Summary on unit 1 (update: 10/31/09)

Sec 1.1-1.9. Coord-vector: $\mathbf{r} =< x, y, z >= r\hat{\mathbf{r}}$, $r = \sqrt{x^2 + y^2 + z^2}$, $\hat{\mathbf{r}}$=< $cos\theta_x, cos\theta_y, cos\theta_z$ >.
For 2D case, $r = \sqrt{x^2 + y^2 + z^2}$, $\hat{\mathbf{r}}$=< $cos\theta, sin\theta$ >.

Displacement: $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$

Velocity: Definition of average velocity- $\mathbf{v}_{av} = \Delta\mathbf{r}/\Delta t$.

- Position up date: $\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_{av}\Delta t$.
- Instantaneous velocity: $\mathbf{v} = Lim_{\Delta t \to 0}(\Delta\mathbf{r}/\Delta t)$. Instantaneous acceleration: $\mathbf{a} = d\mathbf{v}/dt$.
- If acceleration $\mathbf{a} = d\mathbf{v}/dt = \mathbf{F}/m$ is const., $\mathbf{v}_{av} = (\mathbf{v}_f + \mathbf{v}_i)/2$. If not const. set $\mathbf{v}_{av} \sim \mathbf{v}_f$.

Momentum: $\mathbf{p} = \gamma m\mathbf{v}$, $\gamma = 1/\sqrt{1-\beta^2}$, $\beta = |\mathbf{v}|/c$.

- Nonrelativistic approximation(NR): $\gamma \to 1$, $\mathbf{p} = m\mathbf{v}$.
- Identity: $\beta = \beta\gamma/\sqrt{1 + (\beta\gamma)^2}$, where $\beta\gamma = p/cm$.

The extended Newton's law of motion: $\mathbf{F} = \Delta\mathbf{p}/\Delta t$.

- If $\mathbf{F} = 0$, $\mathbf{p} = p\hat{\mathbf{p}}$= *constant*. This is Newton's first law.
- In NR case, it leads to $\mathbf{F} = \Delta\mathbf{p}/\Delta t$=m$\mathbf{a}$. This is Newton's second law, with definition $\mathbf{a} = d\mathbf{v}/dt$.
- For 1-d, a=const. Mom-P: $v_f = v_i + a\Delta t$, pos-update $s = (v_i + v_f)\Delta t = v_i\Delta t + (1/2)a\Delta t^2$.
- If $\mathbf{F} \neq 0$, multiplying both sides by Δt, it states that
  it is the impact of the impulse, $\mathbf{F}\Delta t$, which causes the effect of the change of momentum, $\Delta\mathbf{p}$.

Sec 2.1-2.10. Momentum principle: $\Delta\mathbf{p}$= $\mathbf{p}_f - \mathbf{p}_i = \mathbf{F}\Delta t$

For a one-body system: The momentum $\mathbf{p} = \gamma m\mathbf{v}$, the force $\mathbf{F}$ exerts on it.

For a many-body system:

- $\mathbf{p} \to \mathbf{p}_{total} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 + \cdots$,
- $\mathbf{F} \to \mathbf{F}_{total,ext} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots$,
  where the force $\mathbf{F}_i$, is the external force acting on the particle i.
- The total internal force is 0. This is due to Newton's 3d law, i.e. for each pair of particles the action force is opposite to the reaction force. (This is the Principle of Reciprocity).
- Center of mass (effective one particle system): $\mathbf{P}_{total} = \mathbf{P}_{cm} = \gamma_{cm}M\mathbf{v}_{cm}$, M=m$_1 + m_2 + \cdots$
- Conservation of momentum: When $\mathbf{F}_{total} = 0$, $\mathbf{P}_{total,ext} = 0$, e.g. collisions, binary stars $\cdots$

Sec 3.1-3.3. Iterative procedure(3d):

- Begin with the object's momentum and position $(\mathbf{p}_i, \mathbf{r}_i)$, and the force $\mathbf{F}(\mathbf{r}_i)$ at $t = t_i$
- **IL**, Iterative Loop: Take time step $t_f = t_i + \Delta t$. Apply Momentum principle to update $\mathbf{p}_i$ to $\mathbf{p}_f$.
- Position update moves the object from $\mathbf{r}_i$ to $\mathbf{r}_f$.
- Set present $(\mathbf{p}_f, \mathbf{r}_f)$, $\mathbf{F}(\mathbf{r}_j)$, t_f to next step $(\mathbf{p}_i, \mathbf{r}_i)$, $\mathbf{F}(\mathbf{r}_i)$, t_i Go to **IL**.

Spring force(1d): $|F| = k|s|$. With sign: $F = -ks$, $s = L - L_0$. Stretched $s > 0$, compressed $s < 0$.

Four kinds of forces(or interactions): Gravitational, electromangetic, strong and weak.