Impulse (Section 11.1)
1. Impulse delivered by the incline
   • Context in the textbook: This is an exercise of Eq. (11.4) in Section 11.1.

Elastic collisions in one dimension (Section 11.2)
2. Elastic collision—I
3. Elastic collision—II
4. Sledgehammer and a ball
5. Collision with a pendulum

Inelastic collision in one dimension (Section 11.3)
6. Inelastic collision between a bullet and a ball

Collision in two dimensions (Section 11.4)
7. A perfectly inelastic collision in two dimensions
Consider the deflection of a falling ball by a 45° incline. The ball bounces off horizontally. Assume that this is an elastic collision. Determine the impulse delivered by the incline onto the ball:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direction</td>
<td>↑</td>
<td>↗</td>
<td>↗</td>
<td>↘</td>
</tr>
<tr>
<td>Δp</td>
<td>p_i</td>
<td>p_i</td>
<td>√2 p_i</td>
<td>√2 p_i</td>
</tr>
</tbody>
</table>

**Hint:** The impulse vector imparted on the ball by the plane is defined by \( \Delta \mathbf{p} = \mathbf{p}_f - \mathbf{p}_i \). Sketch the vector diagram first.

**Explanation:** From the sketch, we see that Answer = C.
We have two balls with masses $m_1 = m_2 = m$. The ball $m_1$ with a speed $v_1 = v_0$ is approaching $m_2$, which is at rest ($v_2 = 0$). They undergo a head-on elastic collision. Denote the final velocities as $v_1'$ and $v_2'$. Identify the correct pair of $v_1'$ and $v_2'$ values:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_1'$</td>
<td>$v_0/2$</td>
<td>$v_0/3$</td>
<td>0</td>
<td>$-v_0/2$</td>
</tr>
<tr>
<td>$v_2'$</td>
<td>$v_0/2$</td>
<td>$2v_0/3$</td>
<td>$v_0$</td>
<td>$3v_0/2$</td>
</tr>
</tbody>
</table>

**Hint:** For an elastic collision, the following two conditions must be satisfied:
- Conservation of momentum: $p_i = m_1v_1 + m_2v_2 = p_f = m_1v_1' + m_2v_2'$.
- Conservation of kinetic energy: $K_f = K_i$.

**Explanation:** Notice that for all choices (in the $x$ direction), $p_i = m_1v_1 + m_2v_2 = mv_0$ and $p_f = m_1v_1' + m_2v_2' = mv_0$. In other words, momentum is conserved for all four cases. Next check the condition $K_f = K_i$. The initial kinetic energy is given by $K_i = mv_0^2/2$. We need to work out $K_f$ for all four choices. For instance, for the first choice A, $K_f = m(v_0/2)^2/2 + m(v_0/2)^2/2 = mv_0^2/4$. Do the same for the other three choices. All except C fail this condition. So Answer = C.
In Section 11.2, Eq. (11.13) and Eq. (11.14) are given for head-on elastic collisions between two balls of masses \( m_1 \) and \( m_2 \), where the ball with mass \( m_2 \) is initially at rest \( (v_2 = 0) \). These two equations can be written compactly as \( v_i' = 2v_{cm} - v_i \) (where \( i = 1 \) or \( i = 2 \), depending on which ball’s final velocity you are looking for). We will leave the proof as an exercise for the reader. Now envision \( m_1 \) approaching \( m_2 \), where \( m_2 = 2m_1 \) with a speed \( v_1 = v_0 \). After an elastic collision and using this new compact form, determine the final velocities \( v_1' \) and \( v_2' \):

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_1' )</td>
<td>(-v_0/3)</td>
<td>(v_0/3)</td>
<td>(-v_0)</td>
<td>(-v_0/4)</td>
</tr>
<tr>
<td>( v_2' )</td>
<td>(2v_0/3)</td>
<td>(v_0/3)</td>
<td>(v_0/2)</td>
<td>(v_0/2)</td>
</tr>
</tbody>
</table>

**Extra:** Find the ratio of kinetic energies, \( K_f/K_i \).

**Explanation:** The velocity of the center of mass is given by \( v_{cm} = mv_1/(3m) = v_1/3 = v_0/3 \). Based on the relation \( v_i' = 2v_{cm} - v_i \), \( v_1' = 2v_0/3 - v_0 = -v_0/3 \), and \( v_2' = 2v_0/3 \). Answer = A.

Among the incorrect choices, the reader should verify that

- Choice B corresponds to a perfectly inelastic collision.
- Both choices C and D violate the conservation of momentum.

**Explanation—extra:** Based on Answer A, \( K_f = m(v_0/3)^2/2 + 2m(2v_0/3)^2/2 = (1/9 + 8/9)K_i \) or \( K_f/K_i = 1 \). This is expected because the present case corresponds to an elastic collision.
Consider an elastic, head-on collision involving a sledgehammer of mass $m_1 = 10$ kg hitting a golfball of mass $m_2 = 10$ g. Assume that $v_1 = 10$ m/s and $v_2 = 0$. Using the relations $m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$ and $v_i' = 2v_{cm} - v_i$ (see the previous question), find the final speed $v_2'$ of the golfball:

<table>
<thead>
<tr>
<th>Option</th>
<th>Equation</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>$v_2' \approx v_1 = 10$ m/s</td>
</tr>
<tr>
<td>B</td>
<td>$v_2' \approx 2v_1 = 20$ m/s</td>
</tr>
<tr>
<td>C</td>
<td>$v_2' &gt; 3v_1 = 30$ m/s</td>
</tr>
</tbody>
</table>

**Hint:** Because $m_1 \gg m_2$, verify that $v_{cm} \approx v_1$.

**Explanation:** Using $v_i' = 2v_{cm} - v_i$ with $i = 2$ and $v_{cm} \approx v_1 = 10$ m/s (because $m_1 \gg m_2$), we have $v_2' = 2v_{cm} \approx 2v_1 = 20$ m/s. Answer = B.

**Digression:** It is instructive to look at the process in the frame that is moving along with the sledgehammer. In this frame, the hammer is stationary. We can visualize that the initial velocity of the ball is $-v_1$, and the final velocity of the ball is $+2v_1 - v_1 = +v_1$. In other words, in this frame, the ball elastically bounces off the stationary hammer. Now consider the case in which the mass of the hammer is arbitrary. In the frame moving along with the center of mass (that is, in the cm frame), the initial velocity of the ball is $-v_{cm}$, and the final velocity of the ball in the cm frame is shifted from its lab frame value $v_2'$ by an amount $-0$ cm, i.e. $v_2' - v_{cm} = (2v_{cm} - 0) - v_{cm} = v_{cm}$. In other words, the ball bounces off the center of mass with its velocity changing from $-v_{cm}$ to $+v_{cm}$—that is, it bounces elastically from the center of mass.
A ball of mass $m_1$ with initial kinetic energy $K_1$ is colliding with a pendulum bob of mass $m_2$ initially at rest. The pendulum bob is attached to a stiff rod of length $L$ and is free to rotate around its other endpoint. Let $m_1 = m_2 = m$. Assume that the rod has negligible mass. Determine the minimum kinetic energy of the ball such that the bob can barely pass over the top at point $B$:

$$A \quad B \quad C \quad D$$

| Min. $K_1$ | $3mgL/2$ | $2mgL$ | $5mgL/2$ | $3mgL$ |

**Hint:** Immediately after a head-on elastic collision between two objects of equal mass where one is initially at rest, the initially moving object is stationary, and the initially stationary object moves off with kinetic energy equal to the kinetic energy of the originally moving object.

**Extra:** What is the minimum $K_1$ if the stiff rod is replaced by a string?

**Explanation:** In moving from $A$ to $B$, energy is conserved—that is, the sum of kinetic energy plus potential energy remains constant, or

$$K_A + U_A = K_B + U_B.$$  

With $K_B \approx 0$, $K_A = U_B - U_A = mg2L$. So the minimum value of $K_1 = K_A = 2mgL$. Answer = B.

**Explanation—extra:** At $B$, the resultant force responsible for the centripetal force is given by $mg + T = mv_B^2/L = 2K_B/L$, where $T$ is the tension of the string. When the ball just barely passes through the point $B$, at that moment the tension of the string $T = 0$. It follows that $K_B = mgL/2$. So the minimum value of $K_1$ can be determined as follows:

$$K_1 = K_A = U_B - U_A + K_B = mg2L + mgL/2 = 5mgL/2.$$
A bullet with initial speed $v_1$ and mass $m_1$ is approaching a ball of mass $m_2$, initially at rest. Let $m_2 = 2m_1$. After piercing the ball, the bullet’s final velocity is $v_1' = v_1/2$. Find $v_2'$, the final velocity of the ball:

$$v_2' = \frac{v_1}{2}$$

**Hint:** Use conservation of momentum: $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$.

**Extra:** Find the ratio $K_f/K_i$.

**Explanation:** Using $m_2 = 2m_1$, $v_2 = 0$, and $v_1' = v/2$, the conservation of momentum equation $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$ implies that $v_1 = v_1' + 2v_2' = v_1/2 + 2v_2'$. Solving for $v_2'$ leads to $v_2' = v_1/4$. Answer = B.

**Explanation—extra:** The ratio of the kinetic energies may be determined as follows: $K_f = m(v_1/2)^2/2 + 2m(v_1/4)^2/2 = (1/4 + 1/8)[mv_1^2/2]$, where $K_i = \frac{mv_1^2}{2}$. So $K_f/K_i = 3/8$.
Consider the collision of two balls of masses \( m_1 \) and \( m_2 \), where \( m_1 = m_2 = m \). Ball 1 is initially moving in the positive \( x \) direction with a speed \( v_1 = 3v_0 \). Ball 2 is initially moving in the positive \( y \) direction with a speed \( v_2 = 4v_0 \). After the collision, \( m_1 \) and \( m_2 \) are stuck together. What is the final speed of the combined system?

\[ \begin{array}{ccc}
A & B & C \\
\frac{v_f}{5v_0/2} & \frac{5v_0}{2} & \frac{7v_0}{2} \\
\end{array} \]

**Hint:** For a perfectly inelastic collision, \( \mathbf{p}_f = \mathbf{p}_{cm} = \mathbf{p}_1 + \mathbf{p}_2 \).

**Extra:** Find \( K_f/K_i \).

**Explanation:** From the vector sum shown,

\[ \mathbf{p}_{cm} = \mathbf{p}_1 + \mathbf{p}_2 \] gives \( \mathbf{p}_{cm} = mv_0\sqrt{(3^2 + 4^2)} = 5mv_0 \), or \( v_f = \frac{\mathbf{p}_{cm}}{(m_1 + m_2)} = \frac{5v_0}{2} \). Answer = A.

**Explanation—extra:** \( K_i = K_1 + K_2 = m(3v_0)^2/2 + m(4v_0)^2/2 = 25mv_0^2/2 \).
\( K_f = (2m)(5v_0/2)^2/2 = 25mv_0^2/4 = K_i/2 \), or \( K_f/K_i = 1/2 \).

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100 PhysiQuiz 7. A Perfectly Inelastic Collision in Two Dimensions