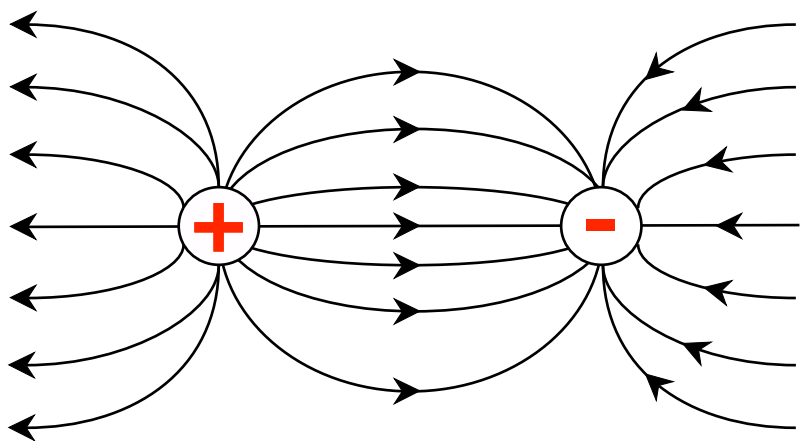
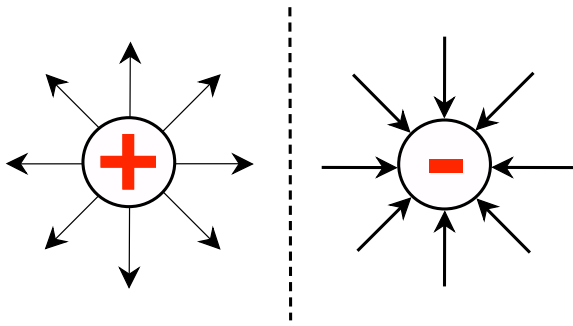


Lecture 4

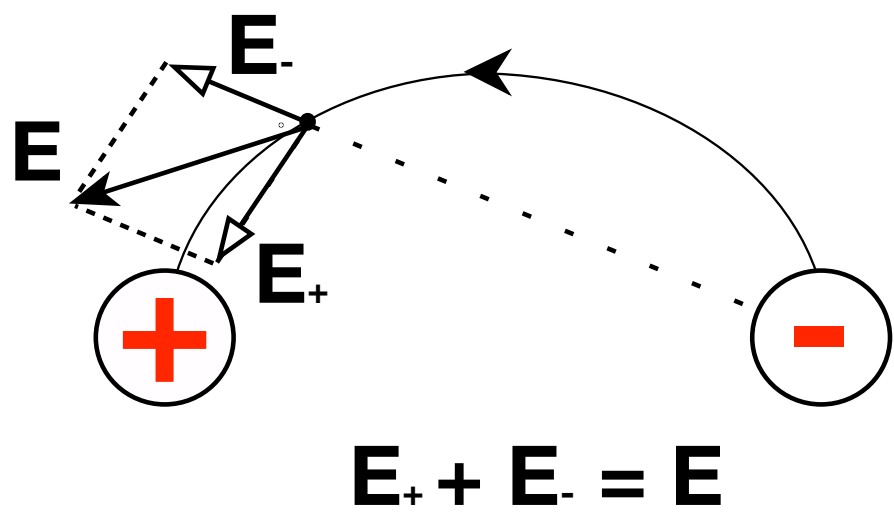
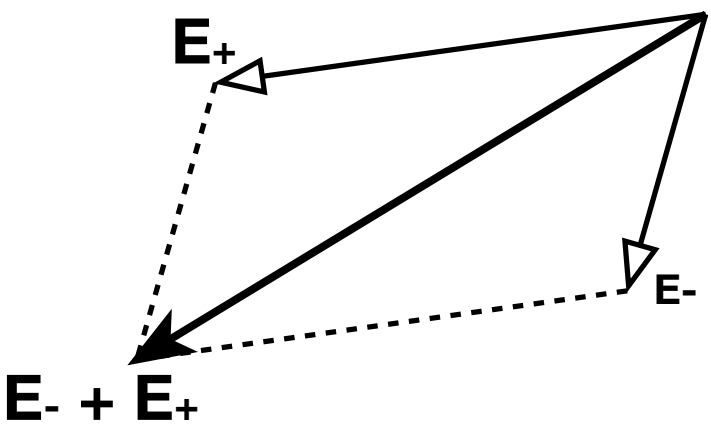
1. Review superposition principle and dipole field pattern



Field Pattern of Charge:

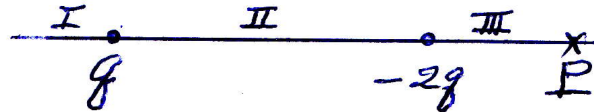


Method of Parallelogram:



Clicker 4-1

Given two source charges q and $-2q$ as shown in the figure. They divide the x-axis into three region I, II and III



Consider the field in region III at an arbitrary point P.

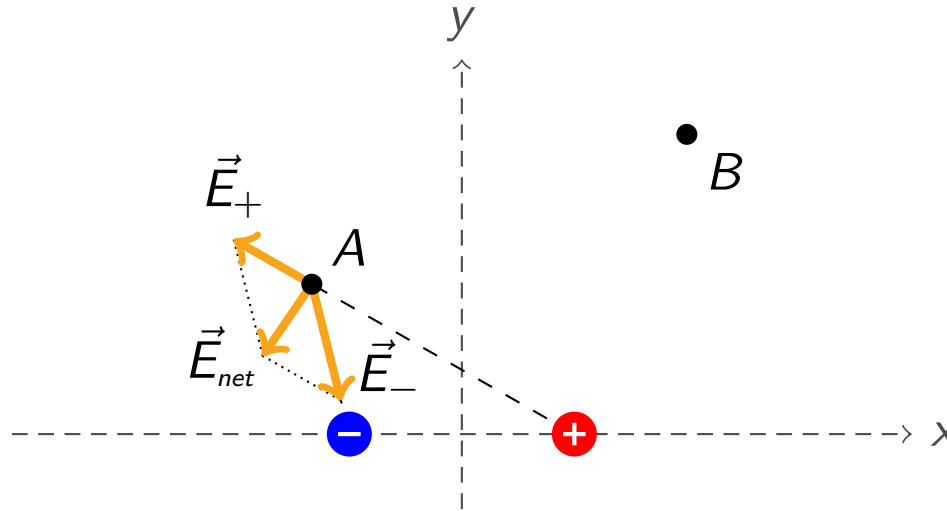
Compare the magnitude of the field contributed by q and that contributed by $-2q$.

	$ E_P^q $	vs	$ E_P^{-2q} $
1.		>	
2.		=	
3.		<	
4.	Can be >, or =, or <		

Clicker 401: Dipole field pattern and dipole interaction

Lec5-1 Sketch a dipole field pattern

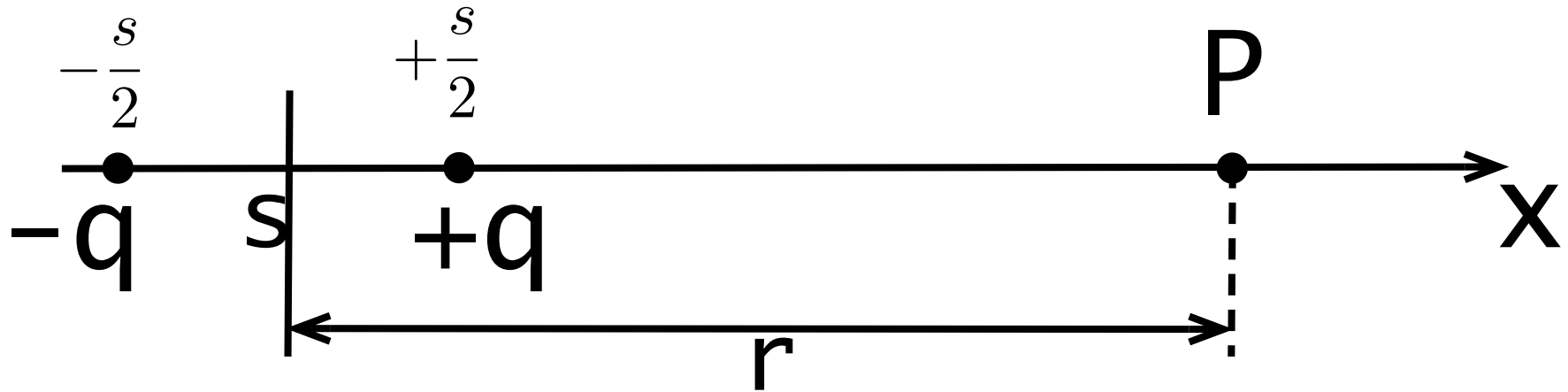
Consider a dipole centered at the origin. A parallelogram of $\mathbf{E}$ field vectors due to the dipole charges has been constructed at point A.



Please sketch in your notebook the parallelogram formed by the field vectors at B. Wait. At the appropriate time discuss the parallelogram construction with your neighbour. Complete the parallelogram construction and label the three field vectors. Enter your clicker answer.

Choice	Have you completed your parallelogram construction at B?
1	Yes
2	No

Field at a point P



$$E_P = E^+ + E^- \quad \text{where: } \epsilon = \frac{s}{2r}$$

$$\begin{aligned} &= \frac{kq}{\left(r - \frac{s}{2}\right)^2} - \frac{kq}{\left(r + \frac{s}{2}\right)^2} \\ &= \frac{kq}{r^2} \left[\frac{1}{(1 - \epsilon)^2} - \frac{1}{(1 + \epsilon)^2} \right] \end{aligned}$$

Digression on small epsilon expansion

$$(1 - \epsilon)^2 = 1 - 2\epsilon + \epsilon^2 = 1 - 2\epsilon + O(\epsilon^2) \approx 1 - 2\epsilon$$

$$(1 - \epsilon)^3 = 1 - 3\epsilon + 3\epsilon^2 - \epsilon^3 = 1 - 3\epsilon + O(\epsilon^2) \approx 1 - 3\epsilon$$

General Formula:

$$(1 - \epsilon)^a = 1 - a\epsilon + O(\epsilon^2) \approx 1 - a\epsilon$$

Where a is negative: $\frac{1}{1 - \epsilon} = 1 + \epsilon$

Product: $(1 - \epsilon)(1 + \epsilon) \approx (1 - \epsilon^2) \approx 1$

So the general formula above works for both positive and negative!

$$\boxed{(1 - \epsilon)^a \approx 1 - a\epsilon}$$

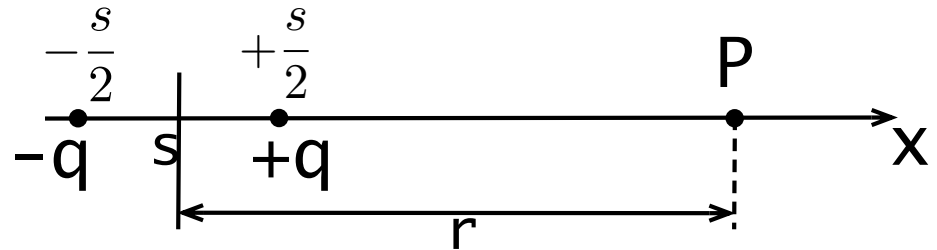
Digression on small epsilon expansion continued...

Examples:

$$\frac{1}{(1 - \epsilon)^2} = \frac{1}{1 - (2\epsilon)} \approx 1 + 2\epsilon$$

$$\frac{1}{(1 + \epsilon)^2} = \frac{1}{1 + 2\epsilon} \approx 1 - 2\epsilon$$

At P (from slide 2):

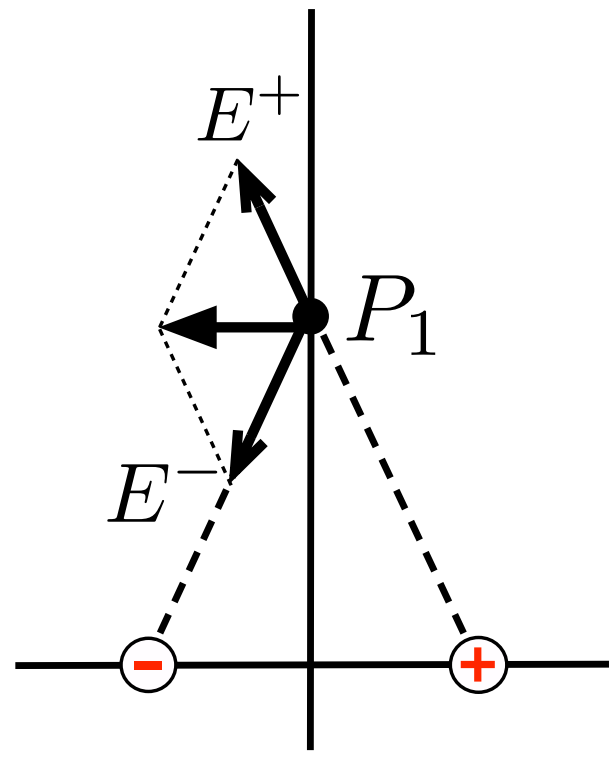


$$E_P = \frac{kq}{r^2} \left[\frac{1}{(1 - \epsilon)^2} - \frac{1}{(1 + \epsilon)^2} \right] \quad \epsilon = \frac{(s/2)}{x}$$

$$[] = (1 + 2\epsilon) - (1 - 2\epsilon) = 4\epsilon = f\left(\frac{s}{2x}\right) = \frac{2s}{x}$$

$$E = \frac{kq}{x^2} \cdot \frac{2s}{x} = \frac{2kqs}{x^3} = \frac{2kp}{x^3} \quad \text{Dipole: } p = qs$$

More on dipoles...



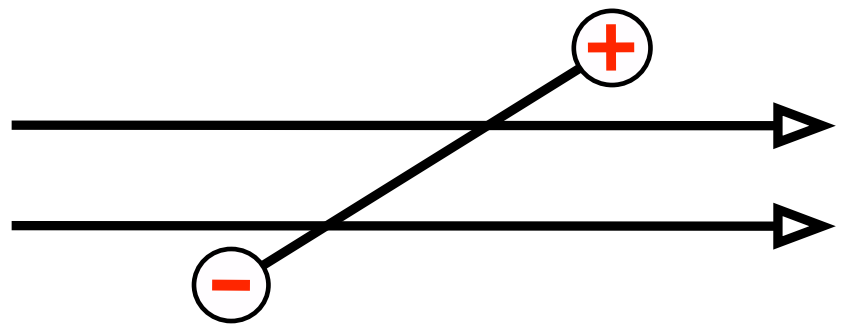
$$\vec{E}_{P1}^{dipole} = \vec{E}_{P1}^+ + \vec{E}_{P1}^-$$

dir of $E_{P1} : (-\hat{x})$

$$\vec{E} = 2E^+ \cos \alpha$$

$$= 2 \times \frac{kq}{r^2} \cdot \frac{s/2}{r} = \frac{kqs}{r^3}$$

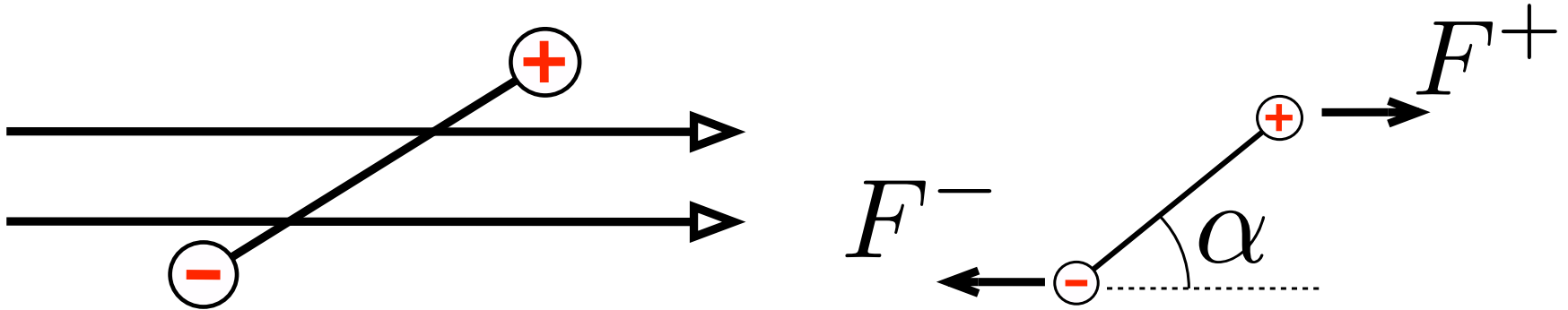
Dipole in Uniform E:



$$\vec{F}_{dipole} = \vec{F}_{+q} + \vec{F}_{-q}$$

$$= q\vec{E} - q\vec{E} = 0$$

Uniform dipole in uniform E-field continued...



$$\tau = Fd = Eqd = Eq s \sin \alpha$$

$$\vec{\tau} = \tau \hat{n}$$

$$= (Ep \sin \alpha) \hat{n}$$

$$= \vec{p} \times \vec{E}$$

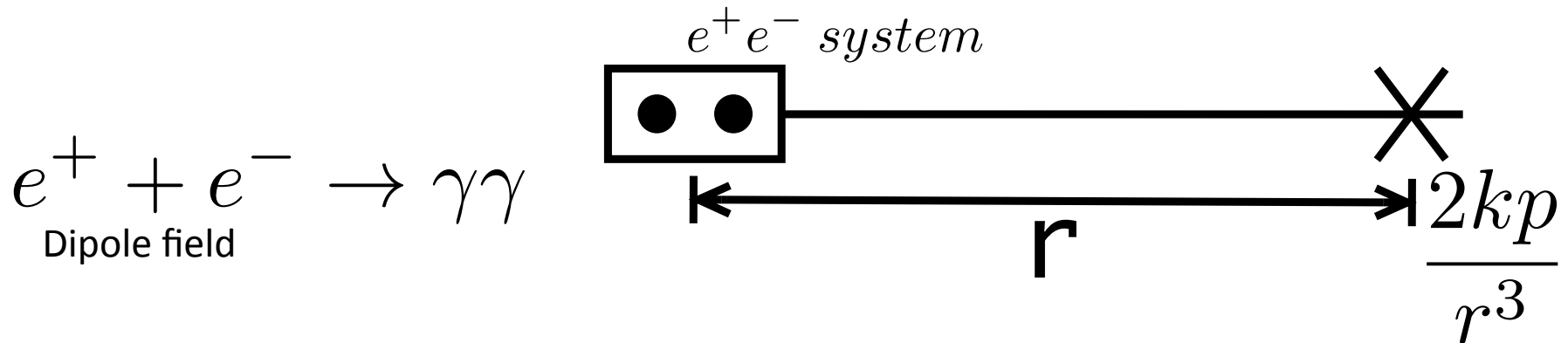
Direction: CW

Is electric field real?

Source creates field: field spreads out with the speed of light.

Time taken to spread out 1 foot:

$$\Delta t \approx \frac{0.3}{3 \times 10^8} = 10^{-9} \text{ sec} = ns$$



At $t=0$, reaction occurs $\rightarrow$ dipole system dissolves

At P: Electric field due to the dipole continues to be present for $t=0 \rightarrow t_1 = r/c$