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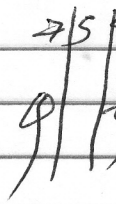
Lecture: 25 (iq22) Ch20 Circuit elements

1. Capacitor and capacitance:
    a. Spherical capacitor: large r, s<<r limit
    b. One sphere capacitor
2. Energy density: Fill the gap with dielectric medium. E'=E/K.
    a. Case 1: Fixed Q case
    b. Case 2: Fixed V case. (see the figure)
3. RC circuit: Charging case.
4. An exercise which involves a one capacitor system and a connected two capacitor system.
5. Ohm's law, and power dissipated in a resistor.
6. An exercise on Ohm's laws.

Lec 22

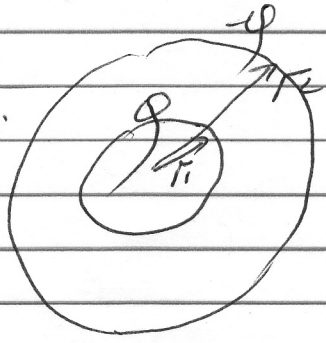
Circuit elements

Capacitors: ||-plate we have discussed earlier.



$$C = \frac{Q}{V} = \frac{Q}{Es} = \frac{\epsilon_0 A}{\left(\frac{Q}{\epsilon_0 A}\right) s} = \frac{\epsilon_0 A}{s}$$

Spherical capacitor:



$$V = \Delta V = \frac{kQ}{r_1} - \frac{kQ}{r_2}$$

$$C = \frac{Q}{V} = \frac{1}{\frac{k}{r_1} - \frac{k}{r_2}} \quad \text{(see Lec)}$$

Large r, $s \ll r$, $r_1 = r$, $r_2 = r + s$

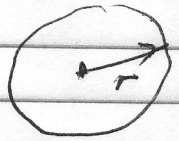
$$C = \frac{1}{\left(\frac{k}{r}\right)} \cdot \frac{1}{1 - \frac{1}{(1+\epsilon)}} \approx \frac{1}{\frac{4\pi k s}{r^2}} = \frac{4\pi r^2 \epsilon_0}{s} = \frac{A\epsilon_0}{s}$$

$1 - (1-\epsilon) = \epsilon = \frac{s}{r}$

(same as || plate)

1 sphere capacitor:

$$C = 4\pi\epsilon_0 r$$



Energy stored: $Q = CV$

$$\Delta q \left\{ \begin{array}{c} \frac{q}{V} \\ -q \end{array} \right. = \frac{q+\Delta q}{q-\Delta q}$$

$$\int_0^U \Delta U = V\Delta q = \int_0^Q \frac{q}{C}\Delta q = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

Energy density: $u = \frac{U}{As} \Rightarrow U = \frac{1}{2}\cdot\frac{A\epsilon_0}{s}\cdot sE^2$

$$= \frac{1}{2}\epsilon_0 (As) E^2$$

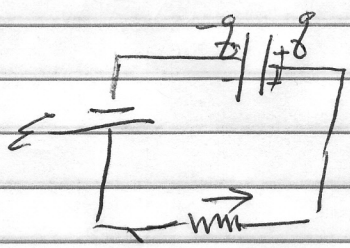
$$\therefore u = \frac{U}{As} = \frac{1}{2}\epsilon_0 E^2.$$

Insert dielectric in the gap: $C' = \frac{Q}{V'} = \frac{Q}{\frac{V}{K}} = KC$

FIX Q case: $U' = \frac{Q^2}{2C'} = \frac{U}{K}$

Fixed V case: $U' = \frac{1}{2}C'V^2 = KU$ (HW) ⇐ See fig ✓

RC circuit:

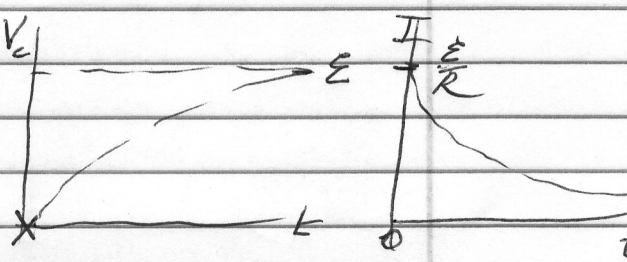


Loop eqn: $\Delta V = 0$

$$\mathcal{E} - IR - \frac{q}{C} = 0$$

$t = 0^+$: $q = 0$, $I = \frac{\mathcal{E}}{R}$

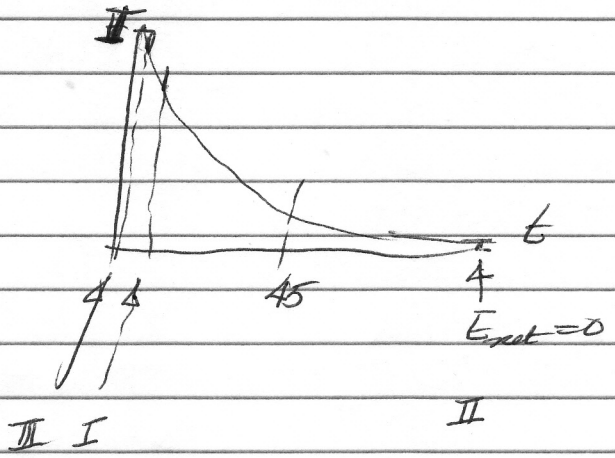
$t = \infty$: $I = 0$, $q = \mathcal{E}C$



Question:

After 45 sec, it goes out

Choose diagrams represent 0.01 s, 55 246 s



Choices: 1) I, II, III

2) III, I, II

3) III, II, I

4) II, III, I

Setup:

Case 1 $\underline{Q_0, V_0}$ ——— C_0

Case 2 $\underline{Q'}$ $\underline{\quad\quad}$ $\underline{Q''}$ $\quad Q'+Q''=Q_0$

V_1 C_0 V_1 C_1

Given: C_0, C_1, V_1

Find: V_0

Hint: $V_0 = \frac{Q_0}{C_0} = \frac{Q'+Q''}{C_0}$

∴ Need Q', Q''.

But $Q'' = C_1 V_1$. Proceed to find Q', then V_0

Ohms Law: (OL)

Two version: OL-1: $\vec{J} = \sigma \vec{E}$ (dimension of resistor does not enter)

or $\vec{E} = \rho \vec{J}$

OL-2: $V = IR$

micro— $i = nAu\vec{E}$, $I = |q| i$

$J = \frac{I}{A} = \frac{|q| nAu E}{A} = \sigma E$, $\sigma = |q| nu$

or $\vec{E} = \rho \vec{J}$

OL2: $\frac{V}{l} = \rho \frac{I}{A}$, $V = \left(\frac{\rho l}{A}\right) I = IR$, $R = \frac{\rho l}{A}$

Brightness of light bulb — Power

$$P = \frac{(\Delta q) V}{\Delta t} = IV = I^2 R = \frac{V^2}{R}.$$

Cut a wire into n equal segments

Then make // connection

$L' \Rightarrow \frac{L}{N}$

$A' \Rightarrow NA$

$\therefore R' = \rho \frac{L'}{A'} = \rho \frac{\left(\frac{L}{N}\right)}{(NA)}$

$= \left(\frac{\rho L}{A}\right) \frac{1}{N^2} = \frac{R}{N^2}$