Summary on unit 1 (update: 9/11/10)

Constants: \( c = 3 \times 10^8 \text{m/s} \), \( 1u \approx m_p \approx m_n \approx 1.7 \times 10^{-27} \text{kg} \), \( G = 6.7 \times 10^{-11} \text{Nm}^2/\text{kg}^2 \).

**Sec 1.1-1.9.** Coord-vector: \( \mathbf{r} =< x, y, z > = r \hat{r} \), \( r = \sqrt{x^2 + y^2 + z^2} \), \( \hat{r} = < \cos \theta x, \sin \theta y, \cos \theta > \).

For 2D case, \( r = \sqrt{x^2 + y^2} \), \( \hat{r} = < \cos \theta x, \cos \theta y > \).

Vectors (Sec. 1.5): Addition, substraction, magnitude of a vector, unit vector, \( \cdots \).

Displacement: \( \Delta \mathbf{r} = \mathbf{r}_f - \mathbf{r}_i \). (Average velocity) = (Total distance traveled)/(travel time), or \( \mathbf{v}_{avg} = \Delta \mathbf{r}/\Delta t = (\mathbf{v}_1 \Delta t_1 + \mathbf{v}_2 \Delta t_2 + \cdots)/(\Delta t_1 + \Delta t_2 + \cdots) \).

Position update: \( \mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_{avg} \Delta t \).

Instantaneous velocity: \( \mathbf{v} = \lim_{\Delta t \to 0} (\Delta \mathbf{r}/\Delta t) \). Instantaneous acceleration: \( \mathbf{a} = d\mathbf{v}/dt \).

For constant acceleration, \( \mathbf{v}_{avg} = \left( \mathbf{v}_i + \mathbf{v}_f \right)/2 \). Otherwise, for sufficiently small \( \Delta t \), use e.g. \( \mathbf{v}_{avg} \sim \mathbf{v}_f \).

**Momentum:** \( \mathbf{p} = \gamma m \mathbf{v}, \) \( \gamma = 1/\sqrt{1 - \beta^2}, \beta = |\mathbf{v}|/c \).

- Nonrelativistic approximation (NR): \( \gamma \to 1 \), \( \mathbf{p} = m \mathbf{v} \).
- Identity: \( \beta = \beta \gamma/\sqrt{1 + (\beta \gamma)^2} \), where \( \beta \gamma = p/cm \). \( \beta = v/c = (\Delta s/c)/\Delta t \). \( \Delta s = \beta c \Delta t \).

The extended Newton’s law of motion: \( \mathbf{F} = \Delta \mathbf{p}/\Delta t \).

- If \( \mathbf{F} = 0 \), \( \mathbf{p} = \mathbf{p} \) is constant, which is Newton’s first law.
- If \( \mathbf{F} \neq 0 \), for NR case, it leads to \( \mathbf{F} = \Delta \mathbf{p}/\Delta t = m \mathbf{a} \), where \( \mathbf{a} = \Delta \mathbf{v}/\Delta t \) is acceleration. This is Newton’s second law. For the R case, \( \mathbf{F} = \Delta \mathbf{p}/\Delta t = (\Delta \gamma/\Delta t) m \mathbf{v} + \gamma m \mathbf{a} \), with \( \mathbf{a} = \Delta \mathbf{v}/\Delta t \).

**Sec 2.1-2.8, also read 2.9.**

**Momentum principle:** \( \Delta \mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = \mathbf{F} \Delta t \)

Momentum Principle is given by a vector equation. The equation is valid for each Cartesian component. A special case: 1d, NR and \( \mathbf{F} \) = constant, or \( a = F/m = constant \).

- From momentum principle: \( \Delta \mathbf{p} = F_{net} \Delta t \). With \( a = F/m \), it leads to \( v_f = v_i + at \).
- For a constant acceleration case, \( \mathbf{v}_{avg} = (v_i + v_f)/2 \). (Why?)
- Position update: \( \Delta s = v_{avg} \Delta t = (v_i + v_f)/2 \Delta t \). This leads to \( \Delta s = v_i \Delta t + (1/2)a \Delta t^2 \). (Derive)

**Sec 3.1-3.5.** Four kinds of forces (or interactions): Gravitational, electromagnetic, strong and weak.

Gravitational force: \( \mathbf{F} = -\frac{GMm}{r^2} \hat{r} \). Near surface of earth (with radius \( R \)) \( \mathbf{F} = mg = m \left( \frac{GM}{R^2} \right) \approx m \left( \frac{GM}{R} \right) \).

Iterative procedure (3d):

- Begin with the object’s momentum and position \( (\mathbf{p}_i, \mathbf{r}_i) \), and the force \( \mathbf{F}(\mathbf{r}_i) \) at \( t = t_i \),
- **IL**, Iterative Loop: Take time step \( t_f = t_i + \Delta t \). Apply **Momentum principle** to update \( \mathbf{p}_i \) to \( \mathbf{p}_f \).
- **Position update** moves the object from \( \mathbf{r}_i \) to \( \mathbf{r}_f \).
- Set present \( (\mathbf{p}_f, \mathbf{r}_f), \mathbf{F}(\mathbf{r}_f), t_f \) to next step \( (\mathbf{p}_i, \mathbf{r}_i), \mathbf{F}(\mathbf{r}_i), t_i \) Go to **IL**.

Principle of reciprocity (Newton’s third law): Force on 2 due to 1 and force on 1 due to 2 satisfies the relationship: \( \mathbf{F}_{1on2} = -\mathbf{F}_{2on1} \).

Spring force (1d): Magnitude satisfies the relationship \( |\mathbf{F}| = k|s| \). With sign: \( \mathbf{F} = -ks \), \( s = L - L_0 \). The stretched case has \( s > 0 \), leading to attraction. The compressed case has \( s < 0 \), leading to repulsion.