Projectile motion (Section 4.3)

1. Which target got hit first?
   • Context of the textbook: Before Example 4.

2. Projectile range
   • A problem comparable to Example 7, except here the initial values given are $v_{0x}$ and $v_{0y}$.

3. Gun and target

4. Gun and target on the moon
   • Questions 4.3 and 4.4 are supplementary problems for projectial motion.

Motion along a circular arc: These two questions are conventional problems in other engineering physics textbooks beyond the uniform circular motion discussed in this section. They may be introduced after Example 9 of Section 4.5.

5. Deceleration of a train

6. Motion of a simple pendulum

Relative velocity and the addition of velocities.
These questions may be used before Example 10 in Section 4.6.

7. Definition of relative velocity
   • A basic conceptual question.

8. Crossing a river
   • A simplified version of Example 10.
Two projectiles are launched simultaneously and with the same launching speed from point O as shown $v_0' = v_0$. The first is launched at angle $\theta$ and hits target A in time $t_A$. The second is launched at a greater angle $\theta' > \theta$ and hits target B in time $t_B$. Compare the flight times for the two projectiles:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_B &gt; t_A$</td>
<td>$t_B = t_A$</td>
<td>$t_B &lt; t_A$</td>
<td></td>
</tr>
</tbody>
</table>

**Explanation:** Consider the projected motion along the vertical direction. The flight time for the first case is given by $t_{\text{flight}} = 2t_{\text{rise}} = 2v_{0y}/g$. For the second case, the y component of the initial velocity $v_{0y} = v_0 \sin \theta$ is replaced by $v'_{0y} = v'_0 \sin \theta'$, where $\theta' > \theta$ and $v_0 = v'_0$ are given. Therefore $v'_{0y} > v_{0y}$, which means that the flight of the second case takes longer. Answer = A.
Given $v_{0x} = 20 \text{ m/s}$ and $v_{0y} = 10 \text{ m/s}$ and $g = 10 \text{ m/s}^2$. Find $R$. Choose one of the following:

\[
\begin{array}{l}
A \quad 2v_0^2/g = 2(20^2 + 10^2)/10 = 100 \text{ m} \\
B \quad v_{0x}v_{0y}/g = 20 \times 10/10 = 20 \text{ m} \\
C \quad 2v_{0x}v_{0y}/g = 2 \times 20 \times 10/10 = 40 \text{ m}
\end{array}
\]

**Hint:** The rising time $t_{\text{rise}} = v_{0y}/g$, $v_0^2 = v_{0x}^2 + v_{0y}^2 = 500 \text{ m}^2$.

**Extra:** Keep $v_0$ fixed and vary $\theta$ to find the maximum value of $R$.

**Explanation:** $R = v_{0x} (2t_{\text{rise}}) = 2v_{0x}v_{0y}/g$. Answer = **C**

**Explanation—extra:** $R = 2v_0^2 \sin \theta \cos \theta/g = v_0^2 \sin 2\theta/g$. So the maximum value of $R$ when $v_0^2$ is fixed is $R = v_0^2/g = 50 \text{ m}$.
Consider a gun at $O$ aiming at a target located at point $A$. The gun fires at time $t = 0$, and at the same time the target begins to fall from rest. Let $v_{Ox} = 10$ m/s and $v_{Oy} = 20$ m/s. Also, let $OB = 20$ m and $AB = 40$ m. Find the height $BP$ as the bullet passes the vertical line $AB$:

<table>
<thead>
<tr>
<th>$BP$ (in m)</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

**Hint:** In the $x$ direction $t_{OP} = OB/v_{ox} = 2$ s. To find $BP$, use $y = v_{Oy}t - gt^2/2$ for the motion of the bullet along the $y$ direction. Assume $g = 10$ m/s$^2$.

**Explanation:** From point $O$ to point $P$, the time that the bullet spends traveling vertically is the same as the time it spends traveling from $O$ to $P$, $t_{BP} = t_{OP} = 2$ s. Use the equation of motion in the vertical direction: $BP = v_{Oy}t_{BP} - gt_{BP}^2/2 = (20)(2) - (10/2)(2)^2 = 20$ m. Answer $= B$.

**An alternative method:** $BP$ may also be determined by considering the fall of the target. $AP = (1/2)gt_{BP}^2 = (10/2)(2)^2 = 20$ m. $BP = AB - AP = 40 - 20 = 20$ m.
Consider the “gun and target” setup of the previous problem, where the initial velocity vector $v_0$, the length of the base line $OB$, and the height $AB$ are kept fixed. Compare the height $BP$, when the experiment is carried out on Earth to the height $BP'$, when the experiment is done on the Moon:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$BP' &lt; BP$</td>
<td>$BP' = BP$</td>
<td>$BP' &gt; BP$</td>
</tr>
</tbody>
</table>

**Explanation:** Because $v_{0x}$ and $OB$ are the same for the two cases, the time of flight $t = OB/v_{0x}$ is also the same. During this time, on Earth the target falls by a distance $AP = (1/2)gt^2$. Because on the Moon the gravitational acceleration $g'$ is less than $g$, $AP' < AP$. In turn, $BP' > BP$. Answer = C.
A train is moving along a circular track of radius \( r = 100 \) m. At point A, \( v = |v| = 10 \) m/s.

It is slowing down with a tangential deceleration of a magnitude \( a_{\text{tangent}} = |a_{\text{tangent}}| = 1 \) m/s\(^2\).

Sketch \( \mathbf{a}_{\text{total}} \) at A. Which quadrant should it be in?

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadrant</td>
<td>I</td>
<td>II</td>
<td>III</td>
</tr>
</tbody>
</table>

**Extra:** Find the magnitude \( |\mathbf{a}_{\text{total}}| \).

**Explanation:** From the sketch at point A, \( \mathbf{a}_{\text{total}} \) is in the third quadrant. Answer = C.

**Explanation—extra:** At point A, centripetal acceleration \( a_c = v^2/r = 10^2/100 = 1 \) m/s\(^2\). So \( a_{\text{total}} = \sqrt{a_c^2 + a_{\text{tangent}}^2} = \sqrt{1^2 + 1^2} = \sqrt{2} \) m/s\(^2\), or \( \approx 1.4 \) m/s\(^2\).
A simple pendulum consists of a string of length $r$ and a ball attached to its end. When the string makes an angle $\theta$ with the vertical and the tangential velocity of the ball is pointing toward the vertical line, determine the corresponding tangential acceleration:

The angle $\theta$ is measured from the vertical line in a counterclockwise manner.

**Extra:** Compare $\theta$ with $\phi$ in the sketch.

**Explanation:** From the sketch, we see that the tangential acceleration is pointing toward the vertical line; thus it has a sign opposite to that of $\theta$. Answer = C.

**Explanation—extra:** Notice in the top sketch, the centrepetal vector $\mathbf{a}_r = g_r$ where $g_r$ is shown in the second sketch. Since both vectors $\mathbf{a}$ and $\mathbf{g}$ are diagonals of two rectangles with same sides $a_r$ and $g_r$, $\phi = \theta$. One can see this equality visually, if $\mathbf{a}_r$ is drawn to scale.

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6. Motion of a Simple Pendulum
Car $A$ travels at a speed of 30 mph to the right (positive $x$ direction) and car $B$ travels at 10 mph to the left. Consider the velocity $\mathbf{v}_{AB} = \mathbf{v}_{AB} \hat{i}$ to be the velocity of car $A$ observed by the driver in car $B$ (in other words, $\mathbf{v}_{AB}$ is the velocity of $A$ relative to $B$). Given the $x$ axis orientation as shown, find $\mathbf{v}_{AB}$ and $\mathbf{v}_{BA}$.

Choose one (in mph):

<table>
<thead>
<tr>
<th></th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbf{v}_{AB}$</td>
<td>20</td>
<td>20</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>$\mathbf{v}_{BA}$</td>
<td>20</td>
<td>-20</td>
<td>40</td>
<td>-40</td>
</tr>
</tbody>
</table>

**Explanation:** Notice that the driver of car $B$ sees that car $A$ is moving toward him or her—that is, along the positive $x$ direction with a speed greater than $\mathbf{v}_A = 30$ mph. The algebra involved is given here:

$\mathbf{v}_{AB} = \mathbf{v}_A - \mathbf{v}_B = 30 - (-10) = 40$ mph.

$\mathbf{v}_{BA} = \mathbf{v}_B - \mathbf{v}_A = -\mathbf{v}_{AB} = -40$ mph. Answer = D.
The diagram here shows a boat attempting to cross a river. Assume that the boat’s speed relative to the water $v_{bw} = 10 \text{ m/s}$ and that the current (the water’s speed relative to the Earth) $v_{we} = 5 \text{ m/s}$.

Find $\theta$ such that the boat crosses the river at a right angle to the bank:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>$30^\circ$</td>
<td>$45^\circ$</td>
<td>$60^\circ$</td>
</tr>
</tbody>
</table>

**Extra:** What is $v_{be}$, the boat’s speed relative to the Earth?

**Explanation:** Because $\sin \theta = v_{we}/v_{bw} = 5/10 = 0.5$, $\theta = 30^\circ$.

**Explanation—extra:** $v_{be} = v_{bw} \cos 30^\circ \approx 8.7 \text{ m/s}$. 

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