Lecture33 iq29

1. Inductance
2. LR circuit
    a. Build up case and
    b. Decay case
3. Energy stored in C and in L.
    a. U_C and U_L
    b. u_E and u_B
4. Comments on selected problems in Ch23.h2
    a. 005 Work in pulling a loop
    b. 006. The emf vs time curve
    c. 015 A nuance in shorting a bulb when the induced emf is present.

Announcement:

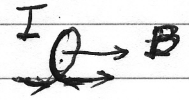
Prepare for midterm3

- Review unit 3 problems have been posted.
- Wednesday (4/10)
    - Inclass review: Review on the part I of unit3
    - Review on the part II of unit 3 will be from 5 to 6pm (Location is to be announced)
- Thursday (4/11): Extra office hour 2:30-4:00.

Re: The updated lesson plan
The updated lesson plan is now available on line (the link is on the first line of our homepage). The changes are indicated by red.

ch 29

1. Inductance



Consider the application of Faraday's Law to a simple loop. I generates B, ϕ.

If I changes $\Rightarrow$ ϕ changes. Loop has magnetic inertial, it generates induced B_{in}, or induced flux cancels the change. This is Faraday's Law:

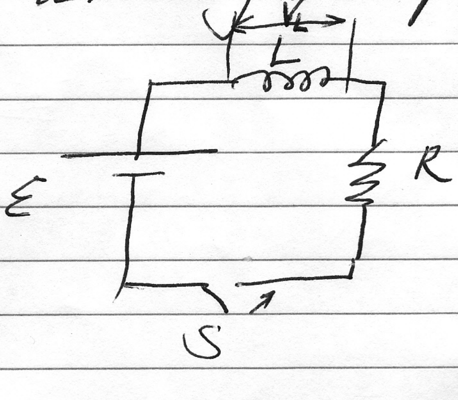
$$\mathcal{E}_{ind} = -\frac{\Delta\phi}{\Delta t}$$

When we work with a circuit, it is convenient simply write the flux as $\phi = const \cdot I$. F.L. becomes $\mathcal{E}_{ind} = -const\frac{dI}{dt} = -\left(\frac{\phi}{I}\right)\frac{dI}{dt}$

This is for 1 loop. For N loop: $\mathcal{E}_{ind} = -N\frac{d\phi}{dt} = -\left(\frac{N\phi}{I}\right)\frac{dI}{dt}$.

Here the inductance $L = \frac{N\phi}{I}$. This is the proportionality constant which tells us how much emf is induced for a given $\frac{dI}{dt}$. The bigger is L, the larger is the emf induced.

2. Solenoid is referred to as the inductor. It in series with a battery and a resistor forms a simple RL circuit.



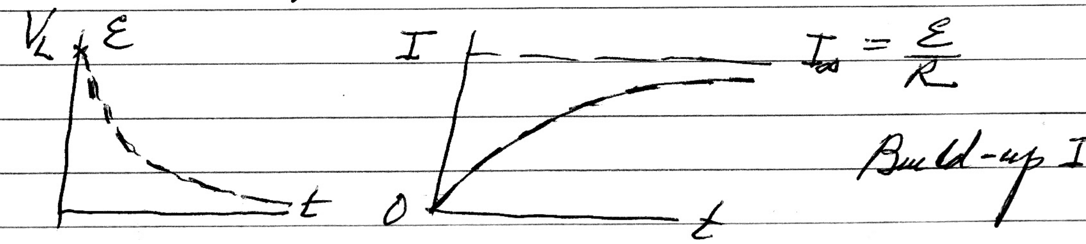
The loop eqn is: $\mathcal{E} - V_L - IR = 0$

where $V_L = L\frac{dI}{dt}$. Faraday's Law says

$$\mathcal{E}_{ind} = -L\frac{dI}{dt}.$$

Close S at $t=0$. Due to magnetic inertia of the solenoid at $t=0^+$, $I=0$, $V_L=\mathcal{E}$.

At $t=\infty$, equilibrium is reached, $V_L = L\frac{dI}{dt}=0$, $I=\frac{\mathcal{E}}{R}$



Analogous to RC case:

$$V_L = \mathcal{E}e^{-t/\tau}, \qquad I = I_\infty\left(1-e^{-t/\tau}\right)$$

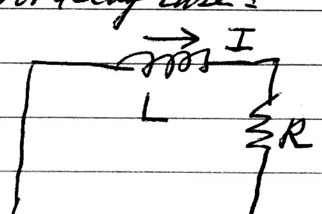
Notice the derivative of the loop eqn gives

$$\frac{d}{dt}\left[\mathcal{E}-V_L-IR\right] = -\frac{dV_L}{dt}-\frac{dI}{dt}R = -\frac{dV_L}{dt}-\underbrace{\left(L\frac{dI}{dt}\right)}_{V_L}\frac{R}{L} =$$

Rearrangement gives: $\frac{dV_L}{dt} = -\frac{V_L}{(L/R)}$.

L/R has the dimension of time & the characteristic time of the circuit

For decay case:



Here battery is removed,

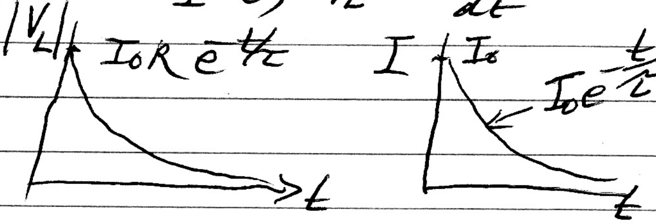
Loop eqn: $-V_L - IR = 0$

At $t=0$, let $I=I_0$,

$$-V_L = I_0R.$$

At $t=\infty$, equilibrium is reached.

$$I=0, \quad V_L = L\frac{dI}{dt}=0$$



3. Energy stored in C + L

Power: $P = \frac{dU}{dt} = \frac{dqV}{dt} = IV$

$$U_C = \int IV_C\,dt = \int \left(\frac{dq}{dt}\right)\left(\frac{q}{C}\right)dt = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$

$$U_L = \int IV_L\,dt = \int I L \frac{dI}{dt}\,dt = L\int I\,dI = \frac{LI^2}{2}$$

Express energy stored in terms of B:

$$LI = \left(\frac{N\phi}{I}\right) I = N\phi = NBA.$$

$$B = \mu_0 \frac{N}{l} I, \quad I = \frac{Bl}{\mu_0 N}$$

$$\therefore U_L = \frac{1}{2}(LI)(I) = \frac{1}{2}(NBA)\frac{Bl}{\mu_0 N} = \frac{1}{2\mu_0}B^2 A l$$

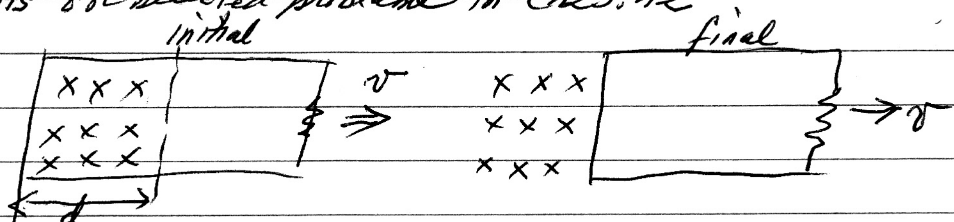
Energy density: magnetic $u_B = \frac{U_L}{Al} = \frac{1}{2\mu_0}B^2.$

Electric

$$u_E = \frac{1}{2}\epsilon_0 E^2.$$

A. Comments on selected problems in Ch23.42

DOS



Find work done.

Hint: Show F_{Ih}^{B} is to the left.

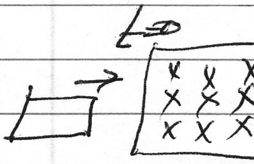
Mechanical work needed to move the loop from init. to fi
is $F_{Ih}^{B} \times d$

Show this work equals to electrical energy consumed, i.e.

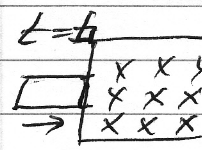
$$W = \text{Power} * \Delta t = IV * \frac{d}{v} = \frac{V^2}{R}\Delta t = \frac{emf^2}{R} * \frac{d}{v}$$

006: Hint - Let us define 4 time points.

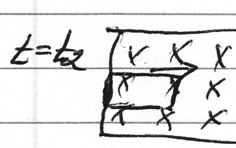
$t=0$, the loop is outside of the field and to the left.



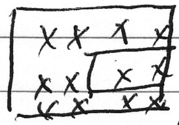
$t=t_1$ it just enters the region



$t=t_2$ the left side enters the field



$t=t_3$ the right side just leave the field



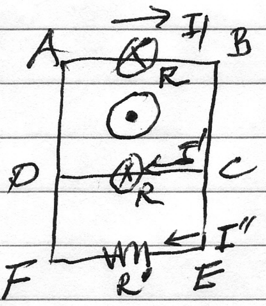
$t=t_3$

Show: From $t=0$ to t_1, $\frac{d\phi}{dt}=0$

From t_1 to t_2, $\frac{d\phi}{dt}=$ const, increasing

From t_2 to t_3, $\frac{d\phi}{dt}=0$.

015 Hint:



Loop eqn for ABCDA

$$emf - I R - I'R = 0 \quad (1)$$

Loop eqn for DCEFD

$$+I'R - I''R' = 0. \quad (2)$$

Here $R'=0$, since it is a wire, resistance is assumed to be very small. (2) leads to $I''R'=0$, or $I'R=$