**Work**

Work in one dimension: *(Section 7.1, part 1)*

The following two problems involve pulleys and may be used after Example 2 in Section 7.1:

1. Atwood machine
2. Raising $m$ by a distance $\Delta x$

Work in two dimensions: *(Section 7.1, part 2)*

3. Work against gravity

   • Context in the textbook: After Example 3 in Section 7.1.

**Work–energy and gravitational potential energy (Section 7.4)**

*Exercises to be used in conjunction with Section 7.4:*

4. Sliding down an incline
5. Comparing final kinetic energies
6. Stopped pendulum
7. Sliding down a dome: The “stay-on” condition
8. Sliding down a dome: Equations of motion
Two blocks with masses $m_1$ and $m_2$ are connected by a light string passing over a light frictionless pulley. Assume $m_2 > m_1$. Determine the potential energy released by the system as block 2 starts from rest and falls by $h/2$:

$$\text{Potential energy released} = \frac{m_2 gh}{2} \quad \left(\frac{m_2}{H11002} \frac{m_1}{11022}\right) \frac{gh}{2} \quad m_1 \frac{gh}{2}$$

**Hint:** Remember that as block 2 descends by $h/2$, block 1 rises correspondingly by a height $h/2$.

**Explanation:** When block 2 falls by a distance $h/2$, block 1 goes up by $h/2$. The potential energy lost by block 2 is $m_2 gh/2$. The potential energy gained by block 1 is $m_1 gh/2$. So the net potential energy lost is $(m_2 - m_1) gh/2$. Answer = B.
Consider the mass–pulley system shown. Determine the distance $d$ covered by the force $F$ as it lifts the block by a height $\Delta x$:

\[
\begin{array}{ccc}
\text{A} & \text{B} & \text{C} \\
\text{Distance of } F & d = \frac{\Delta x}{2} & d = \Delta x & d = 2\Delta x \\
\end{array}
\]

**Explanation:** As the block is lifted by height $\Delta x$, the length of each of the two strings supporting the moving pulley will be reduced by $\Delta x$, so $d = 2\Delta x$. Answer = C.

**Comment:** Notice that conservation of energy implies that the increase of the potential energy equals the input mechanical energy: $mg\Delta x = Fd = 2\Delta x F$. Solving for $F$ gives $F = \frac{mg}{2}$. In other words, the applied force required is half of the weight.
Consider the movement of a block of mass \( m = 1 \) kg in two phases. In the first phase it moves horizontally 4 m from A to B. In the second phase it goes from B to C, a vertical distance of 3 m. Find the work done on the block against the force of gravity for each phase of its journey:

\[
W_{AB} = 40 \text{ J} \quad W_{BC} = 0 \text{ J}
\]

\[
W_{BC} = 0 \text{ J} \quad W_{BC} = 30 \text{ J}
\]

\[
W_{AC} = 40 \text{ J} \quad W_{AC} = 30 \text{ J}
\]

**Hint:** Work is the product of force times displacement parallel to the force.

**Extra:** Consider moving the block directly from A to C along a frictionless inclined plane. Determine the work done, \( W_{AC} \).

**Explanation:** From A to B there is no displacement parallel to gravitation force. So \( W_{AB} = mg \times (0) = 0 \) J. \( W_{BC} = mg \times 3 = 30 \) J. Answer = B.

**Explanation—extra:** Along the incline \( W_{AC} = mg \sin \theta \times AC = mg \times BC = 10 \times 3 = 30 \) J, where \( mg \sin \theta \) is the lifting force along the inclined plane. This illustrates that the work done against a conservative force (that is, gravity) is path independent: The work along the two paths \( A \rightarrow B \rightarrow C \) and \( A \rightarrow C \) is the same.
Consider a block of mass $m = 1$ kg sliding down a frictionless inclined plane from rest at $A$. Given $h = 20$ m, $\theta = 30^\circ$, and $g = 10$ m/s$^2$, find $v_B$, the speed of the block as it passes through point $B$:

\[
\begin{array}{c|c}
\text{A} & v_B = \sqrt{2gh} = 20 \text{ m/s} \\
\text{B} & v_B = \sqrt{gh} = 10\sqrt{2} \text{ m/s} \\
\text{C} & v_B = \sqrt{gh/2} = 10 \text{ m/s} \\
\end{array}
\]

\textbf{Hint:} Use the conservation of energy equation $K_f - K_i = U_i - U_f$. (see Eq. (7.32)), where $K_i = K_A = 0$ and $U_i - U_f = U_A - U_B = mgh$.

\textbf{Extra:} Determine the kinetic energy in joules at $B$.

\textbf{Explanation:} $K_f = K_B = mv_B^2/2$. The conservation of energy equation implies that $mv_B^2/2 = mgh$, that is, $v_B = \sqrt{2gh}$. Answer = A.

\textbf{Explanation—extra:} $K_B = mv_B^2/2 = mgh = 1 \times 10 \times 20 = 200$ J.
Consider the three possible paths marked A, B, and C each for a given mass traveling from height 1 to height 2. A represents a free fall from rest. B represents starting from rest and sliding down a curved slide without friction. C represents a projectile with an initial horizontal speed of \( v_0 \) as shown.

![Diagram of three paths A, B, and C](image)

Which set of relationships is correct for the speeds of the mass at height 2?

\[
\begin{array}{ccc}
A & v_A = v_{By} = v_{Cy} \\
B & v_A = v_B < v_C \\
C & v_A < v_B < v_C
\end{array}
\]

**Hint:** The frictionless surface that constrains the motion for path B neither creates nor dissipates energy. So \( K_f - K_i = U_f - U_i \).

**Extra:** Compare the time taken to reach the ground for these three cases.

**Explanation:** For A and B, the initial kinetic energy \( K_i \) is 0. Conservation of energy implies \( K_f = U_i - U_f = mgh \). In turn, the final kinetic energies are equal: \( K_A = K_B \). Because the masses are the same, the final speeds are equal too: \( v_A = v_B \). For C, \( U_i - U_f \) is still equal to \( mgh \); and because \( K_i > 0 \), conservation of energy implies that \( K_C = U_i - U_f + K_i > K_A \). Answer = B.

**Explanation—extra:** The vertical motion in C is the same as that in A, so the trip time \( t_A = t_C \). Now compare A and B. Because the distance covered in B is longer than the distance covered in A, it takes longer to reach the ground for B than for A.
Consider the simple pendulum shown in which the friction force is negligible. The bob is oscillating between the two extreme points A and C, which have the same height $h$. Now place a stopper at D, located some distance below the initial pivot point. As the bob starts swinging to the right, it hits the stopper and pivots about point D. Denote the new extreme point by E as shown. The height at E is which of the following?

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<tbody>
<tr>
<td>1</td>
<td>Greater than $h$</td>
</tr>
<tr>
<td>2</td>
<td>Equal to $h$</td>
</tr>
<tr>
<td>3</td>
<td>Less than $h$</td>
</tr>
</tbody>
</table>

**Hint:** Use the conservation of energy equation: $U + K = \text{constant}$. This is valid at any point along the path of motion, with or without the stopper.

**Explanation:** Conservation of energy implies that $U_A + K_A = U_B + K_B$, or $K_B = U_A - U_B = mgh$, because $K_A = 0$. It also implies that $U_B + K_B = U_E + K_E$. Because E is the new extreme point, $K_E = 0$, the potential difference between $E$ and $B$ is $U_E - U_B = K_B = mgh = U_A - U_B$. In other words, point E has the same height as A. Answer = B.
A small box of mass $m$ is initially at the top of a hemisphere with a radius $R$. Starting from rest, the box slides without friction down along the surface of the hemisphere. At any $\theta$, the centripetal force is $F_c = \frac{mv^2}{r}$. What is the condition for the block to stay on the surface?

- A: $mg \cos \theta < F_c$
- B: $mg \sin \theta < F_c$
- C: $mg \cos \theta > F_c$
- D: $mg \sin \theta > F_c$

**Explanation:** When the radial component of the weight is larger than the centripetal force, there is a net force directed radially inward that presses the block against the surface. This is the condition for the block to stay on the surface. Answer = C.
Consider a block starting at rest from point $O$, sliding down the surface of a smooth sphere (friction is negligible) and flying off at point $C$. Here two principles are at work:


II. At the flying-off point the surface exerts no force on the block. In other words, at point $C$ the normal force $N = 0$. Define the following relations:

For I:
- $mgR(1 - \cos \theta) = \frac{mv^2}{2}$  \hspace{1cm} la
- $mgR(1 - \sin \theta) = \frac{mv^2}{2}$  \hspace{1cm} lb

For II:
- $mg \cos \theta = \frac{mv^2}{R}$  \hspace{1cm} ll\hspace{1cm}a
- $mg \sin \theta = \frac{mv^2}{R}$  \hspace{1cm} ll\hspace{1cm}b

Determine the correct pair of relations:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
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<tr>
<td>For I</td>
<td>la</td>
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<td>lb</td>
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<tr>
<td>For II</td>
<td>ll\hspace{1cm}a</td>
<td>ll\hspace{1cm}b</td>
<td>ll\hspace{1cm}a</td>
<td>ll\hspace{1cm}b</td>
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**Explanation:** For I, the vertical distance between $O$ and $C$ is $R(1 - \cos \theta)$, so choice la is correct. For II, the radial component of $mg$ is $mg \cos \theta$. Here the choice ll\hspace{1cm}a is correct. Combining the two leads to Answer A.

**Comment:** Based on the two relations at $C$, we may solve for $\cos \theta$. Verify that $\cos \theta = 2/3$. 

66 PhysiQuiz 8. Sliding Down a Dome: Equations of Motion