Lecture: 21 (iq19)

Applications of Ampere’s law
1. Amperian loop with a circular shape
   - Long wire
   - Long thick wire with radius R (h4-002)
   - Coaxial cable (h4:010-011): clicker: Area of a ring
   - Cylindrical conductor with a hole (h3:016): clicker: B_{hole} alone at x?
2. Comments on selected problems in h4.
   - h4:004-006 Torid: See p920, fig22.52, fig22.53.
   - h4:012-014: Currents inside and outside of an Amperian loop
     See clicker 20-1 (Read Lec20-1 slides, understand fig20-1a, fig 20.1b).
   - h4:008: Window. Generalized Amp. law: More than one
     current is encircled and S can take on an arbitrary shape.
   - h4:009: Tornado.
3. Amperian loop with a rectangular shape.
   - Wire sheet: h4-001
     B-pattern. LHS, RHS. Solve for B_{r}
   - Solenoid: h4:003,007, Text p919. Solve for B inside of the
     solenoid based on fig22.20.
     B-pattern. LHS, RHS. Solve for B_{r}

Announcement:
- Regular office hours: MWF 9:15 to 10:15.
- Review unit2:
  - Work through the summary of unit2 line by line.
  - Study today’s discussion on Ampere’s law and complete Ch18.h4.
- Next Wed lecture will be devoted to review unit 2.
- An extra review session: Wed. 5-6pm, Wel 2.304.
- Extra office hours: Thursday office hour: 2-3:30pm.
Lee 21: Ampere's Law & Applications

1. For a long wire with current I, we have found at P,
   \[ B = \frac{\mu_0 I}{2\pi r} \]  
   (1)

   \[ B \] has a curvy field pattern, it is \( \propto \) at \( P \)
   and \( \propto \) at \( P' \) (see sketch for more).

   \[ \text{Rewrite (1) as } 2\pi r B = \mu_0 I. \]

   \[ \text{LHS} = \oint_S B \cdot dl \int_S \frac{B \cdot dl}{\text{Generalize}} \]

   Ampere's law in the simple case says for
   cylindrically symmetric current, choose "perimeter loop" to be a circle
   \[ \oint_S B \cdot dl = \mu_0 I \]
   \[ B_0 \text{ at } P, \text{ let } S \text{ be the circular loop, LHS} = \oint_S B_0 \cdot dl = 2\pi r B_0 \]

   \[ \text{RHS} = \mu_0 I_0 = \mu_0 I. \]

   \[ \text{Current enclosed by } S \]

2. Long thick wire. Assume current density uniform, \( \mu_0 = \text{constant} \)

   \[ \text{Apply Ampere's Law, for } r < R, \text{ where } B = B_0 \]

   \[ \text{For } r > R, \text{ what } B = B_0? \]
1.3 Conical cable: \((k4:10-13)\)

- Let \(I_{in} = I_{out} = I_0\)
- Find \(B_2\) at \(P\), where \(r = r_3\).

**Hint:** Ampere's Law

\[
\text{LHS} = \oint_{bdle} B \cdot dl = 2\pi r_3 B_2
\]

\[
\text{RHS} = \mu_0 I_0 - \mu_0 \mathbf{J} \cdot \mathbf{A}_{\text{ring}} \quad b \rightarrow 5
\]

\[
\mathbf{J} = \frac{I_0}{\text{Area b} \rightarrow a}
\]

\[
\text{RHS} = \mu_0 I_0 \left[ \begin{array}{c}
\mathbf{A}_{\text{ring}} \\
\mathbf{b} \rightarrow 5
\end{array} \right]
\]

\[
\mathbf{A}_{\text{ring}} = \frac{\pi}{4} r_2^2 - \frac{\pi}{4} r_1^2
\]

\[
\text{Area of a ring: } r_1 \text{ to } r_2.
\]
1.4 Cylindrical conductor with hole - \((2.3-16)\)

**Superposition:**

\[ I_0 = I_{cyl} - I_{hole} \]

\[ J = \frac{I_0}{A_0} \]

\[ J_{cyl} = \frac{I_0}{A_1} \]

\[ J_{hole} = \frac{I_0}{A_2} \]

\[ A_0 = A_1 + A_2 \]

\[ J_{cyl} = \frac{I_0}{A_1} \]

\[ J_{hole} = \frac{I_0}{A_2} \]

\[ B_{cyl} = \frac{\mu_0 I_{cyl}}{2\pi (R)} \]

\[ B_{hole} = \frac{\mu_0 I_{hole}}{2\pi (R)} \]

\[ B = \frac{\mu_0 I_{hole}}{2\pi (x - \frac{R}{2})} \]

**Questions:**

1. \( B \) at \( x \gg R \) due to hole only?

2. \( B \) at \( x \gg R \) due to both cylinders?

**Answers:**

1. \( \frac{\mu_0 I_{hole}}{2\pi x} \)

2. \( \frac{\mu_0 I_{cyl} - \mu_0 I_{hole}}{2\pi (x - R)} \)

3. \( \frac{\mu_0 I_{hole}}{2\pi (x - \frac{R}{2})} \)
2. Comments on selected problems in 1.4.

064-066: Toroid, see p. 176, fig. 23.51.

017-044: See choker 20-1 (calr. Sec 24-1, fig. 20-1a, fig. 20-1b).

008: Generalized Ampere's Law:

\[ \mathbf{B} \cdot d\mathbf{l} = \mu_0 \sum \mathbf{m} + \int_{\Gamma_s} \mathbf{I}^\parallel (k) \cdot d\mathbf{l}_{\perp} - \int_{\Gamma} \mathbf{I}^\parallel (k) \cdot d\mathbf{l} \]

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069: Tornado:

3. General Ampere's Law - \( \mathbf{L} \) can be of arbitrary shape. e.g.

3.1 Wire sheet - B-pattern

\[ \mathbf{LHS} = \oint \mathbf{B} \cdot d\mathbf{r} = 0 + B_2 b + 0 + B_1 b \]

\[ \mathbf{RHS}: n = \frac{\Delta N}{\Delta x}, \quad \mathbf{RHS} = \mu_0 (n/b) I \]

\[ \mathbf{LHS} = \mathbf{RHS} \Rightarrow B_2 b = \mu_0 (n/b) I \]

3.2 Solenoid.

\[ \mathbf{LHS} = O + O + O + B_2 d \]

\[ \mathbf{RHS} = \mu_0 I = \mu_0 \frac{N}{L} I \]

\[ \mathbf{LHS} = \mathbf{RHS} = B_2 d = \mu_0 \frac{N}{L} I, \quad B_2 = \mu_0 \frac{N}{L} I \]