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Lecture 10 Ch16.1-16.8, Ch22.1-22.4 Gauss law iq08

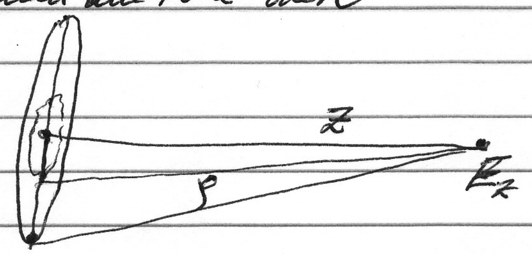
1. Review: Field due to a charged disk
   a. Charged disk: Finite radius, infinite radius limit. Clicker **9-1**
   b. Parallel plate capacitor – a device stores eleectric energy. Clicker **9-2**
2. Field due to spherically symmetric charged distribution.
   a. Point charge: A simple example of Gauss's law
   b. Electric flux and the Gauss law -- porcupine-needle analogy.
   c. Spherical charged shell
   d. What is E within the shell?
3. Field due to a long rod derived from Gauss's law.
4. Field due to an infinite sheet derived from Gauss's law.

clicker questions: clicker **10.1**

Class Announcements:

- Please go through line by line the physics content of course summary of unit 1.
- We are looking for an undergrad assistant who can convert present handwritten classnotes posted into editable efiles.
  - The text will be typed in Latex format.
  - The figure will be redrawn using some free download illustrator application so that each figure is editable.
- If you are interested in applying for this position please contact me.
  - The applicant need to submit sample editable files to demonstrate he/she has the computer skill for the job.
  - The applicant also needs to estimate the total time needed to complete the project. There are approximately 40 lectures in total.
  - The deadline for the application is on Friday, Feb. 15.

1. Field due to a disk



$$E_z = (k\sigma 2\pi z)\, I$$

$$I = \int_0^R \frac{r\,dr}{\rho^3}$$

Change of variable of integration: $\rho^2 = r^2 + z^2$, $\rho\, d\rho = r\, dr$

$$I = \int \frac{\rho\, d\rho}{\rho^3} = -\frac{1}{\rho}\Big|_z^{\sqrt{z^2+R^2}} = -\frac{1}{\sqrt{z^2+R^2}} + \frac{1}{z} = \frac{1}{z}\left(1 - \frac{1}{\sqrt{1+\epsilon}}\right)$$

$$\epsilon = \frac{R^2}{z^2}$$

$$\cancel{E_z} = (k\sigma 2\pi \cancel{z})\, \frac{1}{\cancel{z}}\left(1 - \frac{1}{\sqrt{1+\epsilon}}\right)$$

$$1 - \left(1 - \frac{\epsilon}{2}\right) = \frac{\epsilon}{2} = \frac{R^2}{2z^2}.$$

Lec 9-1

$z \gg R$

$$= (k\sigma 2\pi)\, \frac{R^2}{2z^2} = \frac{k\sigma\pi R^3}{z^2} = \frac{kQ}{z^2},$$

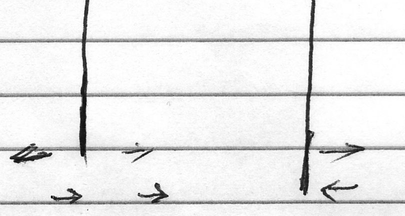
$z \ll R$

$$E_x = (k\sigma 2\pi z)\left(\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right)$$

$$\simeq (k\sigma 2\pi) = \frac{\sigma 2\pi}{4\pi\epsilon_0} = \frac{\sigma}{2\epsilon_0}$$

2. Parallel plate capacitor

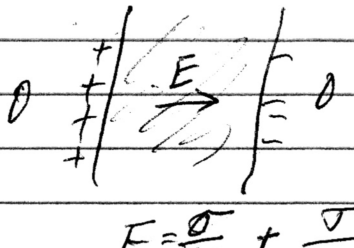
Lec 9-2



	I	II	III
left	$-\frac{\sigma}{2\epsilon_0}$	$+\frac{\sigma}{2\epsilon_0}$	$+\frac{\sigma}{2\epsilon_0}$
right	$\frac{\sigma}{2\epsilon_0}$	$\frac{\sigma}{2\epsilon_0}$	$-\frac{\sigma}{2\epsilon_0}$
	0	$\frac{\sigma}{\epsilon_0}$	0

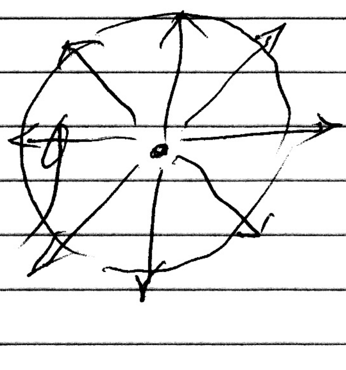
Capacitor a device stores electric energy

Energy density $= \frac{U}{\Delta vol} = \frac{1}{2}\epsilon_0 E^2$,

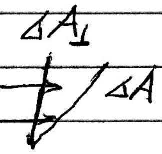


$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$; $U_{stored} = \left(\frac{1}{2}\epsilon_0 E^2\right) A d$.

3. Point charge Q, E-lines, E-flux, Gauss Law



$\Delta\Phi = E \Delta A_\perp$



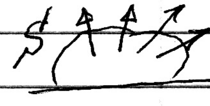
$\Phi = \frac{kQ}{r^2} \times 4\pi r^2$

$= \frac{Q}{\epsilon_0}$ Simple example of Gauss Law

$\Phi_S = \oint_S \vec{E} \cdot \Delta\vec{A}$

$= \oint_S \frac{kQ}{r^2} \times 4\pi r^2 =$ $\quad k = \frac{1}{4\pi\epsilon_0}$,

$= \oint_S \frac{kQ}{r^2} \times 4\pi r^2 = \frac{Q_S}{\epsilon_0}$.

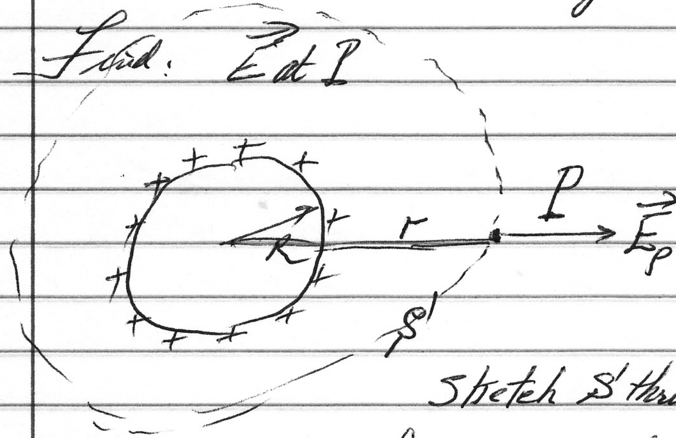


Electric flux through S: $\Phi_S = \frac{Q_S}{\epsilon_0}$ ← charge enclosed by S — Gauss Law

Porcupine-needle analogy

Given: a spherical conducting shell with Q on the surface

Find: $\vec{E}$ at P



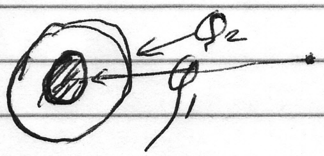
Sketch S' thru P

$$\Phi_S = \oint_S \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$\oint_S \vec{E}\cdot d\vec{A} = E\,\Delta A_\perp$$

$$E\oint_S dA_\perp = E_?\, 4\pi r^2 = \frac{Q_S}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

$$\therefore E_? = \frac{Q}{4\pi\epsilon_0 r^2}$$



$$E_?\, 4\pi r^2 = \frac{\overbrace{(Q_1 + Q_2 + \cdots)}^{=Q}}{\epsilon_0}$$

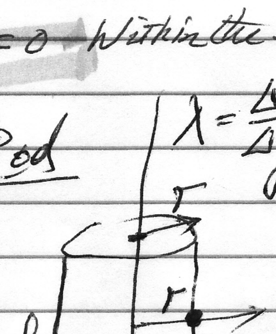
$E_? = 0$ within the shell. $E_? = \dfrac{Q}{4\pi\epsilon_0 r^2}$

$\dfrac{\Delta Q}{\Delta y} = \lambda$, $Q = \lambda h$

Long Rod $\quad \lambda = \dfrac{\Delta Q}{\Delta y}$

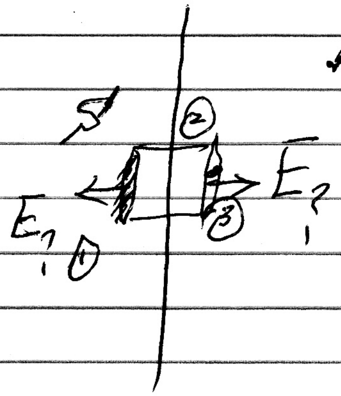
$Q_S = \lambda h$

Lec 10-2



$$\oint_S \vec{E}\cdot d\vec{A} = \oint_S E\,dA_\perp = E_? \underbrace{\oint_S dA_\perp}_{h 2\pi r} = \frac{Q_S}{\epsilon_0} = \frac{\lambda h}{\epsilon_0}$$

$$E_? = \frac{\lambda}{2\pi r \epsilon_0}$$ Agrees with near field expression.



Infinite sheet: Given: $\sigma = \frac{Q}{A}$

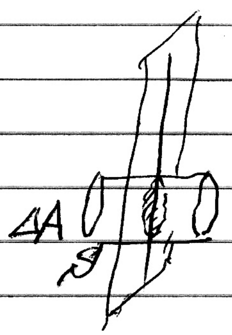
Find: $E_?$ near the surface

Cylindrical Gaussian surface

$$\Phi_S = \oint E\,\Delta A_\perp = \Phi_{1} + \Phi_{2} + \Phi_{3}$$

$$= E_?\,\Delta A + 0 + E_?\,\Delta A = 2E_?\,\Delta A$$

$$= \frac{Q_S}{\epsilon_0} = \frac{\sigma \Delta A}{\epsilon_0}$$



Charge enclosed by Gaussian surface:

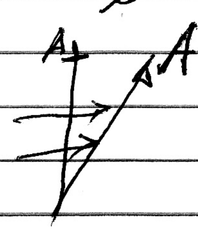
$$Q_S = \Delta Q = \sigma \Delta A$$

$$\therefore\ 2E_?\,\Delta A = \frac{\sigma \Delta A}{\epsilon_0} \qquad \therefore\ E_? = \frac{\sigma}{2\epsilon_0}.$$

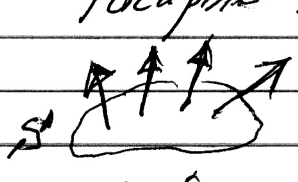
Recap:

Gauss Law: $\Phi_S = \oint_S E\,dA_\perp = \frac{Q_S}{\epsilon_0}$

$\Delta\Phi = E\,\Delta A_\perp$



Porcupine–Needle analogy



$$\Phi_S = \frac{Q_S}{\epsilon_0}$$