

Maxwell equations: GL, Mag. GL, FL and Ampere-Maxwell law: $\oint_{path} \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(I_{path} + \epsilon_0 \frac{d}{dt} \int_{path} \mathbf{E} \cdot d\mathbf{A} \right)$.

Displacement current: Within a capacitor: $\phi_E = \left(\frac{Q/A}{\epsilon_0} \right) A = \frac{Q}{\epsilon_0}$, $I_{displacement} = \frac{dQ}{dt} = \epsilon_0 \frac{d\phi_E}{dt}$.

Electromagnetic Radiation Propagation direction: $\mathbf{E} \times \mathbf{B}$, $\mathbf{E} \perp \mathbf{B}$, $E = cB$, $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$.

Moving slab (pulse) Solution to Maxwell equations in vacuum, where there are no charge and no current.

FL: $|emf| = \frac{d\phi_B}{dt}$ leads to $Eh = Bvh$, (1), A-ML: $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \epsilon_0 Evh$, (2). (1) and (2) leads to $v^2 = \frac{1}{\mu_0 \epsilon_0} = c^2$.

Sinusoidal travel. waves: $E = E_{max} \cos(kx - \omega t + \phi)$. Periodic: $k\lambda = 2\pi$, $\omega T = 2\pi$, with frequency $f = 1/T$.

For fixed phase $(kx - \omega t)$, wave velocity $v = \frac{dx}{dt} = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$. For fixed phase $(kx + \omega t)$, what is v ?

E_{rad} formula. Field at r due to acceleration of charge q at the origin: $\mathbf{E}_{rad} = \frac{1}{4\pi\epsilon_0} \frac{-qa_{\perp}}{c^2 r}$, where $a_{\perp} = a \sin\theta$.

Derive \vec{E}_{rad} . See Sec. 24.11. In Fig24.102, define $\tan\alpha = \frac{E_{\perp}}{E_{coul}}$. Geometry gives $\tan\alpha = \frac{vT \sin\theta}{ct} = \frac{a_{\perp} r}{c^2}$, since $v = at$, $r = cT$ and $a_{\perp} = a \sin\theta$. In turn $E_{rad} = \frac{kq}{r^2} \times \left[\frac{a_{\perp} r}{c^2} \right] = \frac{kqa_{\perp}}{c^2 r}$. \vec{E}_{rad} & \vec{a}_{\perp} are in opposite directions.

Energy, momentum, and re-radiation (scattering) of em waves

Energy density: Equal partition between \mathbf{E} and \mathbf{B} . Show $u_E = u_B$ and $u \equiv u_E + u_B = \epsilon_0 E^2 = \frac{B^2}{\mu_0}$.

Energy flux (Poynting vector): $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = I \hat{S}$, where $I = \frac{U}{A \Delta t} = \frac{Power}{A} = \frac{U}{Ac \Delta t} c = uc$.

Momentum-flux. Particle with m : $p = mv\gamma$, $U = mc^2\gamma$, $p = \left(\frac{v}{c^2} \right) U$. Photon $p = \frac{U}{c}$. mom-flux: $\frac{\vec{S}}{c} = p \hat{S}$.

Time depend. of I and u: $I(t) = \frac{U(t)}{A \Delta t} = u(t)c$. $I(t)$ oscillates rapidly. Will look at only time averaged values.

Time averaged sinusoidal em waves. Let $E = E_{max} \sin(\omega t + \phi)$, $I(t) = \epsilon_0 c E_{max}^2 \sin^2(\omega t + \phi)$.

Intensity: $I \equiv \overline{I(t)} = \overline{u(t)c} = \epsilon_0 c \overline{E(t)^2} = \epsilon_0 c \overline{E_{max}^2 \sin^2(\omega t + \phi)} \equiv \epsilon_0 c E_{rms}^2$, where $E_{rms} \equiv \sqrt{\overline{E(t)^2}} = \frac{E_{max}}{\sqrt{2}}$.

EM radiation pressure ("Press", time averaged): $Press \equiv \frac{F}{A} = \frac{dp/dt}{A} = \frac{dU}{cA dt}$, where $p = \frac{U}{c}$ was used.

100% reflective case: $dp = 2p$, $Press = 2u = 2\frac{I}{c}$. 100% absorptive case, $dp = p$, $Press = u = \frac{I}{c}$.

Polarization of em waves: Direction of polarization is defined to be along the oscillations of \mathbf{E} .

Upon incident on a sheet of parallel metal strips, \mathbf{E} can be decomposed into two components. The one aligned with the strips, it drives the induced current in the strips. There is strong absorption which prevents the transmission of this component. The other is transverse to the strips, there is a negligible induced current transverse to the strip. So the transverse component can be transmitted. Unpolarized light is uniformly polarized in all directions. It is 50%-50% polarized along any pair of \perp axes. The strips can serve as a "polarizer". When an unpolarized light incident on it, the outgoing light is polarized. It is along the direction of transmission axis. For an polarized incident light, the intensity of the transmitted outgoing waves satisfies Malus rule, $I_{out} = I_{in}^2 \alpha$, where α is the angle between the incident and outgoing polarizations.

Scattering of light: $I \propto E_{rad}^2 \propto a_{\perp}^2$. Electron oscillations about an equilibrium: $y = y_0 \sin\omega t$, $a_y = -\omega^2 y$.

As light traverses in medium, more scattering leads to more removal of high frequency light from the beam.

Geometric optics Light enters a medium. $v = c/n$. Same frequency. Wavelength: $\lambda' = v/f = \lambda/n$.

Snell's law. $n_1 \sin\theta_1 = n_2 \sin\theta_2$ Critical angle occurs when the incident angle $\theta_2 = \theta_c$ with $\theta_1 = 90^\circ$.

Total reflection: The total reflection region for θ_2 , incident angle of ray2, is defined by $\theta_c < \theta_2 < 90^\circ$.

Color dispersion The index of refraction n increases as the wavelength of the light decreases. When a white ray passes through a prism, outgoing rays are color dispersed with a rainbow spectrum.

Thin lens & small angle approx: At fixed y , deflection angle is independent of the value of the incident angle.

Relabel text variables: $p = d_1$, $q = d_2$. Geometry of Fig. 24.75 and 24.77 leads to $\theta = \alpha + \beta$ or $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$.

Here $f > 0$. Repeat same procedures for the $f < 0$ case. Review Ray-tracing method and qp-plot method.

Corrective Lenses (c-lenses) in eyeglasses: ($diopter = m^{-1}$). Clear vision range of a normaleye is from 25cm to ∞ . Nearsighted range is from $a_0 < 25cm$ to a_1 . For an object at a_1 the c-lens forms an image at ∞ . Farsighted range is from b to ∞ . For an object at 25cm, the c-lens forms an image at b .

Wave-optics Interference between two rays with same amplitudes and same frequency. Denote the difference between the two path lengths as Δl , with distance between two sources d .

Case1: Circular screen. Let $\Delta l = d \sin \theta$. Given d , determine θ at each of the maxima. Given d count the total number of maxima.

Case2: A distant flat screen. Small angle approximation gives $\Delta l/d = \sin \theta \sim \theta = x/L$. Find position of x for m th maximum. What happens to x , when λ changes?, when d changes?, or when L changes?