Go to: Course homepage, Lectures

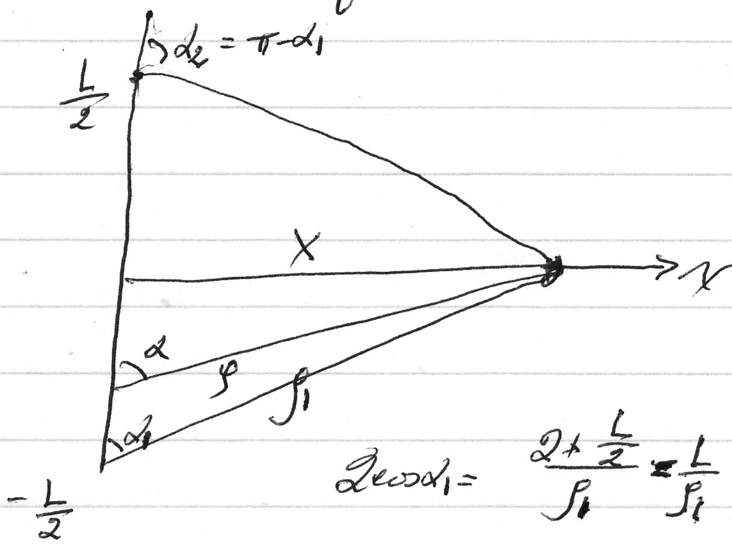
Lecture 9 Ch16.3-6 iq07

1. Force between a long rod and a dipole , discussion on clicker
   h1-003.
2. Field along z due to a charged ring.
   a. Far field, at r>>R
   b. Near field, at r<<R
3. For positively charged ring, find the sign of test charge leading to oscillations near z=0.
   A new clicker question. (See Lec 9-2a)
4. Field along z due a disk
   a. Building up the disk by concentrated rings
   b. Evaluate the I-integral
   c. The far field
      Use the small argument expansion to show point charge case is recovered. clicker 9-1.
   d. The near field, and the field in intermediate region.
5. Fields associated with a parallel plate capacitor
   - Within the gap
   - Outside the gap

   clicker **9-2**.

Class Announcements:

ig07



At P:

$$E_x = \frac{k\lambda}{x}\left(-\cos\alpha\right)\Big|_{\alpha_1}^{\alpha_2}$$

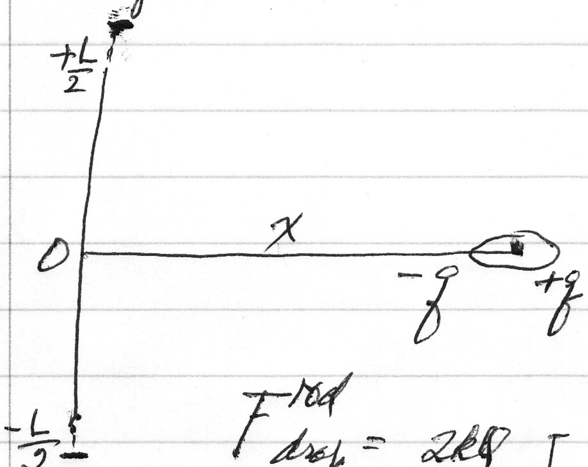
$$= \frac{k\lambda}{r}\, 2\cos\alpha_1 = \frac{k\lambda}{r}\frac{L}{\rho_1}$$

$$2\cos\alpha_1 = \frac{2 * \frac{L}{2}}{\rho_1} = \frac{L}{\rho_1}$$

$$\rho_1 = \sqrt{x^2 + \left(\frac{L}{2}\right)^2}$$

h1-003

Long wire:



$\frac{L}{2} \gg x$ (Nearfield case) $E_x = \frac{k\lambda L}{\cancel{x}\,\frac{L}{2}} = \frac{2kQ}{xL}$

Find: F^{rod}_{drop}

$-q$ at: $x - \frac{s}{2} = x(1-\epsilon)$ $\quad \epsilon = \frac{s}{2x}$

$+q$ at: $x + \frac{s}{2} = x(1+\epsilon)$,

$$F^{rod}_{drop} = \frac{2kQ}{xL}\left[-\frac{1}{1-\epsilon} + \frac{1}{1+\epsilon}\right]$$

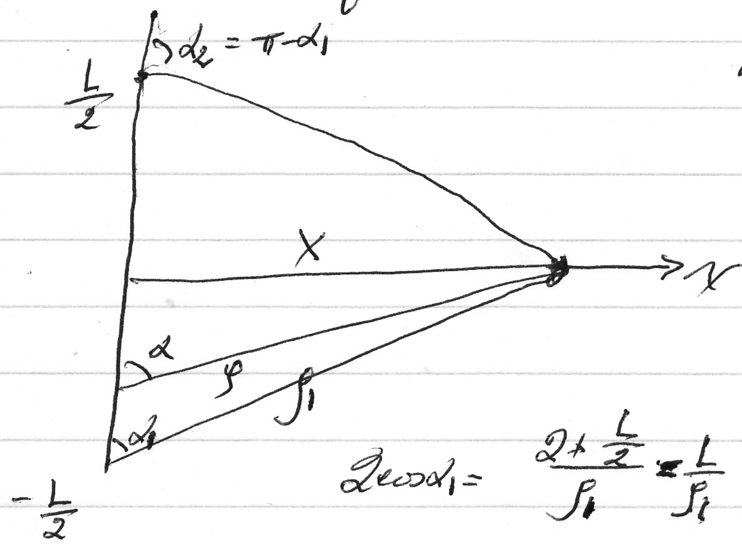
$$-2\epsilon = (-2)\left(\frac{s}{2x}\right)$$

Attractive

$$F^{rod}_{drop} \propto \frac{1}{x^2}$$

ig07

Lec 9-1



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h1-003

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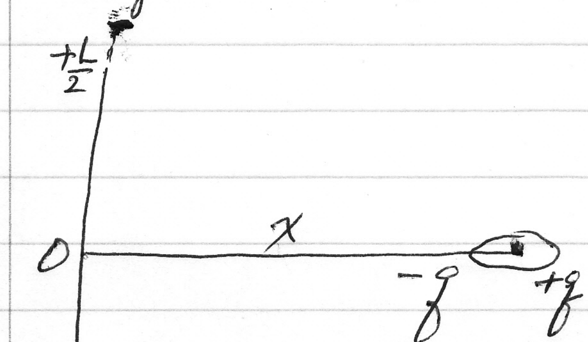

$-\frac{L}{2}$

$-q$ at: $x - \frac{s}{2} = x(1-\epsilon)$ $\quad \epsilon = \frac{s}{2x}$

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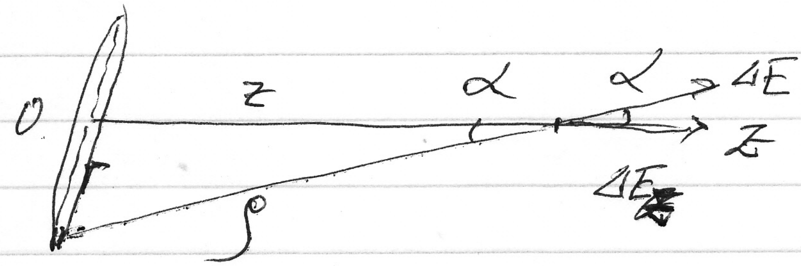
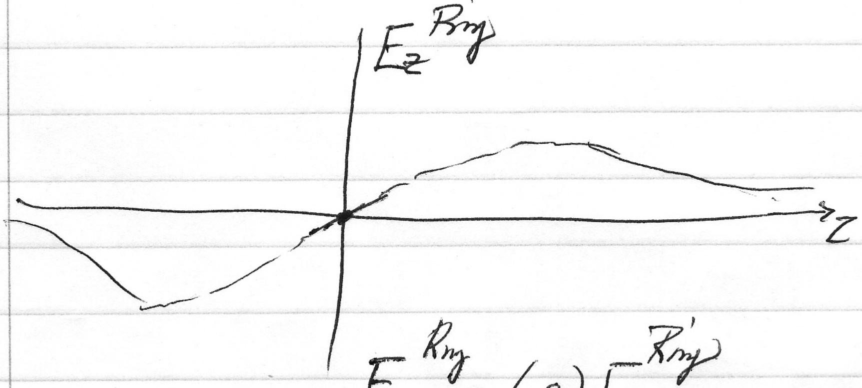
Ring: Q, r



$$\cos\alpha = \frac{\Delta E_z}{\Delta E}, \qquad \Delta E_z = \cos\alpha \, \Delta E$$

$$\cos\alpha \to \frac{z}{\rho}, \qquad \Delta E \to \frac{k\Delta Q}{\rho^2}$$

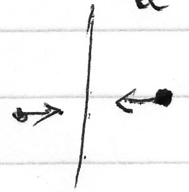
$$E_z^{Ring} = \int \frac{kz\,dQ}{\rho^3} = \frac{kzQ}{\rho^3}$$



$z \gg R$, $\rho = \sqrt{z^2 + r^2} \approx$

$$E_z = \frac{kz}{z^3} Q = \frac{kQ}{z^2}$$

$$F_{el}^{Ring} = (-e) E^{Ring}$$

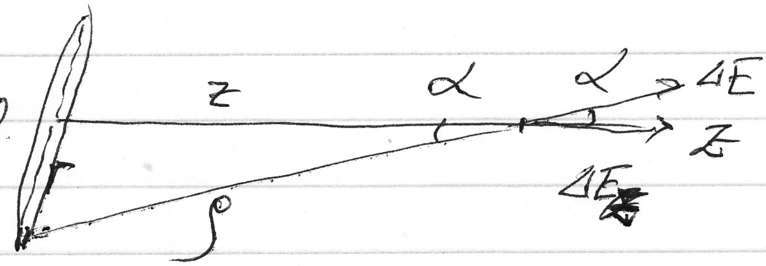


Small displacement toward equilibrium position at $z=0$

∴ ~~Small~~ Oscillate in small z about $z=0$

eg For positive charged ring determine sign of test charge placed near $z=0$ which will lead to small oscillation

Ring : Q, r



$$\cos\alpha = \frac{\Delta E_z}{\Delta E}, \qquad \Delta E_z = \underbrace{\cos\alpha}_{\frac{z}{\rho}} \; \underbrace{\Delta E}_{\frac{k\Delta Q}{\rho^2}}$$

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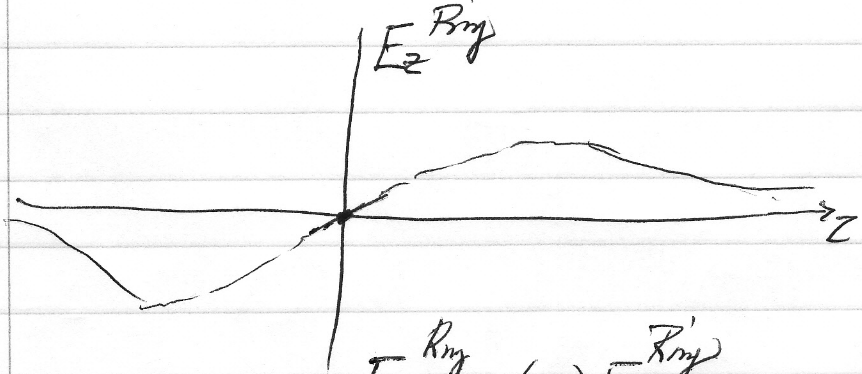

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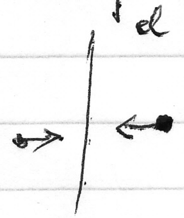


Small displacement toward equilibrium position at $z=0$

$\therefore$ ~~Small~~ Oscillation in small z about $z=0$

ie For positive charged ring determine sign of test charge placed near $z=0$ which will lead to small oscillation

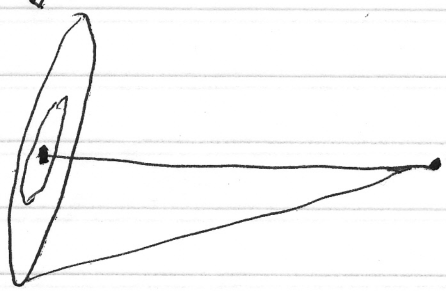
Lec 9 2nd

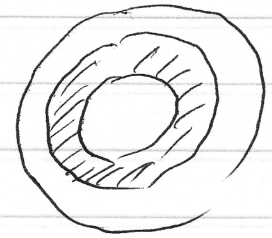
Added clicker question to Lecture 9

For a positively charged ring, determine the sign of the test charge placed near z=0, which will lead to small z oscillations about z=0.

	Sign of test charge
1	positive charge
2	negative charge

Uniformly charged disk.





Disk:

$q^{disk} = \sum \Delta q$

Concentric rings.

$$\Delta q^{disk} = \sigma\, 2\pi r\, \Delta r \quad , \quad E^{Ring} = \frac{k\, q^{Ring}\, z}{\rho^3}$$

$$q^{disk} = \sum \Delta q^{disk} = \sum \sigma (2\pi r \Delta r)$$

$$E^{disk} = \sum \frac{k\, \Delta q^{disk}\, z}{\rho^3} = \sum \frac{k \sigma 2\pi r \Delta r\, z}{\rho^3}$$

$$= \sum (k \sigma 2\pi z) \frac{r \Delta r}{\rho^3}$$

$$= (k\sigma 2\pi z) \underbrace{\int \frac{r\,dr}{\rho^3}}_{I}$$

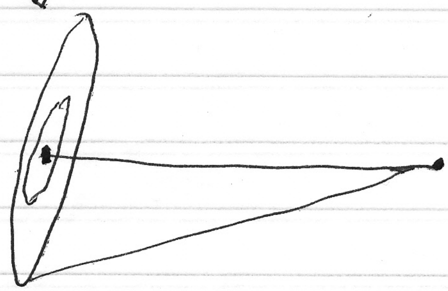
$$I = \int_0^r \frac{r\,dr}{\rho^3} \xrightarrow{\text{change of int variable}} \int \frac{\rho\, d\rho}{\rho^3} = \int \frac{d\rho}{\rho^2} = -\frac{1}{\rho}\Big|_{z}^{\sqrt{z^2+}}$$

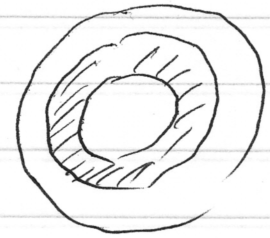
$$\rho^2 = r^2 + z^2, \quad 2\rho\, d\rho = 2r\, dr$$

$$E^{disk} = (k\sigma 2\pi z)\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right]$$

$$= (k\sigma 2\pi)\left[1 - \frac{1}{(1+\epsilon)^{1/2}}\right] \quad \text{where } \epsilon = \frac{R^2}{z^2}.$$

Uniformly charged disk.





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$$E^{disk} = (k\sigma 2\pi)\left[1 - \frac{1}{(1+\epsilon)^{1/2}}\right], \text{ where } \epsilon = \frac{R^2}{Z^2}.$$

Far field:

eq 9-1 $$E^{disk} = (k\sigma 2\pi)\left[1 - \left(1 - \frac{\epsilon}{2}\right)\right]$$

$$\frac{\epsilon}{2} = \frac{R^2}{2Z^2}.$$

$$= k\sigma \cancel{2}\pi\left(\frac{R^2}{\cancel{2}Z^2}\right) = \frac{k\sigma\pi R^3}{Z^2} = \frac{kQ}{Z^2}.$$

Near field:

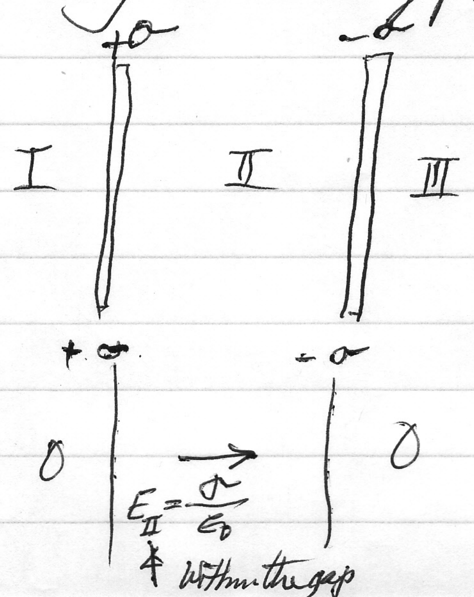
$$E^{disk} = (k\sigma 2\pi \cancel{z})\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right], \quad z \ll R$$

$$\frac{1}{z} - \frac{1}{R} \approx \frac{1}{z}$$

$$= k\sigma 2\pi = \frac{\sigma}{2\epsilon_0}$$

Capacitor field within the gap:

eq 9-2



	I	II	III
E^{left}	$-\frac{\sigma}{2\epsilon_0}$	$+\frac{\sigma}{2\epsilon_0}$	$+\frac{\sigma}{2\epsilon_0}$
E^{right}	$\frac{\sigma}{2\epsilon_0}$	$\frac{\sigma}{2\epsilon_0}$	$-\frac{\sigma}{2\epsilon_0}$
$E^{resultant}$	0	$\frac{\sigma}{\epsilon_0}$	0