Go to: Course homepage, Lectures

Lecture 8 16.1-4 iq06

1. Revisit the charged rod problem. See class notes **bullet 2**.
   - Change variable of integration from linear variable to angular variable.
   - Verify the closed form result agrees with the the result of integration table.
2. Field along x-axis (more generally along r-axis) due to a charged rod Clicker **8-3**
   - Far field, at r>>L/2
   - Near field, at r<<L/2.
3. Force between a long rod and a dipole , discussion on clicker for **h1-003**.
4. Field along z due to a charged ring.
   - Far field, at r>>R
   - Near field, at r<<R

4. z-component of the field in the intermediate region.
5. For positively charged ring, find the motion of an electron near z=0.

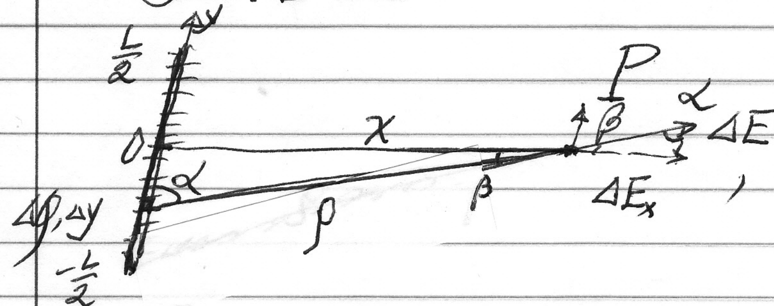
Class Announcements:

ig06

Uniformly charged rod, Q, L, linear charge density $\frac{\Delta Q}{\Delta y} = \lambda = \frac{Q}{L}$.

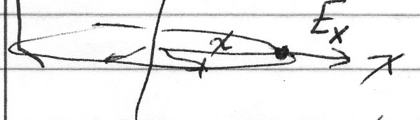
$\therefore \Delta Q = \lambda \Delta y$

Find: $\vec{E}$ at P



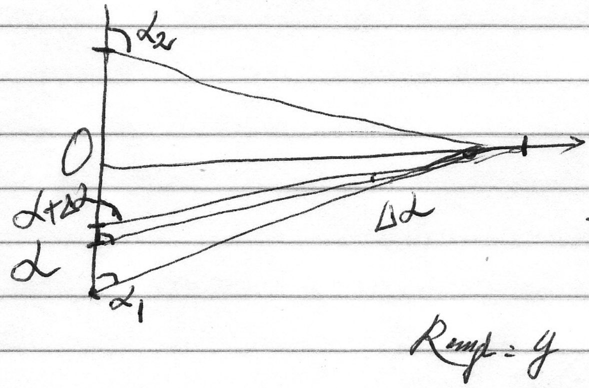
$\sin\alpha = \frac{\Delta E_x}{\Delta E} = \frac{x}{\rho}$

[Cylindrically symmetric: At the end replace x by r]



$$\Delta E = \frac{k(\lambda \Delta y)}{\rho^2}, \quad \Delta E_x = \Delta E \sin\alpha$$

$$E_x = \sum \frac{k\lambda \Delta y}{\rho^2}\sin\alpha \longrightarrow k\lambda \int \frac{dy}{\rho^2}\sin\alpha$$



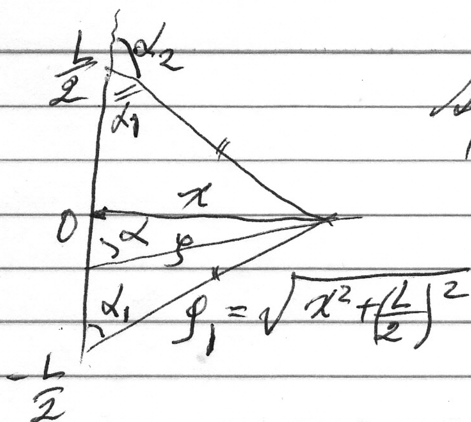
Change variable $\Delta y \to \Delta\alpha$

I.D. $\frac{dy}{\rho} \to \frac{d\alpha}{x}$

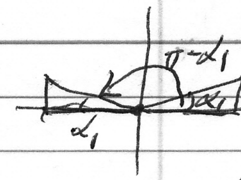
Range: $y \quad -\frac{L}{2} \to +\frac{L}{2}$

$\alpha: \quad \alpha_1 \to \alpha_2$

$$E_x = k\lambda \int_{\alpha_1}^{\alpha_2} \frac{d\alpha}{x}\sin\alpha = \frac{k\lambda}{x}(-\cos\alpha)\Big|_{\alpha_1}^{\alpha_2} = \frac{k\lambda}{x}(\cos\alpha_1 - \cos\alpha_2$$



Symmetry $\alpha_2 = \pi - \alpha_1$



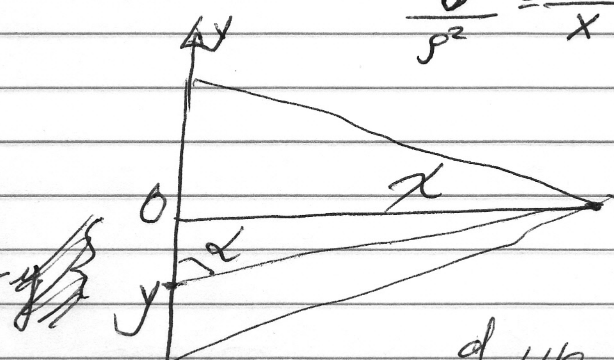
$$\cos\alpha_1 - \cos\alpha_2 = 2\cos\alpha_1 = 2\frac{(L/2)}{\rho_1}$$

$$= \frac{L}{\rho_1}$$

$$\therefore E_x = \frac{k\lambda}{x_1} \cdot \frac{L}{\rho_1}$$ (See p634)

$$= \frac{kQ}{x_1 \rho_1}.$$

Derive Math ID: $\dfrac{dy}{\rho^2} = \dfrac{d\alpha}{x}$



$$\tan\alpha = \frac{x}{(-y)}$$

$$\frac{d}{d\alpha} LHS = \frac{d}{d\alpha}\tan\alpha = \frac{1}{\cos^2\alpha} = \frac{1}{\left(\frac{y}{\rho}\right)^2} = \frac{\rho}{y}$$

$$\frac{d}{d\alpha} RHS = \frac{d}{d\alpha}\frac{\overset{\text{fixed}}{x}}{(-y)} = (-x)\frac{d}{d\alpha}\frac{1}{y} = (-x)\left(\frac{-1}{y^2}\right)\frac{d}{d\alpha}$$

$$= \frac{x}{\cancel{y^2}}\frac{dy}{d\alpha} = \frac{d}{d\alpha}LHS = \frac{\rho^2}{\cancel{y^2}}$$

$$\therefore \boxed{\frac{dy}{\rho^2} = \frac{d\alpha}{x}}$$

Back to E at P:

$$E_r = \frac{kQ}{r\sqrt{r^2+\left(\frac{L}{2}\right)^2}}$$

Far field: $r >> \frac{L}{2}$. $\quad E_r = \frac{kQ}{r^2}$ As expected

Near field: $r << \frac{L}{2}$

$$E_r = \frac{kQ}{r\left(\frac{L}{2}\right)}$$

. Later on we will see from Gauss Law One finds the near field is given by $\frac{\lambda}{2\pi r \epsilon_0}$

Check:

RHS: $\frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{\frac{rL}{2}}$

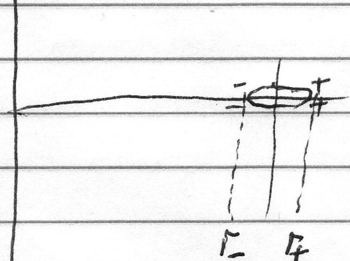
$= \frac{1}{2\pi\epsilon_0} \cdot \frac{\lambda}{r}$. ✓

W1.003

$\frac{\Delta Q}{\Delta y} = \lambda > 0$

$$F_{drop}^{rod} = (-q)E_{-}^{rod} + qE_{+}^{rod} < 0$$

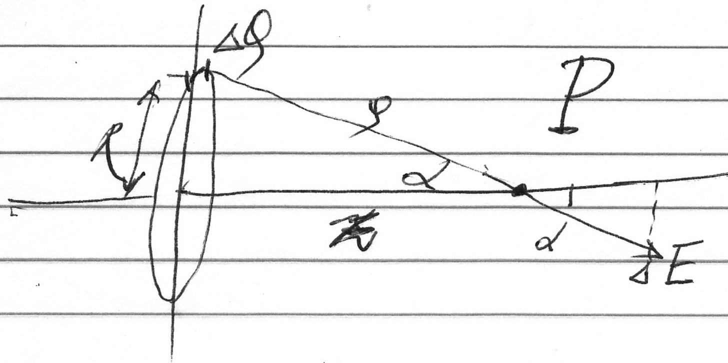
$$E_r = \frac{2k\lambda}{r}$$



$$\left|F_{drop}^{rod}\right| = q\,\frac{2k\lambda}{r}\left|-\frac{1}{(1-\epsilon)} + \frac{1}{1+\epsilon}\right|$$

$$-(1+\epsilon) + (1-\epsilon) = -2\epsilon$$
$$= -2\frac{(S/2)}{r}$$
$$= -\frac{S}{r}$$

$\therefore \left|F_{drop}^{rod}\right| \propto \frac{1}{r^2}$



Given: Unif. charged ring

Q, R

Find: E at P

$$\Delta E = \frac{k\Delta Q}{\rho^2}, \qquad \Delta E_z = \Delta E \cos\alpha = \Delta E \cdot \frac{z}{\rho}$$

$$E_z = \int dE_z = \int \frac{k\Delta Q}{\rho^2} \cdot \frac{z}{\rho} = \frac{kQz}{\rho^3}$$

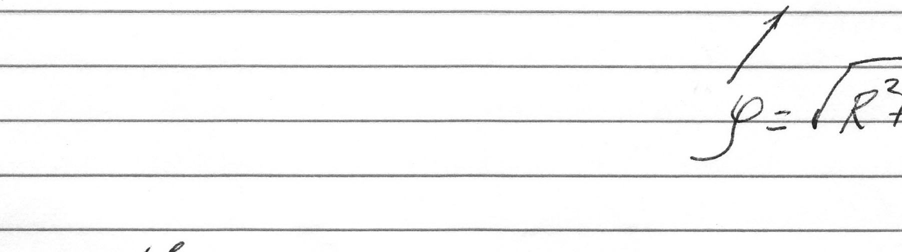
$$\rho = \sqrt{R^2 + z^2}$$

Farfield: $z \gg R$

$$E_z = \frac{kQz}{z^3} = \frac{kQ}{z^2} \text{ as expected } \checkmark$$

Nearfield: $z \ll R$ $\quad E_z = \dfrac{kQz}{R^3}$

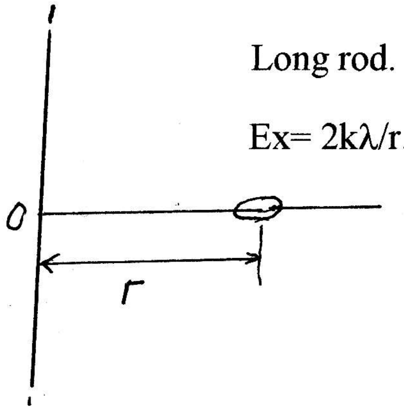
$$E_z = \frac{kQz}{|z|^3}$$



Oscillation of e. $\quad F_z = (-e)E_z$

SHM: $\ddot{z} = -\omega z \qquad \ddot{a} = a_z = -\frac{2kQ}{R^3} z = -\left(\frac{ekQ}{R^3}\right) z$

clicker h1-003.



Long rod. $\lambda=\Delta Q/\Delta y > 0$.

$E_x= 2k\lambda/r$.

Sign of F^{rod}_{drop}: Choices:

1	Repulsive
2	Attractive

Magnitude of the force: $|F^{rod}_{drop}|$

1	$\lvert F^{rod}_{drop}\rvert \propto 1/r^2$
2	$\lvert F^{rod}_{drop}\rvert \propto 1/r^3$
3	$\lvert F^{rod}_{drop}\rvert \propto 1/r^4$