eq. (2):

$$T = -\frac{d M_F}{d t} V_e + (p_e - p_a) A_e \qquad 5)$$

the negative sign arises because $\frac{d M_F}{d t}$ is negative i.e. the propellant mass M_F is decreasing.

The last term in eqs. (3), (4) and (5) arises because of the difference between the pressure on the exhaust plane of the nozzle and the local ambient pressure. As discussed later in this section, the nozzle is usually designed so that $p_e = p_a$ under a chosen set of conditions which are usually termed the "design conditions". In most cases, the pressure term in the above equations is much smaller than the momentum term and can, therefore, often be neglected. In this case, the rocket thrust is given by:

$$T = -\frac{d\,M_F}{d\,t}\,V_e \tag{6}$$

Specific Impulse

Another term that is widely used in defining the performance of a rocket engines is the Specific Impulse, I. This can either be defined as the thrust per unit mass flow rate of propellant i.e. as

$$I_m = \frac{T}{\dot{m}} = \frac{T}{-dM_F/dt} \tag{7a}$$

or it can be defined as the thrust per unit weight flow of propellant i.e. as:

$$I_w = \frac{T}{\dot{w}} = \frac{T}{\dot{m}\,g} = \frac{I_m}{g} \tag{7b}$$

The specific impulse is mainly dependent on the type of propellant used. An important parameter used in defining the overall performance of a propellant is the Total Impulse, I_T, which is equal to integral of thrust over the time of operation of the engine, t, i.e.

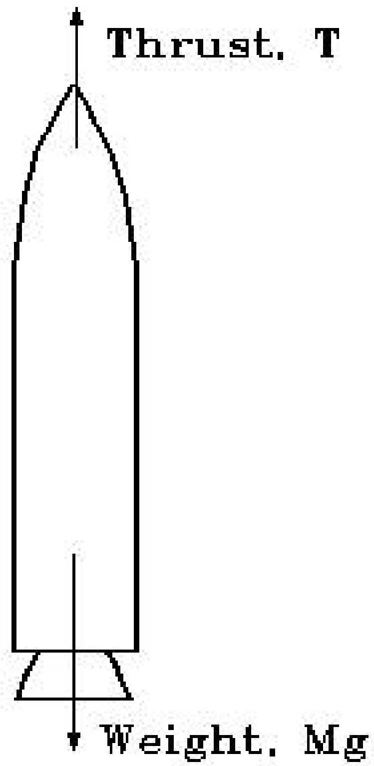
$$I_T = \int_0^t T\,d\,t = -\int_0^t I_m \frac{d\,M_F}{dt}\,d\,t \qquad 8)$$

If the thrust is constant, the total impulse is given by:

$$I_T = T\,t = -I_m \frac{d\,M_F}{dt}\,t = I_m\,M_{F0} = \frac{I_w\,M_{F0}}{g} \qquad 9)$$

where M_{F0} is the initial mass of propellant.

Example. A rocket with a mass of 1000 kg is placed vertically on the launching ramp. If the rate at which the propellants are consumed is equal to 2 kg/s, find the rocket exhaust velocity required if the rocket just begins to rise.



If the rocket just begins to rise, the thrust must be essentially equal to the weight i.e.

$$T = M g$$

i.e. using eq. (6):

$$-\frac{d M_F}{d t} V_e = M g$$

But $\frac{d M_F}{d t}$ = -2 kg/s, M = 1000 kg and g = 9.8 m/s^2:

$$V_e = \frac{1000 \times 9.8}{2} = 4900 \ m/s$$

The required exhaust velocity is, therefore, 4900 m/s. As will be seen later, this is within the range that is typical of chemical rockets.

Note: $I_{sp} = I / (m_p\ g) = v_{ex} / g$, where m_p is the propellant mass and g is Earth's surface gravity. In English units, I_{sp} is thus measured in seconds and is a force per weight flow. Often today, however, specific impulse is measured in the SI units meters/second [m/s], recognizing that force per mass flow is more logically satisfying. The specific impulse is then simply equal to the exhaust velocity, $I_{sp} = v_{ex}$.

Rocket Equation

In defining the performance of a rocket, the so-called Rocket Equation is used. This is derived by noting the if M is the mass of the rocket vehicle at any instant of time and V is its velocity at this time (see Fig. 6).

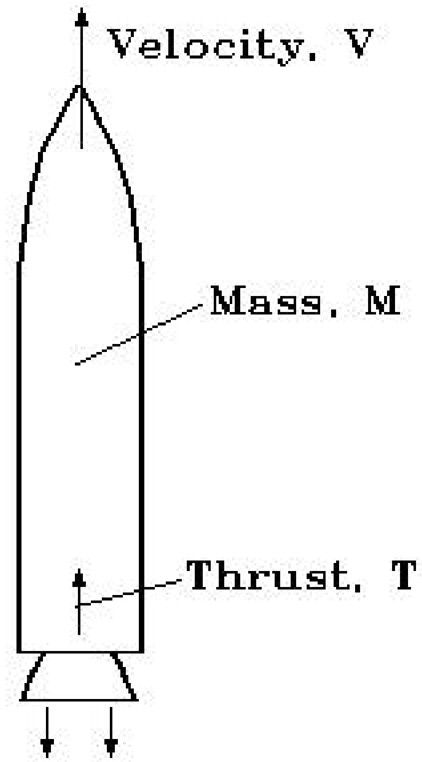


Fig. 6

Force acting on rocket = mass of rocket x acceleration of rocket, so that if the rocket is moving in a vertical direction:

$$T = M \frac{dV}{dt} + Mg \tag{10}$$

Hence using eq. (5):

$$M \frac{dV}{dt} = -\frac{dM_F}{dt} V_e - Mg \tag{11}$$

But

$$M = M_S + M_F$$

where M_S is the mass of the vehicle and motor structure and payload, i.e. the dry mass, which does not change with time and M_F is the fuel mass, it follows that:

$$\frac{dM}{dt} = \frac{dM_F}{dt} \qquad 12)$$

Hence, eq. (11) can be written as:

$$M\frac{dV}{dt} = -\frac{dM}{dt}V_e - Mg \qquad 13)$$

If the exhaust velocity can be assumed constant, this equation can be integrated to give:

$$-V_e\int_{M_i}^{M_f}\frac{dM}{M} = \int_{V_i}^{V_f} dV + g \qquad 4)$$

where M_i and M_f are the initial and final masses of the rocket vehicle and V_i and V_f are its initial and final velocities. Carrying out the integrations then gives:

$$V_e \ln\left(\frac{M_i}{M_f}\right) = (V_f - V_i) + gt \qquad 15)$$

Defining:

$$\Delta V = V_f - V_i \qquad 16)$$

equation (14) shows that:

$$\Delta V = V_e \ln\left(\frac{M_i}{M_f}\right) - g\,t \qquad 17)$$

In the above analysis, it was assumed that g could be treated as constant. This may not always be a justifiable assumption.

If the vehicle initially has a velocity of zero, the velocity achieved when all the fuel has been used, this velocity being termed the burn-out velocity, V_b, is given by eq. (17) as:

$$V_b = V_e \ln\left(\frac{M_i}{M_f}\right) - g\,t \qquad 18)$$

i.e.

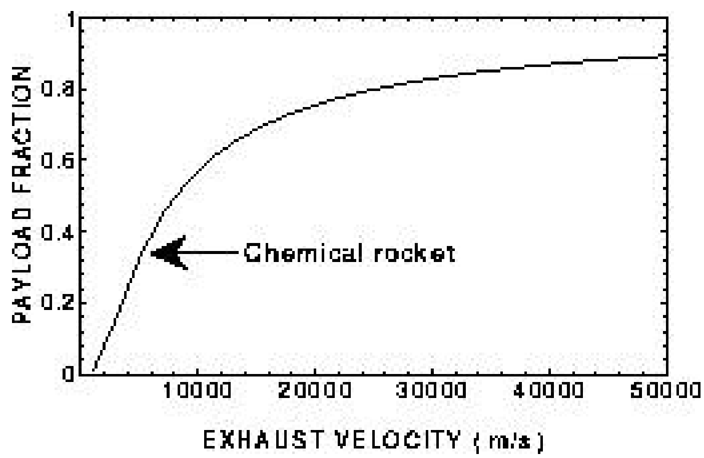
$$V_b = V_e \ln\left(\frac{M_S + M_F}{M_S}\right) - g\,t \qquad 19$$

M_F being the initial mass of fuel and M_S being the mass of the structure and the payload, M_f being equal to M_S since all the propellant has been used at time t. Equation (18) is basically what is referred to as the rocket equation. The quantity

$$\frac{M_i}{M_f}$$

is the rocket mass ratio.

A high exhaust velocity has historically been a driving force for rocket design: payload fractions depend strongly upon the exhaust velocity, as shown (Eqn 17).



It should be realized that in deriving the equations given above for the rocket velocity, the effects of atmospheric drag have been neglected. If this is accounted for, eq. (17), for example, will become:

$$\Delta V = V_e \ln\left(\frac{M_i}{M_f}\right) - g\,t - \int_0^t \frac{D}{M}\,d\,t$$

where D is the drag force acting on the rocket at any instant of time. Its value depends on the size and shape of the vehicle, the velocity, the Mach number and the local properties of the atmosphere through which the vehicle is passing.

Example. A rocket engine which has an exhaust velocity of 3500 m/s is used accelerate a vertically launched rocket vehicle to a speed of 4000 m/s. Find the approximate ratio of the propellant mass to the dry vehicle mass required.

Equation (19) gives:

$$\frac{V_b}{V_e} = \ln\left(1 + \frac{M_F}{M_S}\right) - \frac{g\ t}{V_e}$$

If it is assumed that gt<<, this equation gives approximately:

$$\frac{V_b}{V_e} = \ln\left(1 + \frac{M_F}{M_S}\right)$$

i.e.:

$$\frac{4000}{3700} = \ln\left(1 + \frac{M_F}{M_S}\right)$$

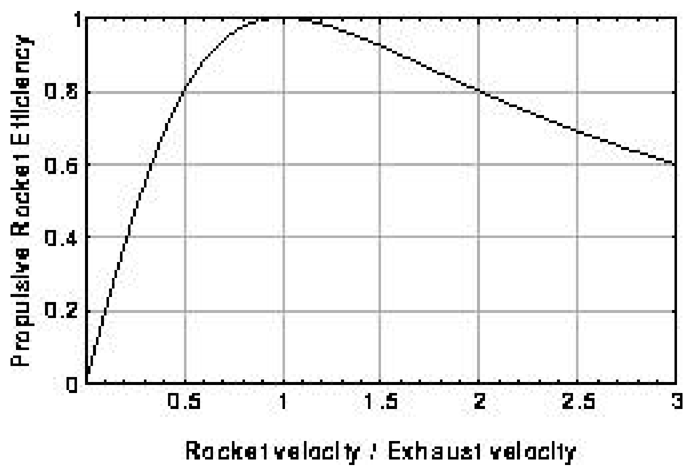
Hence,

$$\frac{M_F}{M_S} = 1.948$$

Therefore, the fuel mass is approximately twice the dry weight of the vehicle.

Efficiency

When the total kinetic energy of the rocket and its exhaust are taken into consideration, the highest efficiency occurs when the exhaust velocity is equal to the instantaneous rocket velocity, as shown in figure.



Trajectories

Spacecraft today essentially all travel by being given an impulse that places them on a trajectory in which they coast from one point to another, perhaps with other impulses or gravity assists along the way. The gravity fields of the Sun and planets govern such trajectories. Rockets launched through atmospheres face additional complications, such as air friction and winds. Most of the present discussion treats this type of trajectory.

Advanced propulsion systems and efficient travel throughout the Solar System will be required for human exploration, settlement, and accessing space resources. Rather than coasting, advanced systems will thrust for most of a trip, with higher exhaust velocities but lower thrust levels. These more complicated trajectories require advanced techniques for finding optimum solutions.

Newton's laws of motion

The fundamental laws of mechanical motion were first formulated by Sir Isaac Newton (1643-1727), and were published in his Philosophia Naturalis Principia Mathematica. They are:

1. Every body continues in its state of rest or of uniform motion in a straight line except insofar as it is compelled to change that state by an external impressed force.

2. The rate of change of momentum of the body is proportional to the impressed force and takes place in the direction in which the force acts.

3. To every action there is an equal and opposite reaction. (dp / dt = F)

Calculus, invented independently by Newton and Gottfried Leibniz (1646-1716), plus Newton's laws of motion are the mathematical tools needed to understand rocket motion.

Newton's law of gravitation

To calculate the trajectories for planets, satellites, and space probes, the additional relation required is Newton's law of gravitation:

Every particle of matter attracts every other particle of matter with a force directly proportional to the product of the masses and inversely proportional to the square of the distance between them.

Symbolically, the force is $F = -G\, m_1\, m_2\, e_r / r^2$, where $G = 6.67 \times 10^{-11}\ m^3\, s^{-2}\, kg^{-1}$, m1 and m2 are the interacting masses (kg), r is the distance between them (m), and e_r is a unit vector pointing between them.

Kepler's laws of planetary motion

The discovery of the laws of planetary motion owed a great deal to Tycho Brahe's (1546-1601) observations, from which Johannes Kepler (1571-1630) concluded that the planets move in elliptical orbits around the Sun. First, however, Kepler spent many years trying to fit the orbits of the five then-known planets into a framework based on the five regular platonic solids. The laws are:

1. The planets move in ellipses with the sun at one focus.

2. Areas swept out by the radius vector from the sun to a planet in equal times are equal.

3. The square of the period of revolution is proportional to the cube of the semi-major axis. That is, $T^2 = \text{const} \times a^3$

Conic sections

In a central-force gravitational potential, bodies will follow conic sections.

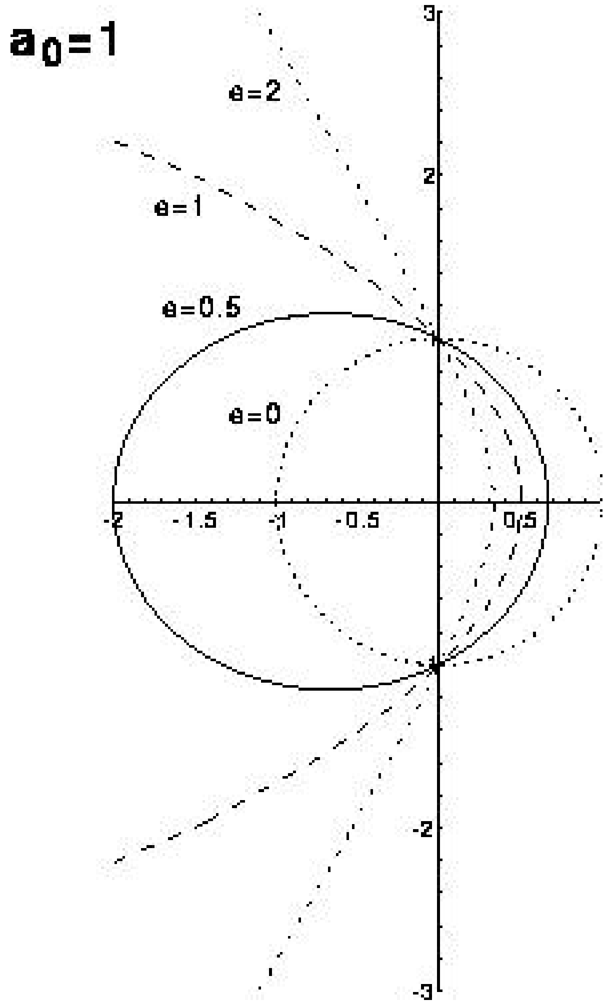
$$r = a_0/(1 + e\cos\theta)$$

$$a = a_0/(1 - e^2)$$

e is eccentricity and a is the semi-major axis.

Special cases (E is the (constant) energy of a body on its trajectory).

e>1	E>0	hyperbola
e=1	E=0	parabola
e<1	E<0	ellipse
e=0		circle



Some important equations of orbital dynamics

Circular velocity

$$v_{cir} = \left(\frac{GM}{R}\right)^{\frac{1}{2}}$$

Escape velocity

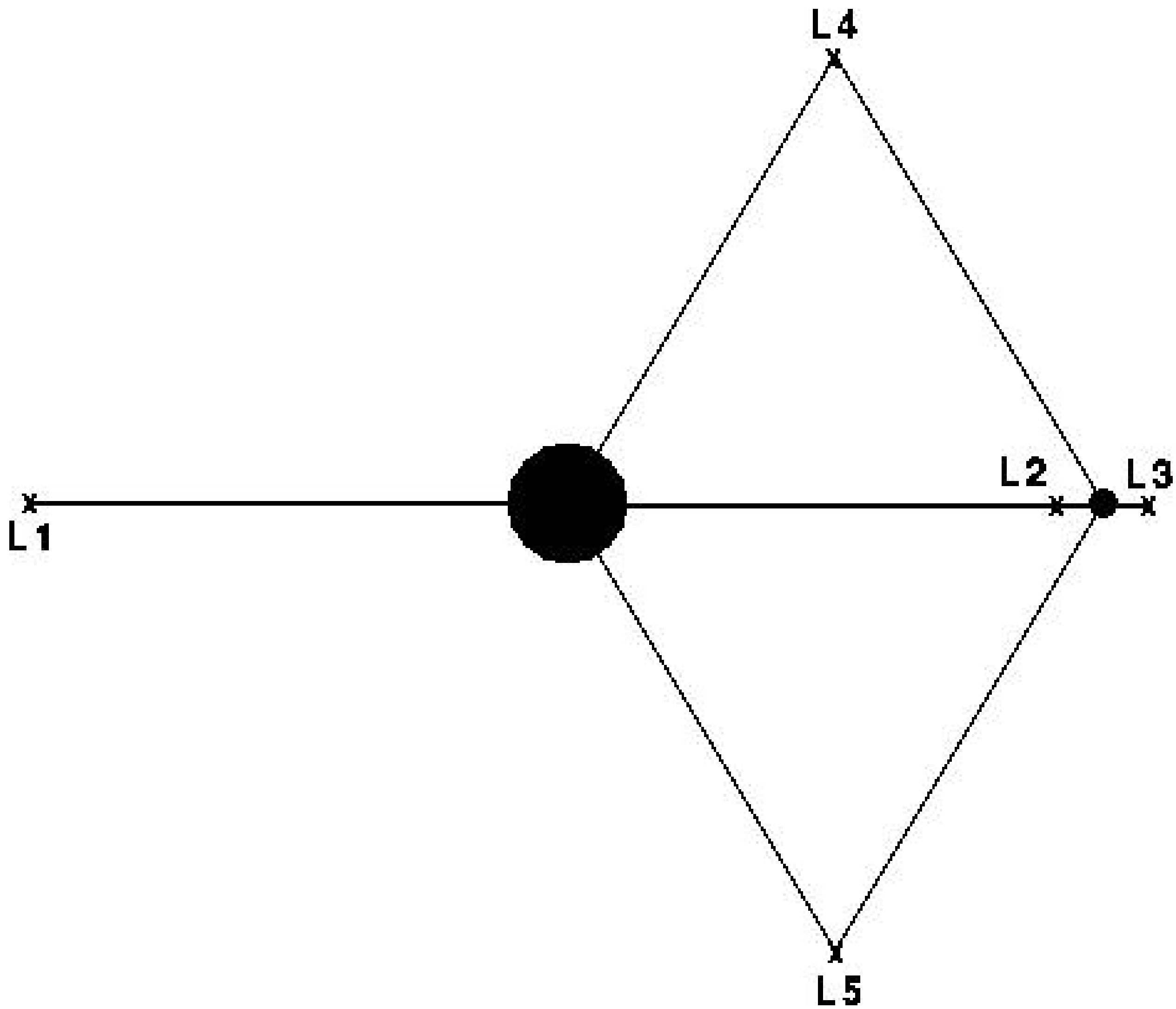
$$v_{esc} = \left(\frac{2GM}{R}\right)^{\frac{1}{2}}$$

Energy of a vehicle following a conic section, where a is the semi-major axis

$$E_{conic} = \frac{-GMm}{2a}$$

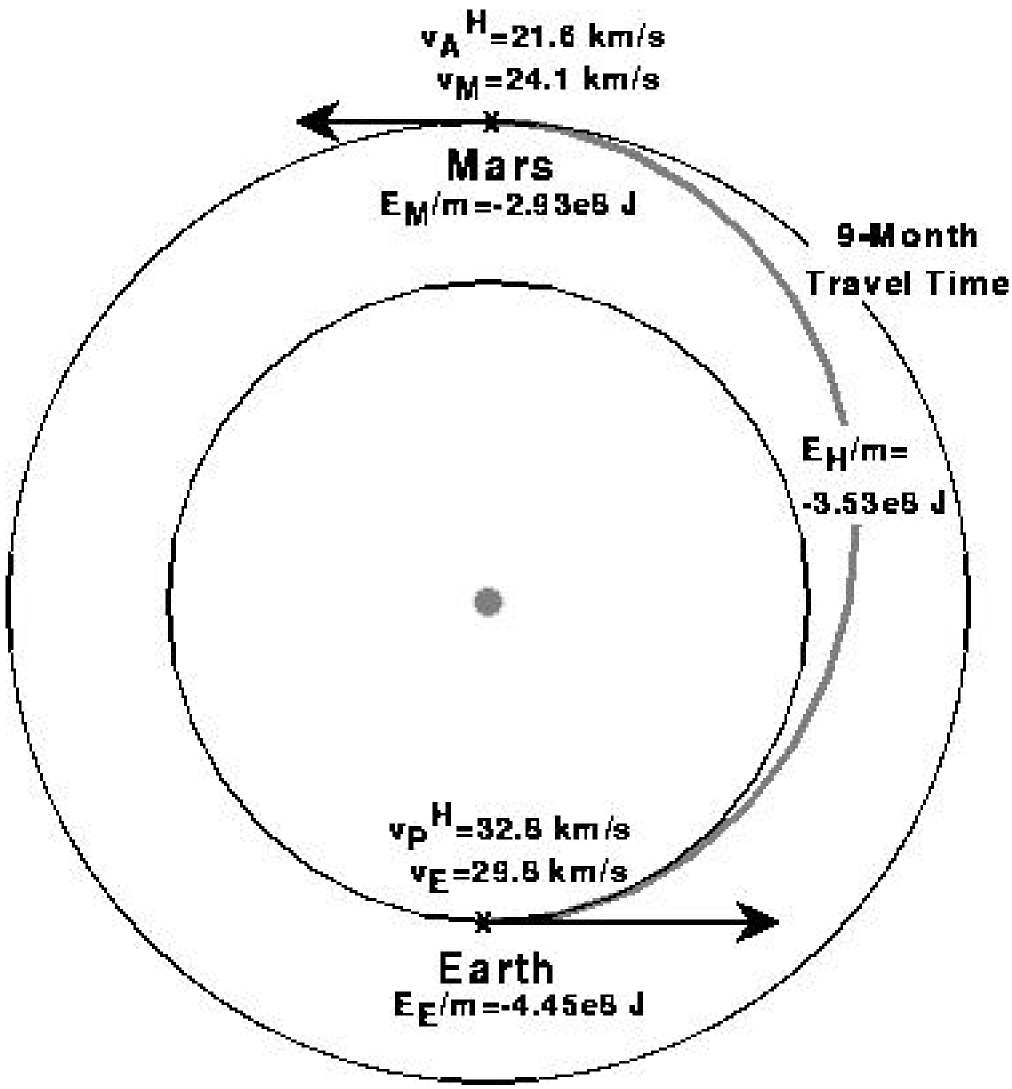
Lagrange points

The Lagrange (sometimes called Libration) points are positions of equilibrium for a body in a two-body system. The points L1, L2, and L3 lie on a straight line through the other two bodies and are points of unstable equilibrium. That is, a small perturbation will cause the third body to drift away. The L4 and L5 points are at the third vertex of an equilateral triangle formed with the other two bodies; they are points of stable equilibrium. The approximate positions for the Earth-Moon or Sun-Earth Lagrange points are shown below.



Hohmann minimum-energy trajectory

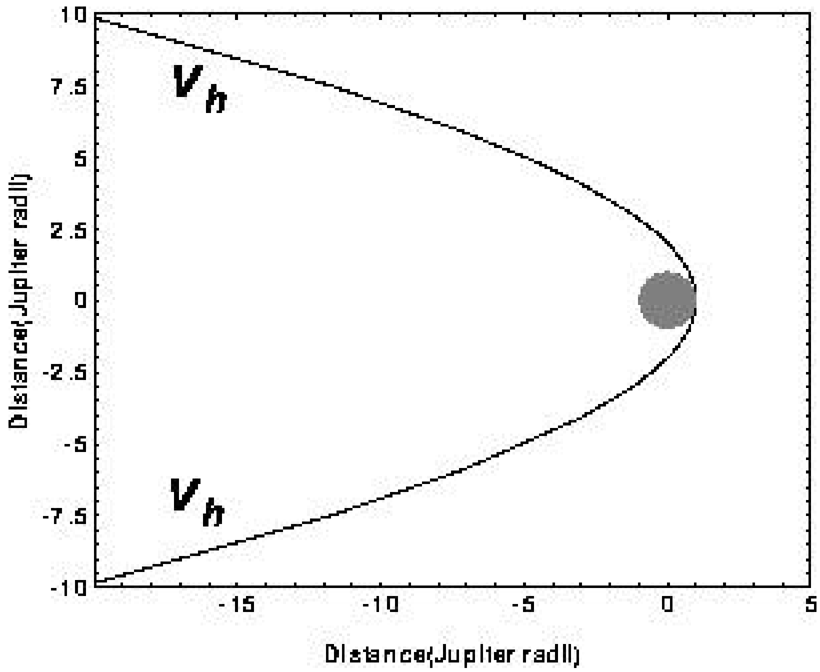
The minimum-energy transfer between circular orbits is an elliptical trajectory called the Hohmann trajectory. It is shown for the Earth-Mars case, where the minimum total delta-v expended is 5.6 km/s. The values of the energy per unit mass on the circular orbit and Hohmann trajectory are shown, along with the velocities at perihelion(closest to Sun) and aphelion (farthest from Sun) on the Hohmann trajectory and the circular velocity in Earth or Mars orbit. The differences between these velocities are the required delta-v values in the rocket equation.



Gravity assist

Gravity assists enable or facilitate many missions. A spacecraft arrives within the sphere of influence of a body with a so-called hyperbolic excess velocity equal to the vector sum of its incoming velocity and the planet's velocity. In the planet's frame of reference, the direction of the spacecraft's velocity changes,

but not its magnitude. In the spacecraft's frame of reference, the net result of this trade-off of momentum is a small change in the planet's velocity and a very large delta-v for the spacecraft. Starting from an Earth-Jupiter Hohmann trajectory and performing a Jupiter flyby at one Jovian radius, as shown, the hyperbolic excess velocity v_h is approximately 5.6 km/s and the angular change in direction is about 160^o.



High-exhaust-velocity, low-thrust trajectories

The simplest high-exhaust-velocity analysis splits rocket masses into three categories:

1. Power plant and thruster system mass, M_w.

2. Payload mass, M_l. (Note that this includes all structure and other rocket mass that would be treated separately in a more sophisticated definition.)

3. Propellant mass, M_p.

Mission power-on time tau

Total mass $M_0 = M_w+M_l+M_p$

Empty mass $M_e = M_w+M_l$

Specific power [kW/kg]

$$\alpha\left[\frac{\mathrm{kW}}{\mathrm{kg}}\right] \equiv \frac{P_w}{M_w} \equiv \frac{P_{thrust}}{M_{power\ system}+M_{thrust\ system}}$$

Propellant flow rate

$$\dot{M} = \frac{M_p}{\tau}$$

Thrust power

$$P_w = \frac{1}{2}\dot{M}v_{ex}^2 = \frac{M_p v_{ex}^2}{2\tau}$$

Thrust

$$F = \dot{M}v_{ex} = \frac{M_p v_{ex}}{\tau}$$

High-exhaust-velocity rocket equation

Assume constant exhaust velocity, v_{ex}, which greatly simplifies the analysis. The empty (final) mass in the Tsiolkovsky rocket equation now becomes M_W+M_l, so

$$\frac{M_f}{M_0} = \frac{M_e}{M_0} = \frac{M_w+M_L}{M_w+M_l+M_p} = \exp\left(\frac{-u}{v_{ex}}\right).$$

where u measures the energy expended in a manner analogous to delta-v. After some messy but straightforward algebra, we get the high-exhaust-velocity rocket equation:

$$\frac{M_L}{M_0} = \exp\left(\frac{-u}{v_{ex}}\right) - \frac{v_{ex}^2}{2\alpha\tau}\left[1-\exp\left(\frac{-u}{v_{ex}}\right)\right]$$

Note that a chemical rocket effectively has $M_W = 0$ ==> alpha = infinity, and the Tsiolkovsky equation ensues. The quantity alpha*tau is the energy produced by the power and thrust system during a mission with power-on time tau divided by the mass of the propulsion system. It is called the specific energy of the power and thrust system.

Relating the specific energy to a velocity through E = $mv^2/2$ gives the definition of a very important quantity, the characteristic velocity:

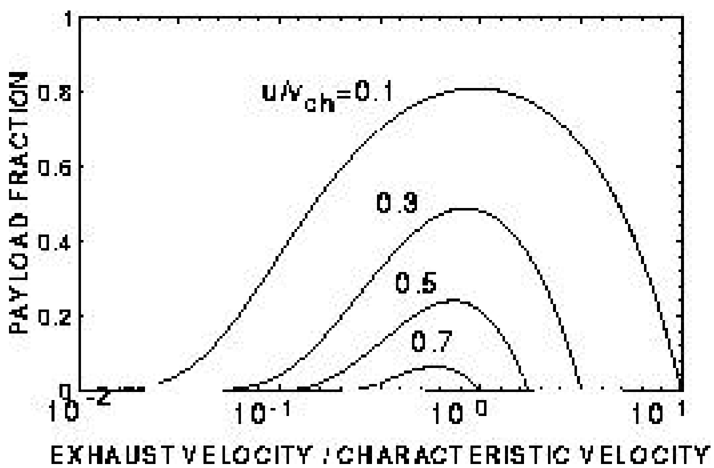
$$v_{ch} \equiv (2\alpha\tau)^{\frac{1}{2}}$$

.

The payload fraction for a high-exhaust-velocity rocket becomes

$$\frac{M_L}{M_0} = \exp\left(\frac{-u}{v_{ex}}\right) - \frac{v_{ex}^2}{v_{ch}^2}\left[1 - \exp\left(\frac{-u}{v_{ex}}\right)\right]$$

which is plotted below.

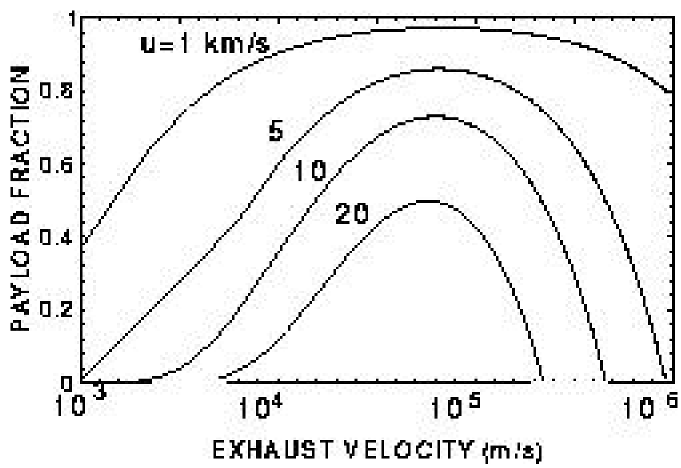


Analyzing a trajectory using the characteristic velocity method requires an initial guess for tau plus some iterations. The minimum energy expended will always be more than the Hohmann-trajectory energy. The payload capacity of a fixed-velocity rocket vanishes at u = 0.81 v_{ch}, where v_{ex} = 0.5 v_{ch}. Substituting these values into the rocket equation gives

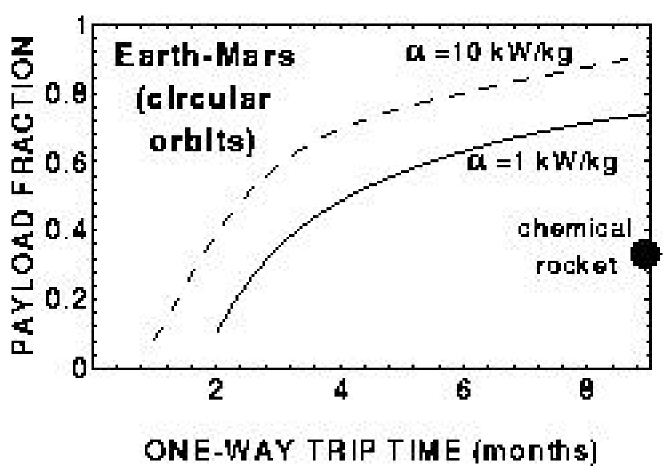
$$\frac{M_0}{M_e} = \exp\left(\frac{u}{v_{ex}}\right) = \exp\left(\frac{0.81}{0.5}\right) \simeq 5.$$

Example: 9 month Earth-Mars trajectory

(alpha = 0.1 kW/kg, alpha tau = 2x109 J/kg.) NB: When the distance traveled is factored into the analysis, only u > 10 values turn out to be realistic.



Trade-off between payload fraction and trip time for selected missions.



Variable exhaust velocity and gravity

Variable exhaust velocity and gravity considerably complicate the problem. When the exhaust velocity is varied during the flight, variational principles are needed to calculate the optimum v(t). The key result is that it is

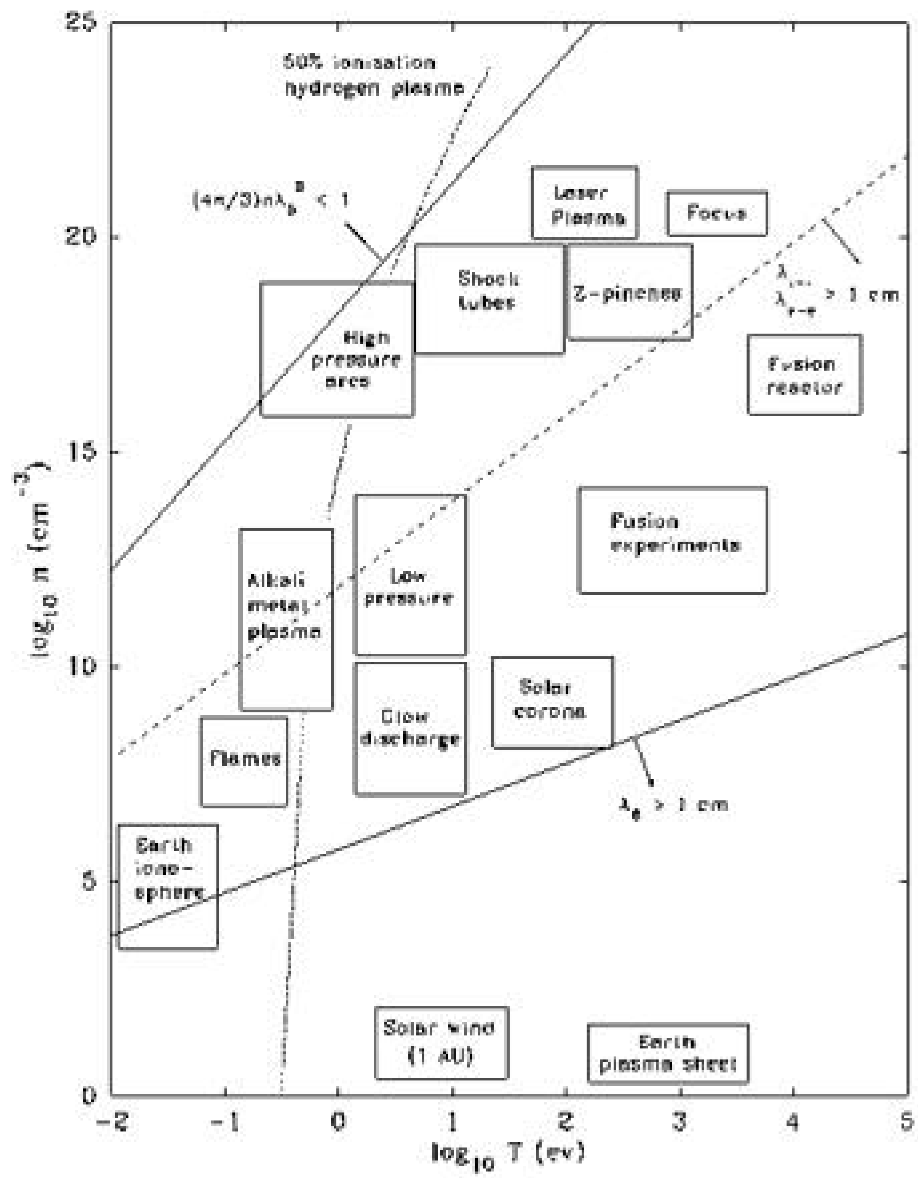
$$\int a^2(t)\, dt$$

necessary to minimize . Even the simplest problem with gravity, the central-force problem, is difficult and requires advanced techniques, such as Lagrangian dynamics and Lagrange multipliers. In general, trajectories must be found numerically, and finding the optimum in complex situations is an art.

Plasma and electric propulsion

Year	People	Event
1906	Robert H. Goddard	Brief notebook entry on possibility of electric propulsion
1929	Hermann Oberth	Wege zur Raumschiffahrt chapter devoted to electric propulsion
1950	Forbes and Lawden	First papers on low-thrust trajectories
1952	Lyman Spitzer, Jr.	Important ion-engine plasma physics papers
1953	E. Saenger	Zur Theorie der Photonrakete published
1954	Ernst Stuhlinger	Important analysis. I ntroduces specific power
1958	Rocketdyne Corp.	First ion-engine model operates
1960	NASA Lewis; JPL	NASA establishes an electric propulsion research program
1964	Russians	Operate first plasma thruster in space (Zond-2)

Plasma physics overview



Key equations Maxwell's equations for the microscopic electric (E) and magnetic (B) fields

$$\nabla \cdot \vec{E} = \rho/\epsilon_0$$
$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} = -\partial\vec{B}/\partial t$$
$$\nabla \times \vec{B} = \mu_0\vec{j} + c^{-2}\partial\vec{E}/\partial t$$

Electrostatic potential definition and Poisson's equation

$$\mathbf{E} = -\nabla\Phi$$
$$\nabla^2\Phi = \rho/\epsilon_0$$

Lorentz force on a particle of charge q

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

The result of this equation is that charged particles spiral along lines of magnetic force with the gyrofrequency (cyclotron frequency)

$$\omega_c = qB/m$$

at a distance called the gyro radius (Larmor radius)

$$v_{th}/\omega_c$$,

where

$$v_{th} = \left(2T/m\right)^{\frac{1}{2}}$$

is the average thermal velocity of a particle.

Because the electron's mass is much smaller than any ion's mass, the electron gyrofrequency is much faster than the ion gyrofrequency and the electron gyro radius is much smaller.

The Lorentz force leads to several charged-particle drifts, even in static electric and magnetic fields. These are:

ExB drift:

$$\vec{v}_E \equiv \frac{\vec{E}\times\vec{B}}{B^2}$$

Grad-B drift:

$$\vec{v}_B \equiv \frac{mv_\perp^2}{2q}\left(\frac{\vec{B}\times\nabla B}{qB^2}\right)$$

Curvature drift:

$$\vec{v}_R \equiv mv_\parallel^2\left(\frac{\vec{R}_c\times\nabla B}{qR_c^2B^2}\right)$$

Plasmas Will Try to Reach Thermodynamic Equilibrium. Neglecting boundary effects, equilibrium is represented by the Maxwell--Boltzmann or Maxwellian distribution of particles in energy,

$$f_M(v) = n_0\left(\frac{m}{2\pi k_BT}\right)^{3/2}\exp\left(\frac{-mv^2}{2k_BT}\right),$$

where n_0 is the average charge density and Boltzmann's constant is $k_B = 1.38 \times 10^{-23}$ J/K = 1.6×10^{-19} J/eV. The latter value is given because it is often convenient to measure plasma energies and temperatures in electron volts, eV, rather than Kelvin, K (1 eV = 11,604 K).

Plasmas are Dynamic Entities

Electrons are extremely mobile. For example, the typical velocities for ions and electrons in a hydrogen plasma are

$$v_i = (2T_i/m_i)^{\frac{1}{2}} = 1.39 \times 10^4 T_i^{\frac{1}{2}}(eV)/(m_i/m_p)^{\frac{1}{2}},\ [\mathrm{m/s}]\ \mathrm{and}$$
$$v_e = (2T_e/m_e)^{\frac{1}{2}} = 5.93 \times 10^5 T_e^{\frac{1}{2}}(eV)\ [\mathrm{m/s}].$$

Debye Shielding

An important consequence of the high plasma mobility is Debye shielding, in which electrons tend to cluster around negative density fluctuations and to avoid positive density fluctuations. The Debye length, or Debye screening distance, gives an estimate of the extent of the influence of a charge fluctuation. It plays an extremely important role in many problems. The Debye length is given by

$$\lambda_D^2 = \epsilon_0 T/n_0 e^2.$$

Plasma Parameter

The number of particles, N, in a Debye sphere (sphere with radius equal to the Debye length) must satisfy N>>1 in order for there to be statistical significance to the Debye shielding mechanism:

$$N = 4\pi \lambda_D^3 n_0/3.$$

In general, the condition N>>1 is necessary for collective effects to be important.

Electrostatic potential sheaths

Near any surface, and sometimes in free space, electron and ion flows can set up electrostatic potential differences, called sheaths. Commonly, these are approximately three times the electron temperature. Physically, sheaths set up in order to conserve mass, momentum, and energy in the particle flows. Sheaths repel electrons, which have high mobility, and attract ions. Free-space sheaths are called double layers.

Quantum mechanics and atomic physics

Quantum mechanics enters the world of plasma thrusters because line radiation--the light emitted when electrons move down energy levels in an atom--can be a significant energy loss mechanism for a plasma.

Other important phenomena include collisions and charge exchange (electron transfer between ions and atoms or other ions). Two important plasma regimes for radiation transport can be analyzed with relative ease:

Local thermodynamic equilibrium (LTE)

High density plasma, so collisional effects dominate radiative ones.

Characterized by the electron temperature, because electrons dominate the collisional processes.

Coronal equilibrium

Optically thin plasma

Collisional ionization, charge exchange, and radiative recombination dominate.

High-Exhaust-Velocity Thrusters

Plasma and electric thrusters generally give a higher exhaust velocity but lower thrust than chemical rockets. They can be classified roughly into five groups, the first three of which are relevant to the present topic and will be discussed in turn.

Electrothermal
- Resistojet
- Arcjet
- RF-heated

Electrostatic
- Ion

Electrodynamic
- Magnetoplasmadynamic (MPD)
- Hall-effect
- Pulsed-plasma
- Helicon

Photon
- Solar sail
- Laser

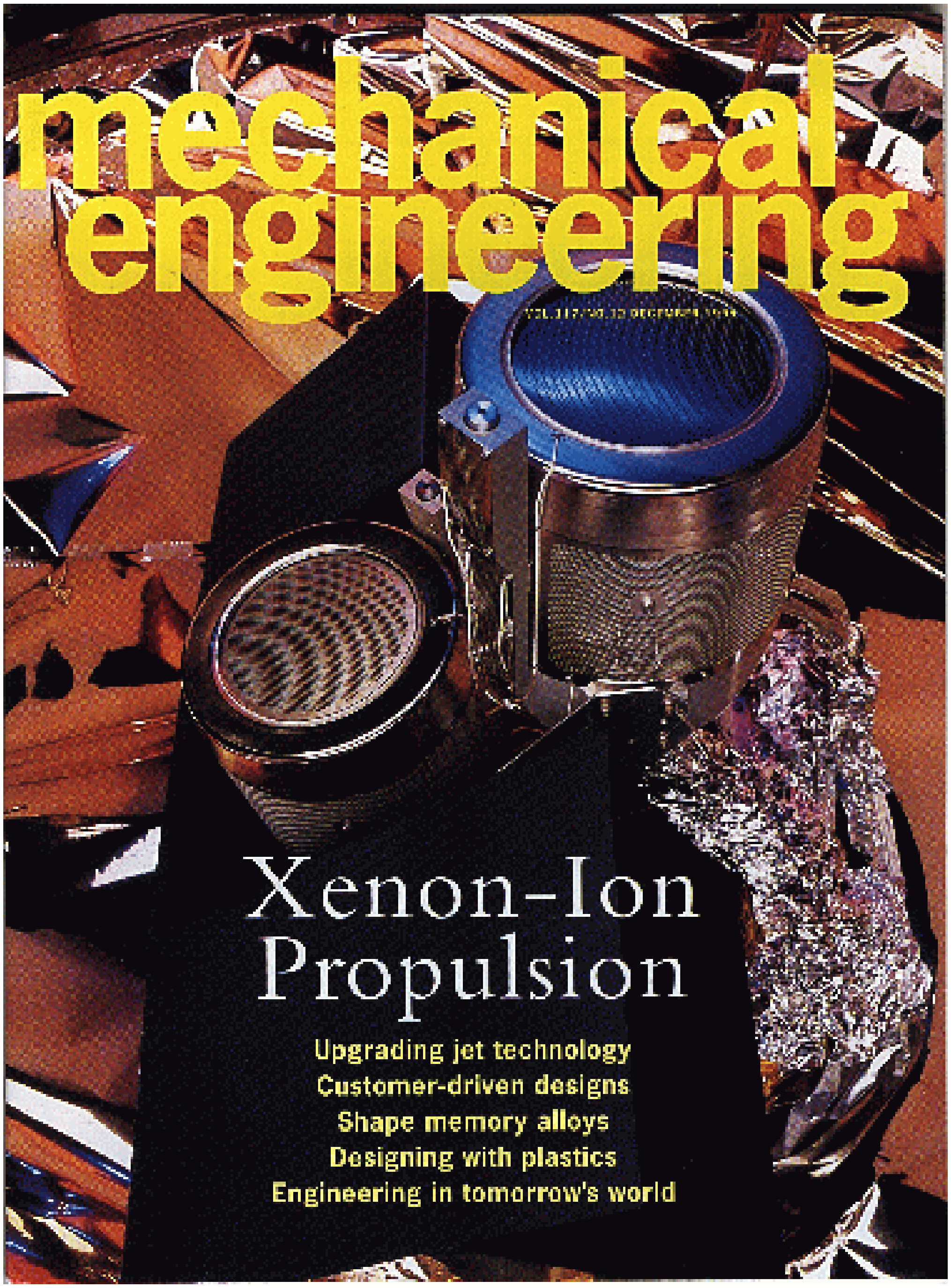
Advanced
- Fusion
- Gas-core fission
- Matter-antimatter annihilation
- Tether
- Magnetic sail

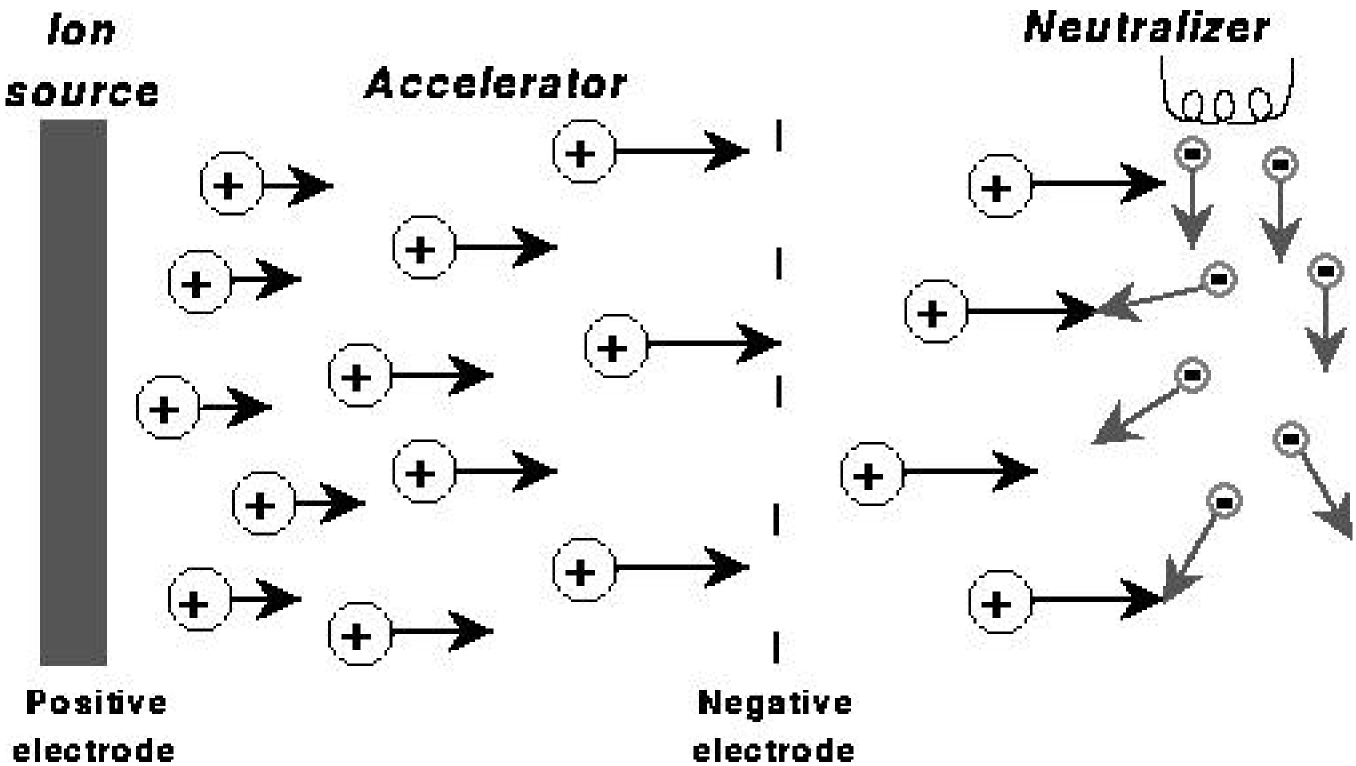
Electrothermal thrusters

This class of thrusters (resistojet, arcjet, RF-heated thruster) does not achieve particularly high exhaust velocities. The resistojet essentially uses a filament to heat a propellant gas (not plasma), while the arcjet passes propellant through a current arc. In both cases material characteristics limit performance to values similar to chemical rocket values. The RF-heated thruster uses radio-frequency waves to heat a plasma in a chamber and potentially could reach somewhat higher exhaust velocities.

Electrostatic thrusters (ion thrusters)

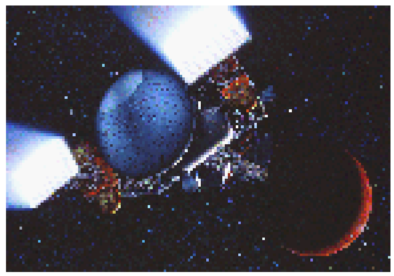
This class has a single member, the ion thruster. Its key principle is that a voltage difference between two conductors sets up an electrostatic potential difference that can accelerate ions to produce thrust. The ions must, of course, be neutralized--often by electrons emitted from a hot filament. The three main stages of an ion-thruster design are ion production, acceleration, and neutralization. They are illustrated in the figure below. The basic geometry of an actual ion thruster appears at right on the cover from a recent Mechanical Engineering (from PEPL home page).


mechanical
engineering
Xenon-Ion
Propulsion
Upgrading jet technology
Customer-driven designs
Shape memory alloys
Designing with plastics
Engineering in tomorrow's world


Ion source
Accelerator
Neutralizer
Positive electrode
Negative electrode

Fission reactors are located at the ends of the long booms in these nuclear-electric propulsion (NEP) systems. These and other designs are shown on NASA Lewis Research Center's Advanced Space Analysis Office's (ASAO) project Web page.

NEP Mars approach



Hydra multiple-reactor NEP vehicle

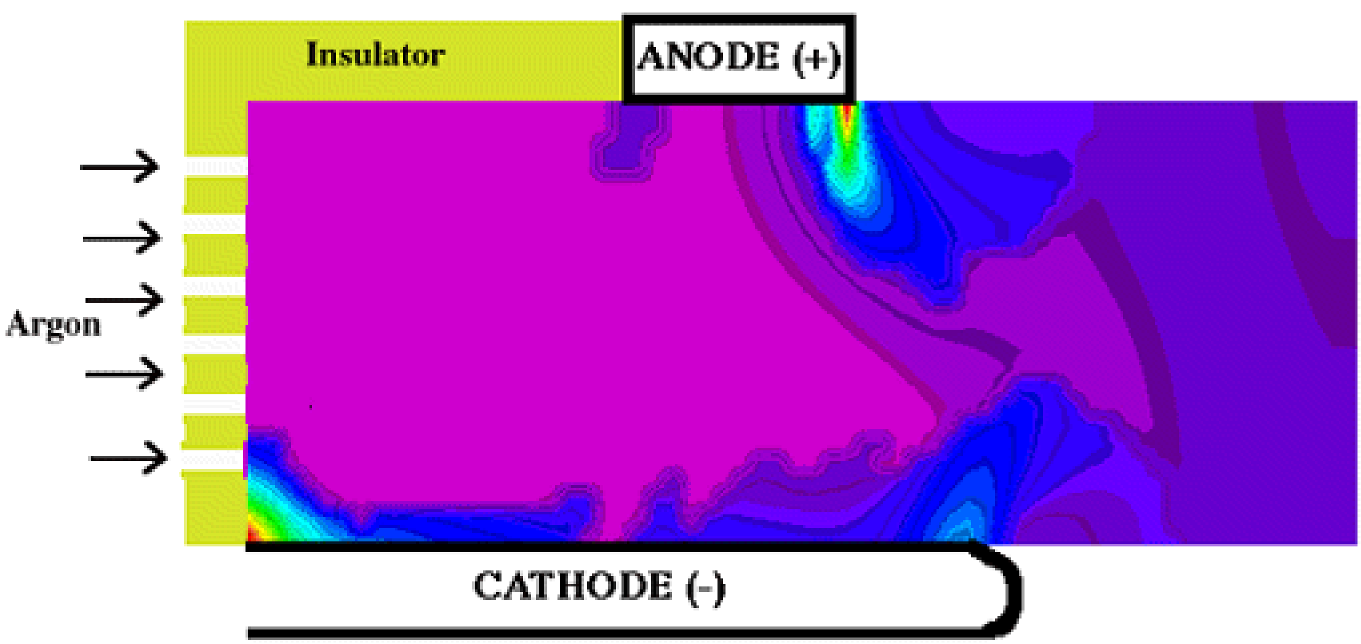
Electrodynamic thrusters

Magnetoplasmadynamic (MPD) thruster

In MPD thrusters, a current along a conducting bar creates an azimuthal magnetic field that interacts with the current of an arc that runs from the point of the bar to a conducting wall. The resulting Lorentz force has two components:

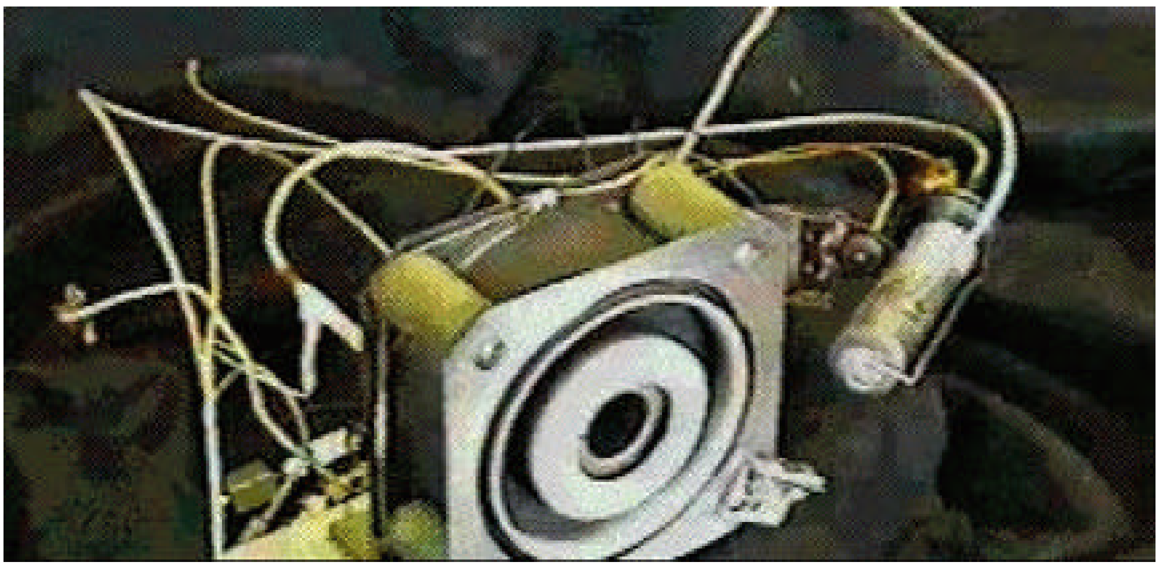
> Pumping: a radially inward force that constricts the flow.
>
> Blowing: a force along the axis that produces the directed thrust.

The basic geometry is shown from a computer simulation done by Princeton University's Electric Propulsion and Plasma Dynamics Lab. Erosion at the point of contact between the current and the electrodes generally is a critical issue for MPD thruster design.



Hall-effect thruster

In Hall-effect thrusters, perpendicular electric and magnetic fields lead to an ExB drift. For a suitably chosen magnetic field magnitude and chamber dimensions, the ion gyro radius is so large that ions hit the wall while electrons are contained. The resulting current, interacting with the magnetic field, leads to a JxB Lorentz force, which causes a plasma flow and produces thrust. The Russian SPT thruster is presently the most common example of a Hall-effect thruster.

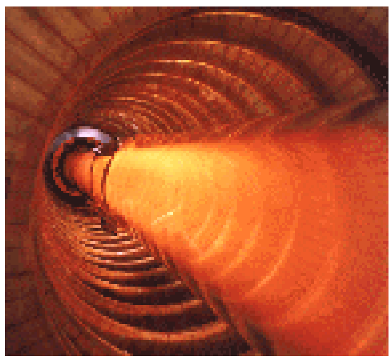


Pulsed-plasma thruster

In a pulsed-plasma accelerator, a circuit is completed through an arc whose interaction with the magnetic field of the rest of the circuit causes a JxB force that moves the arc along a conductor.

Helicon thruster

The principle of the helicon thruster is similar to the pulsed-plasma thruster: a traveling electromagnetic wave interacts with a current sheet to maintain a high JxB force on a plasma moving along an axis. This circumvents the pulsed-plasma thruster's problem of the force falling off as the current loop gets larger. The traveling wave can be created in a variety of ways, and a helical coil is often used. The plasma and coil of a helicon device is shown.



Useful references on plasma and electric thrusters

Robert G. Jahn, Physics of Electric Propulsion (McGraw-Hill, New York, 1968).

Ernst Stuhlinger, Ion Propulsion for Space Flight (McGraw-Hill, New York, 1964).

Fusion propulsion

1950's to 1970's: Conceptual designs were formulated for both D-^{3}He and D-T fusion reactors for space propulsion. These included simple mirror reactors and Toroidal reactors with magnetic divertors. Both type-II superconductivity and fusion power were new concepts in the early 1960's when the first D-^{3}He design was performed. Three recent advances enhance the feasibility of D-^{3}He reactors for space applications.

Lunar ^{3}He resources

More credible physics concepts

Advances in technology

Two interesting early papers on D-^{3}He space-propulsion reactors are Englert (1962) and Hilton, et al. (1964). These papers followed much of the same logic given in the present discussion to propose using the D-^{3}He fuel cycle in linear magnetic fusion reactors. Although we now know that the simple-mirror concept used in the earliest papers cannot achieve a sufficiently high Q (ratio of fusion power out to required input power), which probably must be on the order of 10 or more, they presented many interesting ideas and recognized several important engineering approaches.

Later papers, such as Roth, et al. (1972), examined the idea of adding 'bucking' coils to extract a magnetic flux tube from a toroidal magnetic fusion reactor and exhaust the thrust. Although this geometry may work in relatively low magnetic field toroidal reactors, it would require massive coils and be extremely difficult for the present mainline concept, the tokamak (see lecture 26), where the magnetic fields in conceptual designs approach practical limits of about 20 T.

Magnetic fusion fuels for space applications

Advantages of D-^{3}He magnetic fusion for space applications

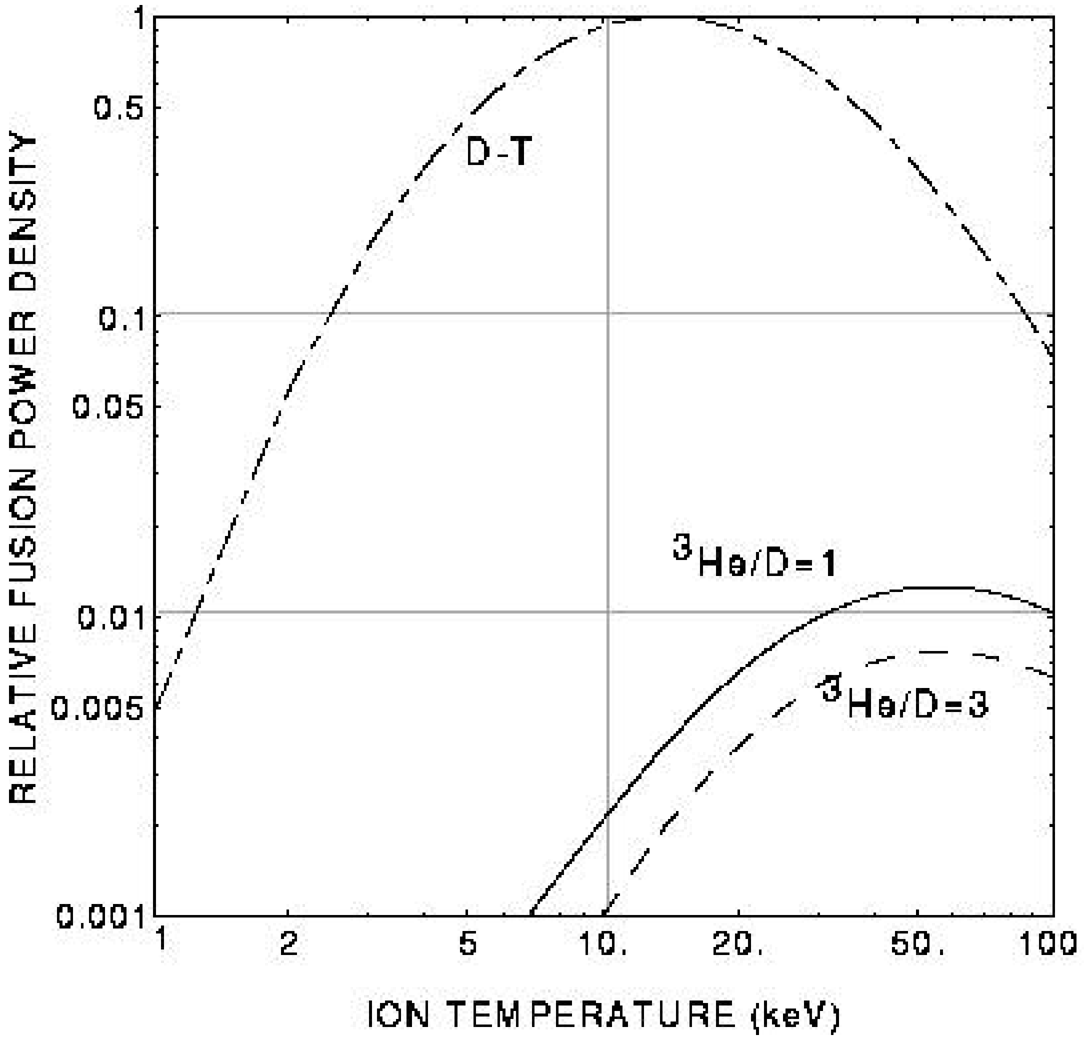
- No radioactive materials are present at launch, and only low-level radioactivity remains after operation.
- Conceptual designs project higher specific power values (1--10 kW-thrust per kg) for fusion than for nuclear-electric or solar-electric propulsion.
- Fusion gives high, flexible specific impulses (exhaust velocities), enabling efficient long-range transportation.
- D-^{3}He produces net energy and is available throughout the Solar System.
- D-^{3}He fuel provides an extremely high energy density.

D-^{3}He fuel is more attractive for space applications than D-T fuel.

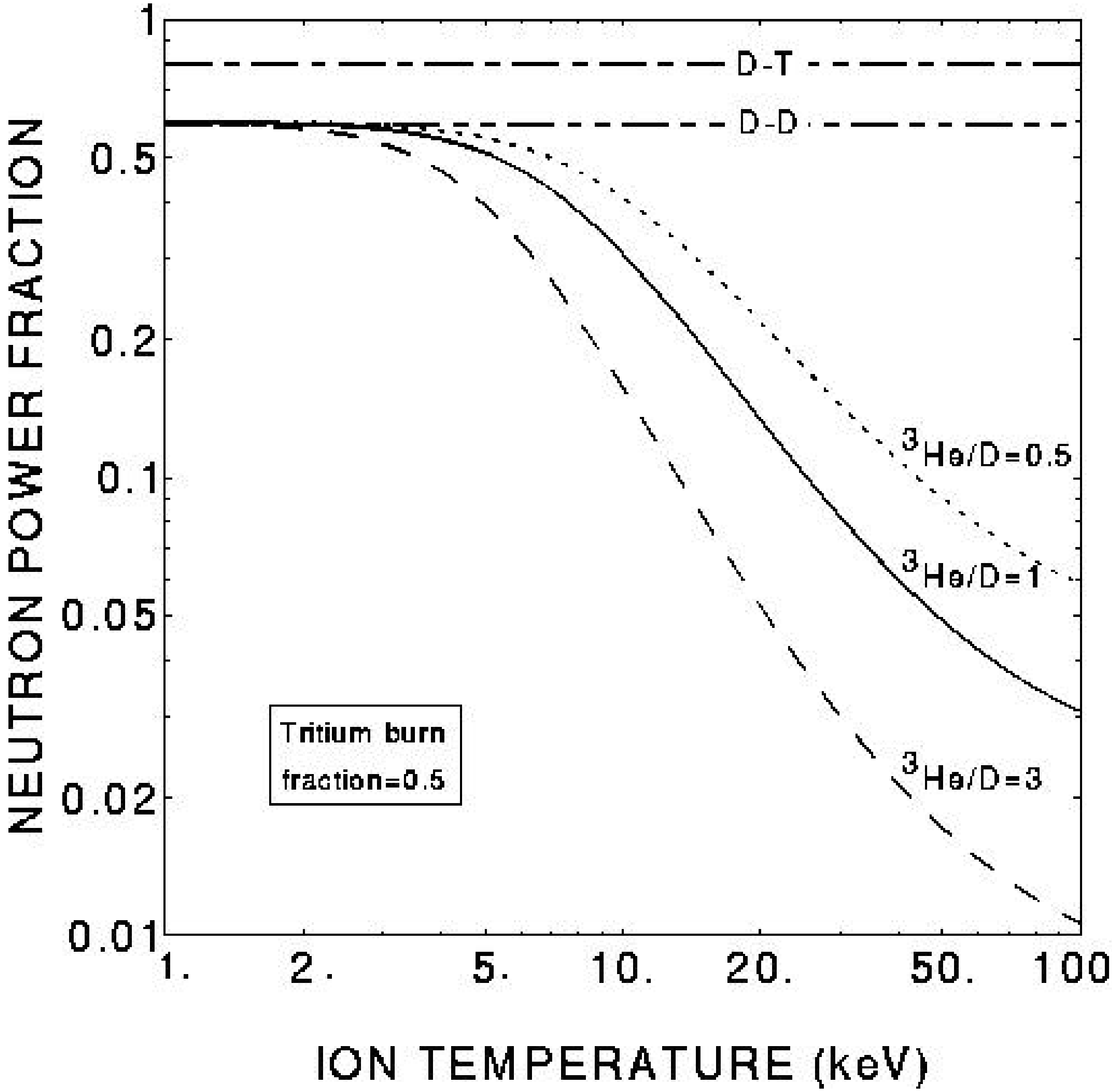
- High charged-particle fraction allows efficient direct conversion of fusion power to thrust or electricity.
  - Increases useful power.
  - Reduces heat rejection (radiator) mass.
  - Allows flexible thrust and exhaust velocity tailoring.
- Low neutron fraction reduces radiation shielding.
- D-^{3}He eliminates the need for a complicated tritium-breeding blanket and tritium-processing system.

Shown below are the fusion power density in the plasma and the fraction of fusion power produced as neutrons for D-T and D-^{3}He fuel.

Fusion Power Density



Neutron Power Fraction



The high fusion power density in the plasma favors D-T fuel, but the reduced neutron power fraction favors D-^{3}He fuel. This trade-off exemplifies the competition between physics and engineering in fusion energy development. In reality, a balance among these and other considerations must be found. For space applications, D-^{3}He fuel has usually been projected to be most attractive. The key reason for this is that the most important factor is not the fusion power density in the plasma (kW-fusion/plasma volume) but is the engineering power density (kW-thrust/mass of reactor and radiators). Several factors contribute to the dominance of D-^{3}He fuel:

Reduced neutron flux helps greatly

Reduced shield thickness and mass

Reduced magnet size and mass

Increased magnetic field in the plasma

Direct conversion can be used to increase the net electric power if plasma thrusters are needed.

Many configurations can increase magnetic fields (B fields) in the fusion core, gaining power density from a B^4 scaling.

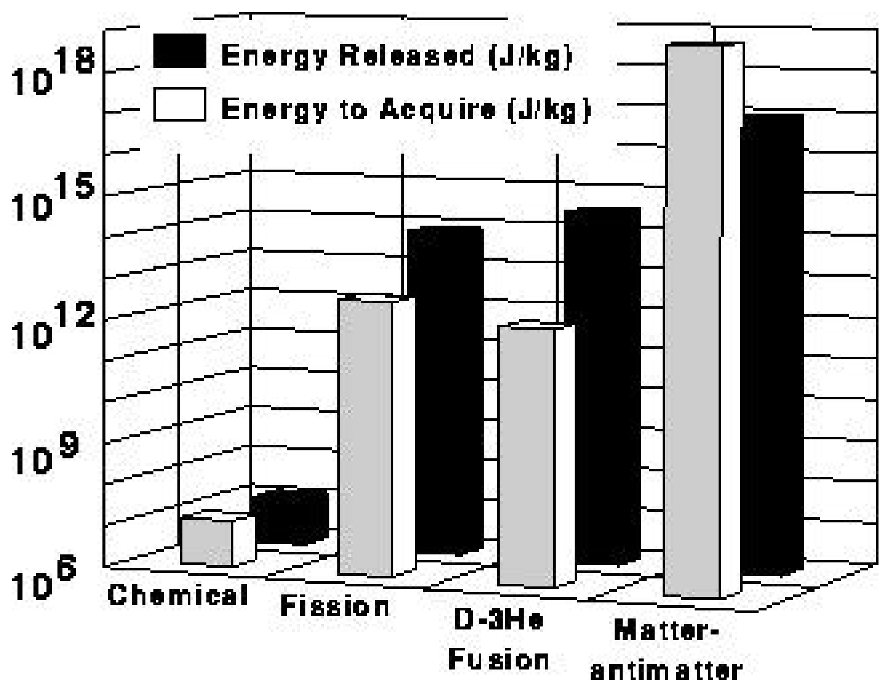
Regarding the last point, magnetic-fusion configurations can be classified as in the following table:

B at limit	**B near limit**	**Relatively low B**
S/c tokamak	Copper tokamak	Field-reversed configuration
Stellarator	Heliotron	Spheromak
Torsatron		Tandem mirror
		Bumpy torus
		Reversed-field pinch

Energy density of space-propulsion fuels

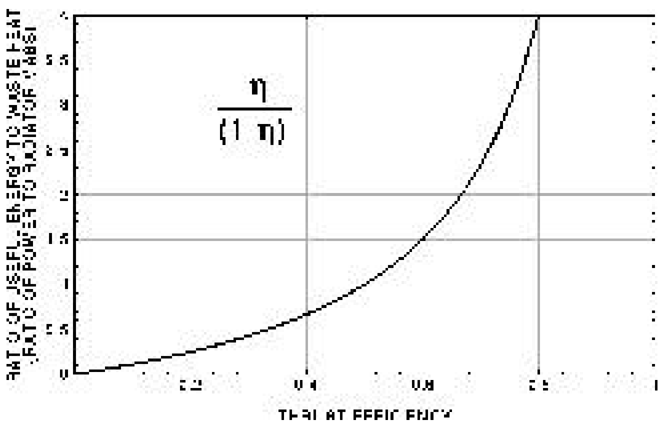
A fundamental limit on the specific power available from a fuel is the energy density of that fuel. A realistic assessment, of course, requires the detailed design of fuel storage and a means of converting fuel energy to thrust. Nevertheless, a high fuel energy density is desirable, because it facilitates carrying excess fuel, which contributes to mission flexibility, and indicates the potential for a high specific power.

The argument is often forwarded that antimatter-matter annihilation is the best space-propulsion fuel. A key difficulty exists, however: antimatter takes much more energy to acquire than it produces when annihilated with matter. Presently the ratio is about 10^4, and there appears little likelihood that ratios below about 10^2 are accessible. Antimatter, therefore, will probably be of limited use for routine access to the Solar System, although it will be the fuel of choice for specialized applications, such as interstellar missions. The energy needed to acquire various fuels is compared with the energy released in burning them in the figure .



High efficiency is critical in space

The ratio of useful thrust energy, which scales with efficiency, to the waste heat, which scales with (1 - efficiency), is a strong function of the efficiency of converting the fusion power to thrust. Because radiators often contribute a substantial fraction of the total rocket mass, efficiency generally is an important parameter.



Fusion Reactor Designs for Space Applications

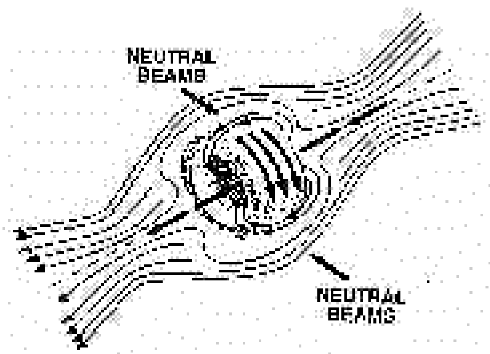
Conceptual designs of magnetic fusion reactors for space propulsion during the past decade have generally calculated specific powers of 1-10 kWthrust/kg reactor. The projected specific powers for selected designs appear in the table below. Note: Widely varying assumptions and levels of optimism have gone into these conceptual designs and the resulting specific powers.

Author	Year	Configuration	
Borowski	1987	Spheromak	10.5
Santarius	1988	Tandem Mirror	1.2
Chapman	1989	FRC	
Haloulakis	1989	Colliding Spheromaks	
Bussard	1990	Riggatron Tokamak	3.9
Bussard	1990	Inertial-Electrostatic	>10
Teller	1991	Dipole	1.0
Carpenter	1992	Tandem Mirror	4.3
Nakashima	1994	FRC	1.0
Kammash	1995	Gas Dynamic Trap	21(D-T)
Kammash	1995	Gas Dynamic Trap	6.4(D-^{3}He)

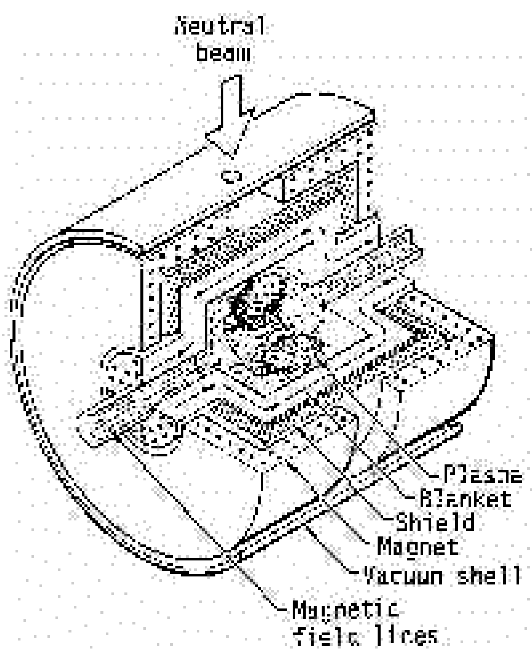
Various fusion-reactor configurations have been considered for space applications. Generally, the key features contributing to an attractive design are

> D-^3He fuel
>
> Solenoidal magnet geometry (linear reactor geometry) for the coils producing the vacuum (without plasma) magnetic field.
>
> Advanced fusion concepts that achieve high values of the parameter beta (ratio of plasma pressure to magnetic-field pressure).

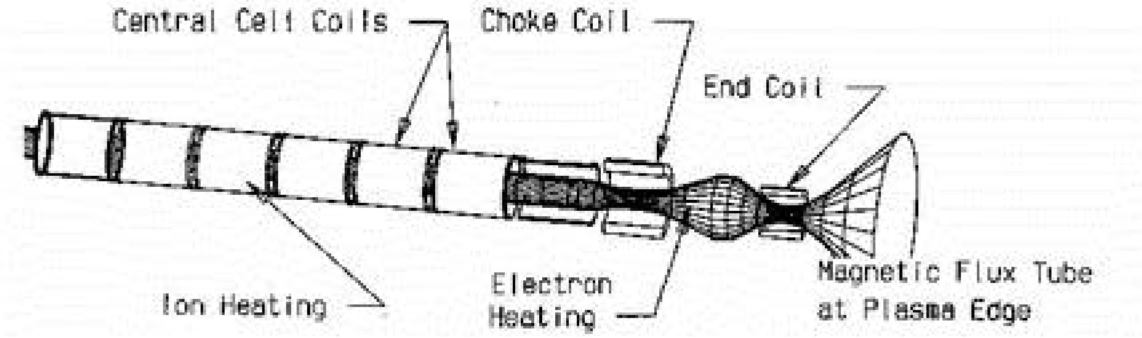
Field-reversed configuration (FRC)



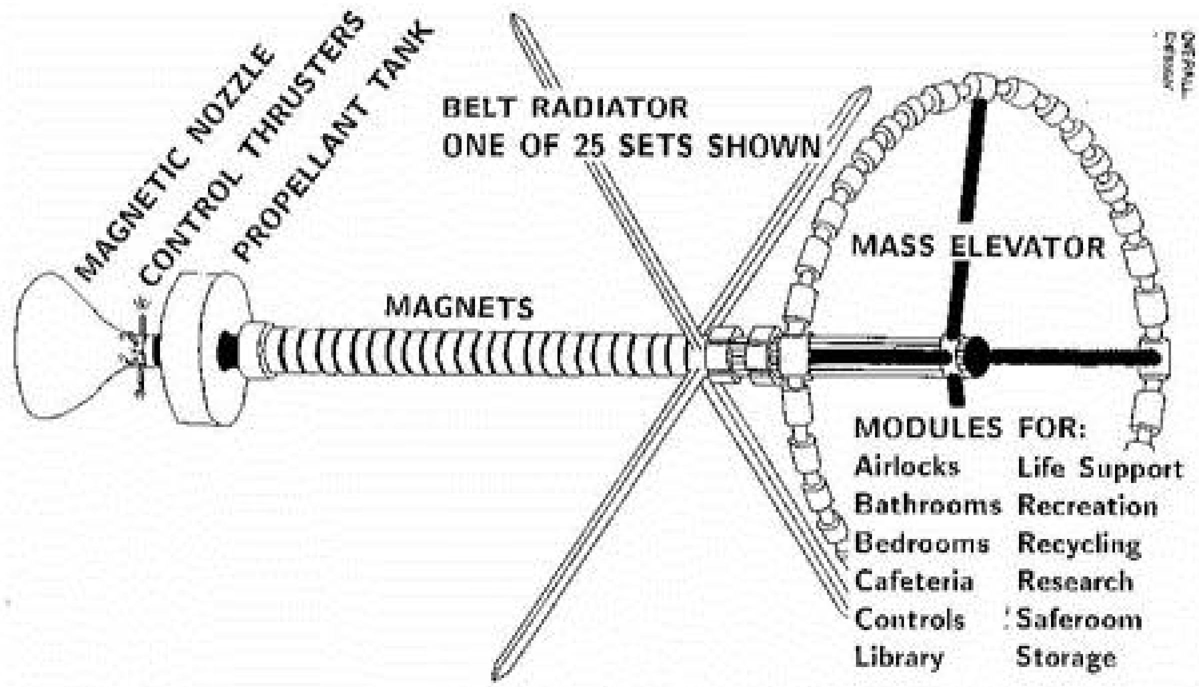
Engineering schematic (DT version)



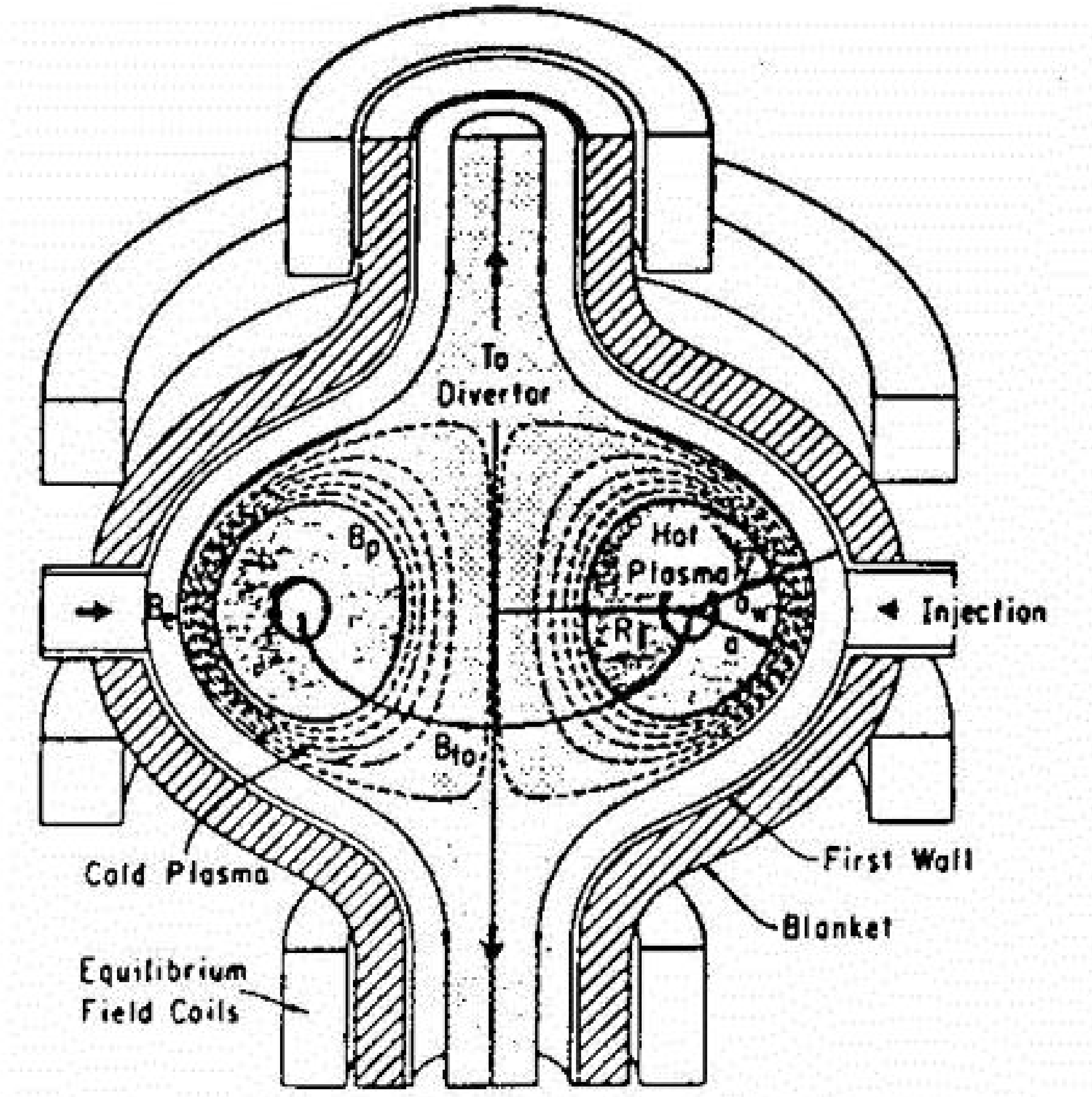
Tandem mirror engine



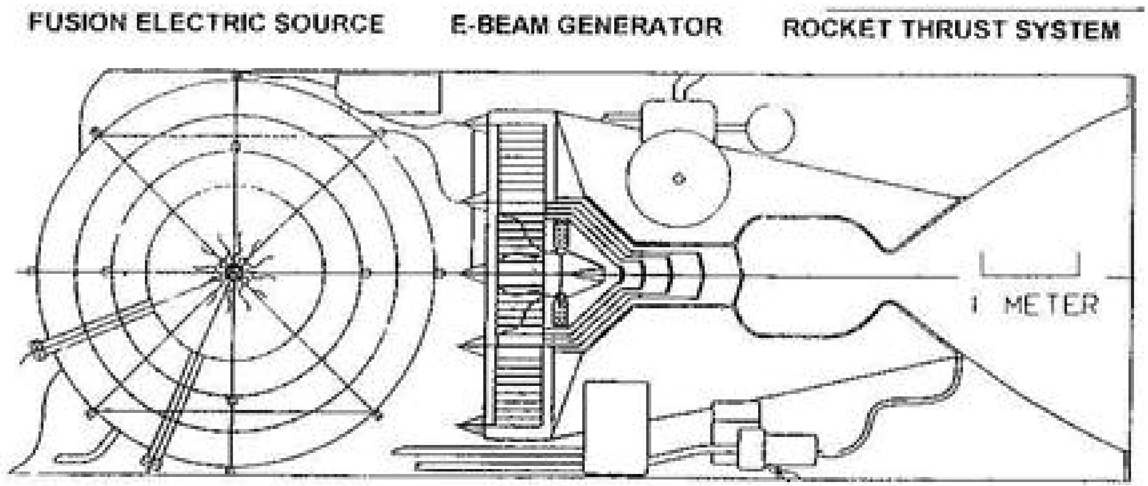
Tandem mirror with life support and other systems



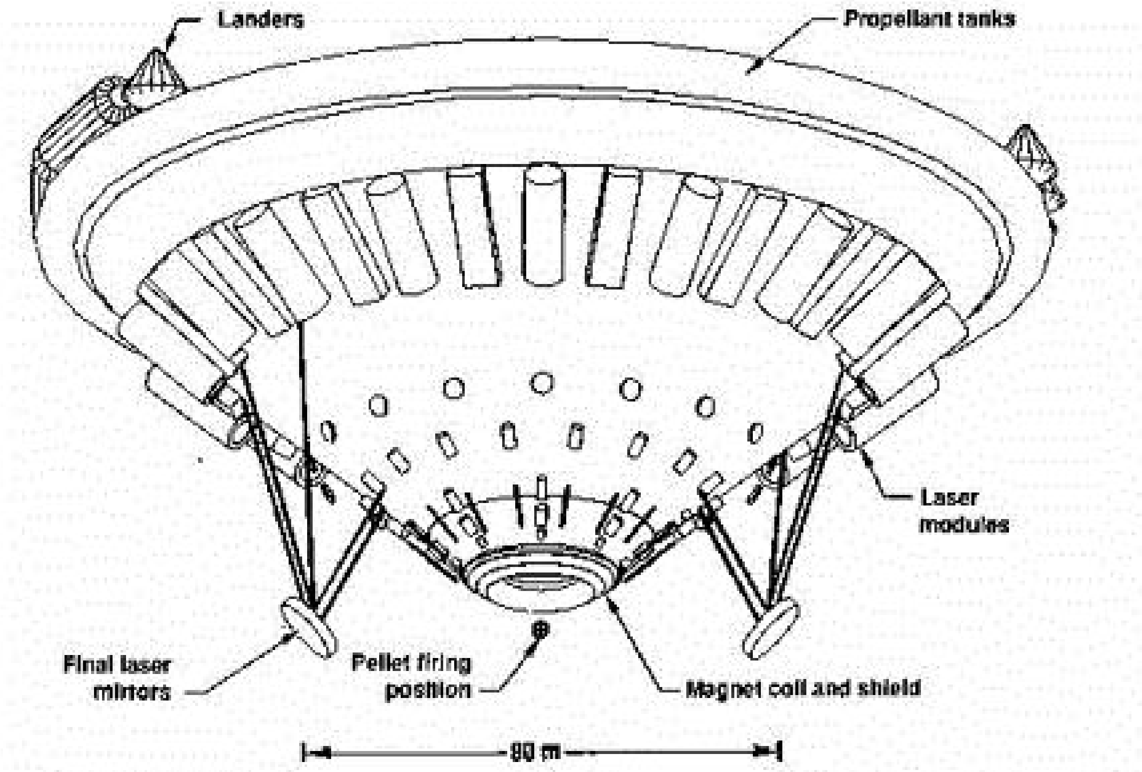
Spheromak


To
Divertor
Hot
Plasma
Injection
Cold Plasma
First Wall
Blanket
Equilibrium
Field Coils

Inertial-Electrostatic Confinement (IEC)



VISTA ICF space-propulsion design



Some Inertial-confinement fusion (ICF) reactors for space propulsion have also been designed. One example is VISTA, shown at right. Because of fusion burn dynamics, D-^{3}He fuel is much harder to use in ICF reactors, and VISTA used the D-T fuel cycle. The British Interplanetary Society's earlier Daedalus study used D-^{3}He fuel, but had to simply assume that the physics would work.

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R. W. Bussard, "Fusion as Electric Propulsion," Journal of Propulsion and Power 6, 567 (1990).

R. W. Bussard and L. W. Jameson, "The QED Engine Spectrum: Fusion-Electric Propulsion for Air-Breathing to Interstellar Flight," Journal of Propulsion and Power 11, 365 (1995).

A. Bond, A. R. Martin, R. A. Buckland, T. J. Grant, A. T. Lawton, et al., "Project Daedalus," J. British Interplanetary Society 31, (Supplement, 1978).

S. K. Borowski, "A Comparison of Fusion/Antiproton Propulsion Systems for Interplanetary Travel," AIAA/SAE/ASME/ASEE 23rd Joint Propulsion Conference, paper AIAA-87-1814 (San Diego, California, 29 June--2 July 1987).

S. A. Carpenter and M. E. Deveny, "Mirror Fusion Propulsion System (MFPS): An Option for the Space Exploration Initiative (SEI)," 43rd Congress of the Int. Astronautical Federation, paper IAF-92-0613 (Washington, DC, 28 August--5 September, 1992).

S. Carpenter, M. Deveny, and N. Schulze, "Applying Design Principles to Fusion Reactor Configurations for Propulsion in Space," 29th AIAA/SAE/ASME/ASEE Joint Propulsion Conference, paper AIAA-93-2027.

R. Chapman, G.H. Miley, and W. Kernbichler, "Fusion Space Propulsion with a Field Reversed Configuration," Fusion Technology 15, 1154 (1989).

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