Lecture35 iq31

RC and RLC circuits

1. Energies and energy densities stored in C and in L
2. Conservation of energy in LC circuit
3. Solution to LC circuit. Simple harmonic Oscillator. Clicker 32.1a.
4. RLC circuit
   - The I^2R term in the power-equation is always positive.
   - Damped envelop in the oscillatory Q-curve and I-curve. (See Fig 23.49)

Announcement:

Midterm 3, class average 71.

The lecture video in our homepage is now available for viewing. Keep in mind it is an experimental project. Your feedback is welcome.

Application of the LA position is now available. For those of you who do well in this course are encouraged to apply the LA job. LAs can play an important part in helping students through their interaction with students. If you are interested in this job opportunity please contact Lisa Gentry*.

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ig 31

1. Energy stored in C and in L

$$\frac{dU}{dt} = P = IV$$

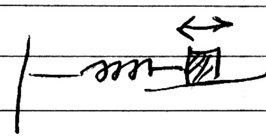
$$U = \int IV\,dt$$

In capacitor: q, $-q$ $\Big\}\ V = \frac{q}{C}$

$$U_C = \int \frac{dq}{dt}\cdot\frac{q}{C}\,dt = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$

$$U_L = \int I\left(L\frac{dI}{dt}\right)dt = L\int_0^I I'\,dI' = \frac{LI^2}{2}$$

	C	L
U	$\frac{Q^2}{2C}$	$\frac{1}{2}LI^2$
u	$\frac{1}{2}\epsilon_0 E^2$	$\frac{1}{2\mu_0}B^2$

Mechanical analogy: Mass-spring system 

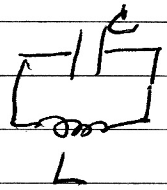
$$F = ma = m\frac{d^2x}{dt^2} = -kx$$

$$KE = \frac{1}{2}mv^2, \quad PE = \frac{1}{2}kx^2$$

$$\downarrow \qquad\qquad \downarrow$$

Correspondence: $\frac{1}{2}LI^2$ $\qquad$ $\frac{1}{2C}q^2$ $\qquad$ "LC oscillator"

2. LC circuit



Basic equations —

Loop eqn: $V_L + V_c = 0$, or $L\frac{dI}{dt} + \frac{q}{C} = 0$

Energy stored: $U_L = \frac{1}{2}LI^2$, $U_C = \frac{1}{2}C\,q^2$

Power eqn: $P = \frac{dU}{dt} = \frac{dq\,V}{dt} = IV$

Power eqn = $I *$ loop eqn $= IV_L + IV_c = 0$

Conservation of energy:

$$0 = I\left(L\frac{dI}{dt}\right) + \frac{dq}{dt}\frac{q}{C} = \frac{d}{dt}\left[\frac{LI^2}{2} + \frac{1}{2C}q^2\right] = \frac{d}{dt}(U_L + U_C)$$

$\therefore$ $U_L + U_C = \text{const}$, energy of the LC system is constant.

When observe mass spring system —

Mech. Analogy: $KE + PE = \text{const}$

$$U_{C\,max} = U_C + U_L = U_{L\,max} \equiv U_0$$

3. Solving the loop eqn. Analogy to mass-spring system –

$$m\frac{dx^2}{dt^2} = -kx$$

Solution: $X = A\cos\omega t$

$$\frac{dx}{dt} = -A\omega\sin\omega t$$

$$\frac{dx^2}{dt^2} = -A\omega^2\cos\omega t = -\omega^2 x$$

General solution – $x = x_{max}\cos(\omega t+\delta)$ initial phase angle of oscillation

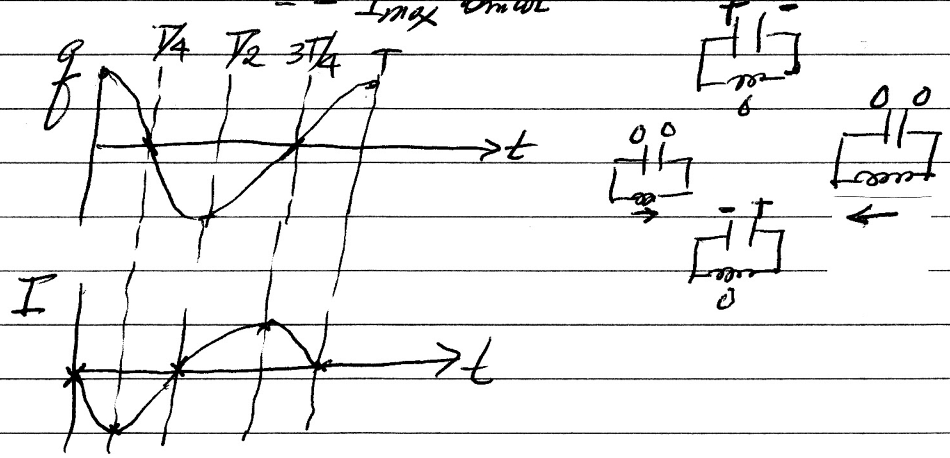
Interpretation: Oscillation can begin at any time

For present illustration set $\delta = 0$.

LC circuit analogy –

$$q = q_{max}\cos\omega t$$

$$I = \frac{dq}{dt} = -q_{max}\omega\sin\omega t$$
$$= -I_{max}\sin\omega t$$



Check $U_L + U_C = \text{const}$

$$LHS = \frac{1}{2}L\left(I_{max}\sin\omega t\right)^2 + \frac{1}{2C}\left(q_{max}\cos\omega t\right)^2$$

$$\frac{1}{2}L I_{max}^2 = \frac{1}{2}L\omega^2 q_{max}^2 = \frac{1}{2}L\frac{1}{LC}q_{max}^2 = \frac{1}{2C}q_{max}^2$$
$$= U_0$$

$$\therefore LHS = U_0\sin^2\omega t + U_0\cos^2\omega t = U_0,$$

where $\sin^2\omega t + \cos^2\omega t = 1$ is used.

4. RLC circuit – Loop eqn.

$$-L\frac{dI}{dt} - \frac{q}{C} - IR = 0$$

Power eqn = I * loop eqn.

$$-\frac{d}{dt}\left[\frac{L}{2}I^2 + \frac{q^2}{2C}\right] - I^2R = 0$$

$$+ LI\frac{dI}{dt} + \frac{q}{C}\frac{dq}{dt} + I^2R$$

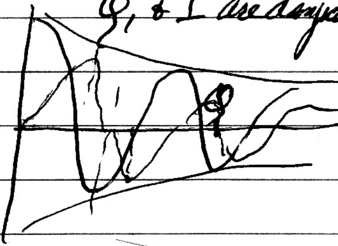
$$\underbrace{P_L + P_C}_{} + \underbrace{P_R}_{} = 0$$

Each term can be positive or negative

Sum slightly negative

always positive

Q, + I are damped



See Fig 23.49