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Lecture: 20 (iq18)

1. Second order small eps-expansion for a quadrupole charge system
2. B along z due to a circular ring: Dipole moment of a loop. **clicker 18-2.**
   Added meaning to RHR3.
3. Magnetic dipole moment of an atom
   - Current loop is like a magnet. It is a dipole source which generates B^{dipole}.
   - Measurement of a magnetic dipole of a bar magnet clicker: **Direction of B^{earth}**
   - Dipole moment of a H-like atom in terms of orbital angular momentum.
   - Quantum unit of atomic dipole moment **clicker 19-4**
   - Order of magnitude estimate of the macro-magnetic-dipole-moment of a bar magnet.
4. Ampere's law
   - B due to a line segment – a simple example of the Ampere's law.
   - Hints to homework problems
     1. A cylindrical wire with a circular hole. **Ch18-h3:016.**
     2. Long solenoid: **clicker 20-3**. Please read it on your own.

Announcement:

- Ch18-h2, 008. The two currents are assumed to have the same magnitude.
- Regular office hours: MWF 9:15 to 10:15.
- Review unit2:

1. Small ϵ expansion for $V(x)$ of a quadrupole system.

Explanation on Ch17-h3:008.

Given: Quadrupole charge system is defined by —

There is $+Q$ at $x = \pm d$, $-2Q$ at $x = 0$.

Find: $V(x)$, where $x \gg d$.

Explanation: $V(x) = kQ\left[\frac{1}{x+d} - 2 + \frac{1}{x-d}\right]$

$$= \frac{kQ}{x}\left[\frac{1}{1+\epsilon} - 2 + \frac{1}{1-\epsilon}\right] \equiv \frac{kQ}{x} f(\epsilon)$$

Notice to first order in ϵ

$$f(\epsilon) \approx (1-\epsilon) - 2 + (1+\epsilon) = 0.$$

Need to work to the order of ϵ^2.

We work with approximation

$$\frac{1}{1-\epsilon} = 1 + \epsilon + \epsilon^2 \quad (1)$$

[Proof: $1 \stackrel{?}{=} (1-\epsilon)(1+\epsilon+\epsilon^2) = (1+\epsilon+\epsilon^2) - (\epsilon + \epsilon^2 + \epsilon^3)$

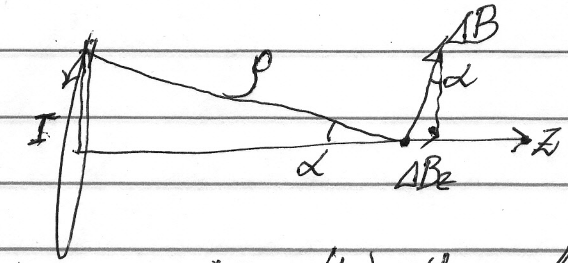
$= 1 - \epsilon^3$. So (1) is satisfied up to ϵ^2-term

Using (1), $f(\epsilon) = \left[\frac{1}{1+\epsilon} - 2 + \frac{1}{1-\epsilon}\right] = 1 + (-\epsilon) + (-\epsilon)^2 - 2 + 1 + \epsilon + \epsilon^2$

$$= 2\epsilon^2 = 2\left(\frac{d}{x}\right)^2$$

$$V(x) = \frac{kQ}{x} \cdot \frac{2d^2}{x^2}$$

2.



B along z due to current loop; I, r

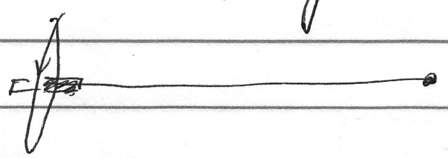
Ring: I, R

$$\Delta B_z = \left(\frac{\mu_0}{4\pi}\right)\frac{I\Delta l}{\rho^2}\cdot \sin\alpha \qquad \sin\alpha = \frac{R}{\rho}$$

$$B_z = \frac{\mu_0}{4\pi}\cdot\frac{I 2\pi R * R}{\rho^3} \approx \left(\frac{\mu_0}{4\pi}\right) 2\,\frac{IA}{z^3} \equiv \left(\frac{\mu_0}{4\pi}\right)\frac{2\mu_{loop}}{z^3}$$

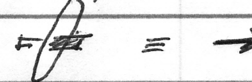
→ 182

$$\mu_{loop} = IA$$



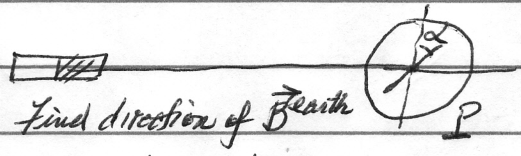
3. Macro-micro correspondance.

atomic dipole moment



$$B_P^{magnet} = \left(\frac{\mu_0}{4\pi}\right)\frac{2\mu^{magnet}}{x^3}, \qquad \mu^{magnet} = N\mu^{atom}$$

Exp.



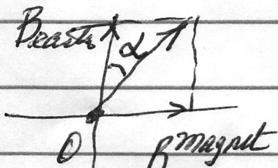
clicker - Find direction of $\vec{B}_{earth}$

For the setup show choose the direction of $\vec{B}_{earth}$

Choices:

1) up
2) right
3) down
4) left

Explanation: Correct vector diagram is



Ans: Choice 1), up.

$$\mu_{atom} = IA = \frac{e}{T}\pi R^2 \quad , \quad T = \frac{2\pi R}{v}$$

$$= \frac{e}{\left(\frac{2\pi R}{v}\right)}\pi R^2$$

✓ $\mu_{atom} = \frac{e}{2} vR$

$$\boxed{\frac{(1.6\times10^{-19})(1.05)\times10^{-34}}{2\times(9\times10^{-31})} \approx 10^{-19-34+30} = 10^{-23}}$$

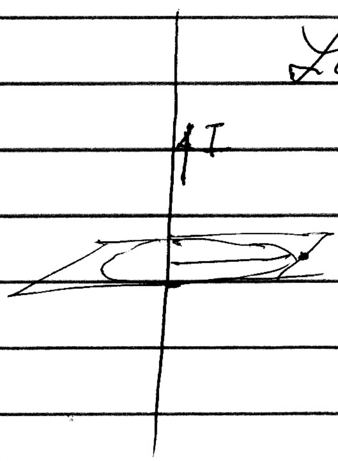
Angular mom.: $L = mvR$

clicker ∴ $\mu_{atom} = \frac{e}{2}\cdot\frac{L}{m} \approx \frac{e}{2}\frac{\hbar}{m}$ ✓

Ground state of H-like atom: $\hbar$ (least value of L)

Ampère's Law:

Long wire:



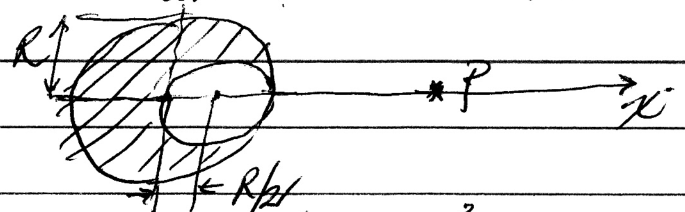
$$B = \frac{\mu_0 I}{2\pi r}$$

$$B\,2\pi r = \mu_0 I$$

Amperian loop: $\oint_{\mathcal{L}} \vec{B}\cdot d\vec{l} = \mu_0 I_{\mathcal{L}}$ -- simple example of Ampère's Law

Discussion on Ch18-P3; 016.

Given: Total current I_0 flows thru a long conductor with a hole (See sketch)



Geometry: $A_{cyl} = \pi R^2 \equiv A_1$

$A_{hole} = \pi \left(\frac{R}{2}\right)^2 = \frac{A_1}{4}$

Area of conducting medium $= A_{cyl} - A_{hole} = \frac{3A_1}{4}, \equiv A_0$

Assume I_0 flows out of the page, current density is constant

$$J = \frac{I_0}{A_0}$$

Magnitudes: $I_{cyl} = JA_1$, $I_{hole} = J\frac{A_1}{4}$

$I_{cyl} = \frac{I_0}{A_0} A_1 = \frac{4}{3} I_0$ (out)

$I_{hole} = \frac{I_0}{A_0} \frac{A_1}{4} = \frac{1}{3} I_0$ (into)

Find: B at P

$B = B_{cyl} - B_{hole}$ (up, RHR$_1$)

$$B_{cyl} = \frac{\mu_0 I_{cyl}}{2\pi x}$$

$$B_{hole} = \frac{\mu_0 I_{hole}}{2\pi \left(x - \frac{R}{2}\right)}.$$