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Lecture: 21 (iq19)

Applications of Ampere's law

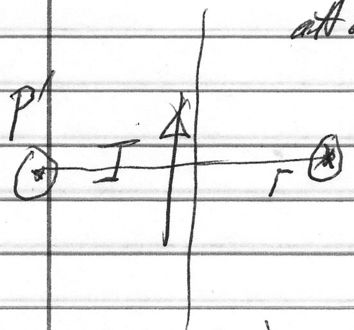
1. Amperian loop with a circular shape
   - Long wire
   - Long thick wire with radius R (h4-002)
   - Coaxial cable (h4: 010-011): clicker: **Area of a ring**
   - Cylindrical conductor with a hole (h3:016): clicker: $\mathbf{B}^{hole}$ **alone at x?**
2. Comments on selected problems in h4.
   - h4:004-006 Torid: See p920, fig22.52, fig22.53.
   - h4: 012-014: Currents inside and outside of an Amperian loop See clicker 20-1 (Read Lec20-1 slides, understand fig20-1a, fig 20.1b).
   - h4: 008:Window. Generalized Amp. law: More than one current is encircled and S can take on an arbitrary shape.
   - h4: 009: Tornado.
3. Amperian loop with a rectangular shape.
   - Wire sheet: h4-001
     B-pattern. LHS, RHS. Solve for $B_?$
   - Solenoid: h4: 003,007, Text p919. Solve for B inside of the solenoid based on fig22.20.
     B-pattern. LHS, RHS. Solve for $B_?$

Announcement:

- Regular office hours: MWF 9:15 to 10:15.
- Review unit2:
  - Work through the summary of unit2 line by line.
  - Study today's discussion on Ampere's law and complete Ch18.h4.
  - Next Wed lecture will be devoted to review unit 2.
  - An extra review session: Wed. 5-6pm, Wel 2.304.
  - Extra office hours: Thursday office hour: 2-3:30pm.

Lec 21 Amperes' Law & Applications

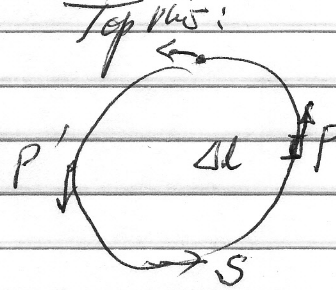
1. For a long wire with current I, we have found at P at distance r from the wire



$$B=\frac{\mu_0 I}{2\pi r}, \quad (1)$$

B has a curly field pattern, it is ⊗ at P and ⊙ at P'. (See sketch for Top view).

Top view:



Rewrite (1) as $2\pi r B=\mu_0 I$.

$$LHS = \underbrace{\oint_{S'}}_{\text{circle}} B\,dl \quad \left[\xrightarrow{\text{Generalize}} \oint_S \vec{B}\cdot d\vec{l}\,\right]$$

Ampere's law in its simple case says for cylindrically symmetric current, choose "amperian loop" to be a circle

$$\oint_{S'} B\,dl = \mu_0 I_{S'}$$

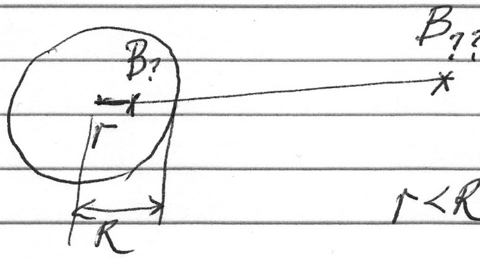
$B_?$ at P. Let S' be the circular loop, $LHS = B_? \oint dl = 2\pi r B_?$

$$RHS = \underbrace{\mu_0 I_S}_{\text{current encircled by } S} = \mu_0 I.$$

1-2. Long thick wire: Assume current density is uniform. (h4-002)

Apply Ampere's Law. For $r<R$, where $B=B_?$

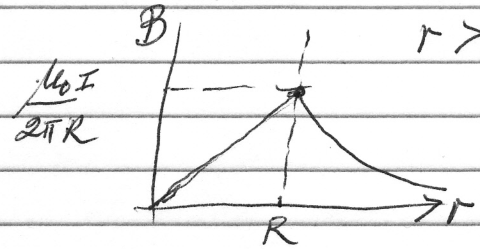
For $r>R$, $B=B_{??}$



Define $J = \frac{\Delta I}{\Delta A} = \frac{I}{\pi R^2}$.

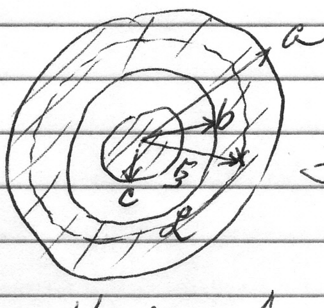
$r < R$: $2\pi r B_? = \mu_0 (J \pi r^2)$

$\therefore B_? \propto r$



$r > R$: $2\pi r B_{??} = \mu_0 I$, $B_{??} \propto \frac{1}{r}$

1.3 Coaxial cable: (h4:10-13)



Let $I_{in} = I_{out} = I_0$

Find: $B_?$ at P, where $r = r_3$.

Hint: Ampere's Law

$$LHS = \oint_{\mathcal{L}} B\,dl = 2\pi r_3 B_?$$

$$RHS = \mu_0 I_{\mathcal{L}} = \mu_0 J \, A_{ring\, b \to r_3}$$

$$J = \frac{I_0}{A_{ring\, b \to a}} \qquad \therefore RHS = \mu_0 I \left[\frac{A^{ring}_{b \to r_3}}{A^{ring}_{b \to a}} \right]$$

$$A^{ring}_{r_1 \to r_2} = \pi r_2^2 - \pi r_1^2$$

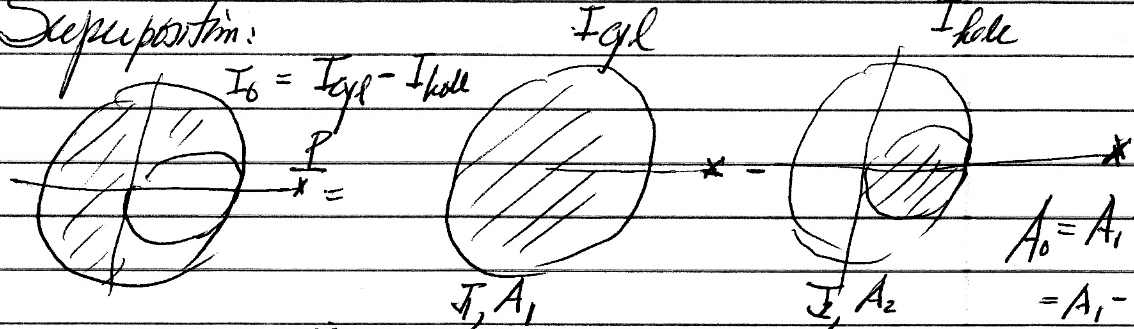
Clicker: Area of a ring: r_1 to r_2.

$A^{ring}_{r_1, r_2}$

1) $2\pi(r_2 - r_1^2)$
2) $\pi r_2^2 - \pi r_1^2$ $\Leftarrow$
3) $\frac{\pi}{2}(r_2^2 - r_1^2)$

1.4 Cylindrical conductor with hole – (h3-16)

Superposition:



$A_0 = A_1 - A_2$
$= A_1 - \frac{A_1}{4}$
$= \frac{3}{4} A_1$

$$J = \frac{I_0}{A_0}, \quad I_{cyl} = J A_1 = I_0 \frac{A_1}{A_0} = I_0 \cdot \frac{4}{3}.$$

$$J_1 = \frac{I_{cyl}}{A_1}, \quad I_{hole} = \frac{I_0}{3}.$$

Clicker: Find B at $x >> R$ due to hole only?

$$B^{hole} = \frac{\mu_0 I_{hole}}{2\pi (x - \frac{R}{2})}$$

Choices:

1) $\frac{\mu_0 I_{hole}}{2\pi x}$

2) $\frac{\mu_0 I_{hole}}{2\pi (x - R)}$

3) $\frac{\mu_0 I_{hole}}{2\pi (x - \frac{R}{2})}$ Ans: 3 by inspection

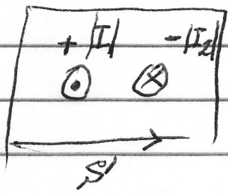
2. Comments on selected problems in h4.

004-006: Toroid, see p920, fig 22.52.

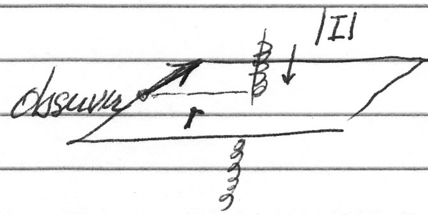
012-014: See clicker 20-1 (online Lec 20-1, fig 20.1a, fig 20.1b)

008: Generalized Ampere's Law:

$$\oint_S \vec{B}\cdot d\vec{l} = \mu_0 \sum_k \pm |I_S^{(k)}|, \quad \begin{array}{l} + \text{ if } I^{(k)} \text{ encircled by } S \text{ along R...} \\ - \text{ if } I^{(k)} \text{ " opposite to "} \end{array}$$



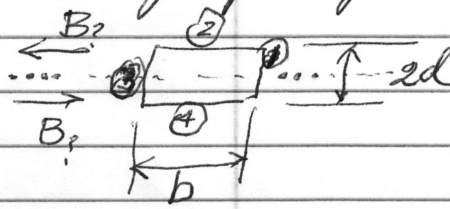
009 Tornado:



3. General Ampere's Law — L can be of arbitrary shape e.g. □

3.1 Wire sheet — B-pattern

h4-001

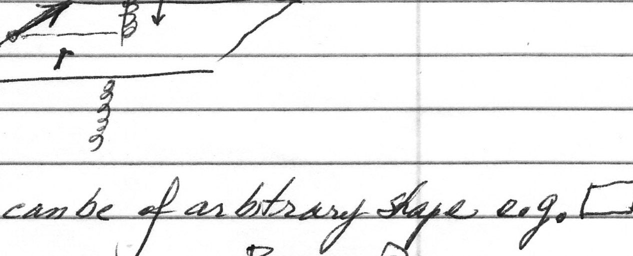


- LHS = $\oint_{\square} B\,dl = 0 + B_? b + 0 + B_? b$
- RHS: $n = \frac{\Delta N}{\Delta x}$, RHS $= \mu_0 (nb) I$
- LHS = RHS $\Rightarrow 2B_? b = \mu_0 (nb) I$. — $B_? = \frac{\mu_0 n I}{2}$.

3.2 Solenoid:

h4: 003, 007. Text p919.

B pattern



- LHS = $0 + 0 + 0 + B_? d$
- RHS = $\mu_0 I_S = \mu_0 \frac{N}{L} d\, I$

LHS = RHS = $B_? d = \mu_0 \frac{N}{L} d\, I$, $B_? = \mu_0 \frac{N}{L} I$.